Are we there yet? Advances in anytime-valid methods for hypothesis testing and prediction
Pérez-Ortiz, M.F.

Curriculum Vitae

Muriel was born in Bogotá in 1993. He spent his high school years between El Colegio Filipense Sagrado Corazón de Jesús in Alcalá de Henares, Spain (2006-2008), and El Colegio Tolimense, in Ibagué, Colombia (2008-2009). He went on to study a BSc in Physics at the University of the Andes (2010-2015) in Bogotá and a BSc in Mathematics (2011-2016) at the same university. After a year working at the Astronomy Group of the University of the Andes, Muriel went to Amsterdam, where he completed a MSc degree (cum laude) in Mathematics at the University of Amsterdam (2016-2018). With the guidance of Peter Grünwald and Wouter Koolen, at the Machine Learning group of *Centrum Wiskunde & Informatica* (CWI), he completed is PhD in 2023.

List of Publications

The contents of this dissertation are based on the following publications.

- Chapter 2 is based on

  M. F. Pérez-Ortiz, T. Lardy, R. de Heide, and P. Grünwald. E-Statistics, Group Invariance and Anytime Valid Testing, Aug. 2022. URL `http://arxiv.org/abs/2208.07610`. arXiv:2208.07610 [math, stat], under submission.

- Chapter 3 is based on

  J. ter Schure, M. F. Pérez-Ortiz, A. Ly, and P. Grünwald. The Safe Logrank Test: Error Control under Continuous Monitoring with Unlimited Horizon, July 2021. URL `http://arxiv.org/abs/2011.06931`. arXiv:2011.06931 [math, stat], under submission.

- Chapter 4 is based on

  M. F. Pérez-Ortiz and W. M. Koolen. Luckiness in Multiscale Online Learning. *Advances in Neural Information Processing Systems*, 35:25160–25170, Dec. 2022. URL `https://papers.nips.cc/paper_files/paper/2022/hash/a0d2345b43e66fa946155c98899dc03b-Abstract-Conference.html`.

- Chapter 5 is based on

  P. D. Grünwald, M. F. Pérez-Ortiz, and Z. Mhammedi. Exponential Stochastic Inequality, Apr. 2023. URL `http://arxiv.org/abs/2304.14217`. arXiv:2304.14217 [math, stat].

Bibliography

P. Alquier. User-friendly introduction to PAC-Bayes bounds, Mar. 2023. URL `http://arxiv.org/abs/2110.11216`. arXiv:2110.11216 [cs, math, stat].

V. Amrhein, D. Trafimow, and S. Greenland. Inferential Statistics as Descriptive Statistics: There Is No Replication Crisis if We Don't Expect Replication. *The American Statistician*, 73(sup1):262–270, Mar. 2019. ISSN 0003-1305. doi: 10.1080/00031305.2018.1543137. URL `https://doi.org/10.1080/00031305.2018.1543137`. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/00031305.2018.1543137.

P. K. Andersen, O. Borgan, R. D. Gill, and N. Keiding. *Statistical Models Based on Counting Processes*. Springer Science & Business Media, 1993. ISBN 978-1-4612-4348-9. Google-Books-ID: fh3SBwAAQBAJ.

S. Andersson. Distributions of Maximal Invariants Using Quotient Measures. *Annals of Statistics*, 10(3):955–961, Sept. 1982. ISSN 0090-5364, 2168-8966. doi: 10.1214/aos/1176345885. URL `https://projecteuclid.org/euclid.aos/1176345885`. Publisher: Institute of Mathematical Statistics.

F. J. Anscombe. Fixed-Sample-Size Analysis of Sequential Observations. *Biometrics*, 10(1):89–100, 1954. ISSN 0006-341X. doi: 10.2307/3001665. URL `https://www.jstor.org/stable/3001665`. Publisher: [Wiley, International Biometric Society].

P. Armitage. The Search for Optimality in Clinical Trials. *International Statistical Review / Revue Internationale de Statistique*, 53(1):15–24, 1985. ISSN 0306-7734. doi: 10.2307/1402871. URL `https://www.jstor.org/stable/1402871`. Publisher: [Wiley, International Statistical Institute (ISI)].

J.-Y. Audibert. PAC-Bayesian statistical learning theory. *These de doctorat de l'Université Paris*, 6:29, 2004.

J.-Y. Audibert. Fast learning rates in statistical inference through aggregation. *The Annals of Statistics*, 37(4):1591–1646, Aug. 2009. ISSN 0090-5364, 2168-8966. doi: 10.1214/08-AOS623. URL `https://projecteuclid.org/euclid.aos/1245332827`.

P. L. Bartlett and S. Mendelson. Empirical minimization. *Probability Theory and Related Fields*, 135(3):311–334, July 2006. ISSN 0178-8051, 1432-2064. doi: 10.1007/s00440-005-0462-3. URL `https://link.springer.com/article/10.1007/s00440-005-0462-3`.

D. J. Benjamin, J. O. Berger, M. Johannesson, B. A. Nosek, E.-J. Wagenmakers, R. Berk, K. A. Bollen, B. Brembs, L. Brown, C. Camerer, D. Cesarini, C. D. Chambers, M. Clyde, T. D. Cook, P. De Boeck, Z. Dienes, A. Dreber, K. Easwaran, C. Efferson, E. Fehr, F. Fidler, A. P. Field, M. Forster, E. I. George, R. Gonzalez, S. Goodman, E. Green, D. P. Green, A. G. Greenwald, J. D. Hadfield, L. V. Hedges, L. Held, T. Hua Ho, H. Hoijtink, D. J. Hruschka, K. Imai, G. Imbens, J. P. A. Ioannidis, M. Jeon, J. H. Jones, M. Kirchler, D. Laibson, J. List, R. Little, A. Lupia, E. Machery, S. E. Maxwell, M. McCarthy, D. A. Moore, S. L. Morgan, M. Munafó, S. Nakagawa, B. Nyhan, T. H. Parker, L. Pericchi, M. Perugini, J. Rouder, J. Rousseau, V. Savalei, F. D. Schönbrodt, T. Sellke, B. Sinclair, D. Tingley, T. Van Zandt, S. Vazire, D. J. Watts, C. Winship, R. L. Wolpert, Y. Xie, C. Young, J. Zinman, and V. E. Johnson. Redefine statistical significance. *Nature Human Behaviour*, 2(1):6–10, Jan. 2018. ISSN 2397-3374. doi: 10.1038/s41562-017-0189-z. URL https://www.nature.com/articles/s41562-017-0189-z. Number: 1 Publisher: Nature Publishing Group.

J. O. Berger and D. Sun. Objective priors for the bivariate normal model. *Annals of Statistics*, 36(2):963–982, Apr. 2008. ISSN 0090-5364, 2168-8966. doi: 10.1214/ 07-AOS501. URL https://projecteuclid.org/euclid.aos/1205420525. Publisher: Institute of Mathematical Statistics.

J. O. Berger and R. L. Wolpert. *The Likelihood Principle*. Institute of Mathematical Statistics, 1984. ISBN 978-0-940600-06-5. Google-Books-ID: a6NUBG4WZPMC.

J. O. Berger, L. R. Pericchi, and J. A. Varshavsky. Bayes Factors and Marginal Distributions in Invariant Situations. *Sankhyā: The Indian Journal of Statistics, Series A (1961-2002)*, 60(3):307–321, 1998. ISSN 0581-572X. URL https:// www.jstor.org/stable/25051210. Publisher: Springer.

R. H. Berk. A Note on Sufficiency and Invariance. *The Annals of Mathematical Statistics*, 43(2):647–650, Apr. 1972. ISSN 0003-4851, 2168-8990. doi: 10.1214/aoms/1177692645. URL https://projecteuclid.org/journals/ annals-of-mathematical-statistics/volume-43/issue-2/A-Note-on-Sufficiency-and-Invariance/10.1214/aoms/1177692645.full. Publisher: Institute of Mathematical Statistics.

J. L. Bhowmik and M. L. King. Maximal invariant likelihood based testing of semilinear models. *Statistical Papers*, 48(3):357–383, Sept. 2007. ISSN 1613-9798. doi: 10.1007/s00362-006-0342-7. URL https://doi.org/10.1007/s00362-006-0342-7.

J. V. Bondar. Borel Cross-Sections and Maximal Invariants. *Annals of Statistics*, 4 (5):866–877, Sept. 1976. ISSN 0090-5364, 2168-8966. doi: 10.1214/aos/1176343585. URL https://projecteuclid.org/euclid.aos/1176343585. Publisher: Institute of Mathematical Statistics.

J. V. Bondar and P. Milnes. Amenability: A survey for statistical applications of Hunt-Stein and related conditions on groups. *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, 57(1):103–128, 1981. Publisher: Springer.

S. Boucheron, G. Lugosi, and P. Massart. *Concentration Inequalities: A Nonasymptotic Theory of Independence.* OUP Oxford, Feb. 2013. ISBN 978-0-19-953525-5. Google-Books-ID: koNqWRluhP0C.

N. Bourbaki. *Integration II: Chapters 7–9.* Elements of Mathematics. Springer-Verlag, Berlin Heidelberg, 2004. ISBN 978-3-540-20585-2. doi: 10.1007/978-3-662-07931-7. URL `https://www.springer.com/gp/book/9783540205852`.

L. Breiman. Optimal Gambling Systems for Favorable Games. *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume 1: Contributions to the Theory of Statistics*, 4.1:65–79, Jan. 1961. URL `https://projecteuclid.org/ebooks/berkeley-symposium-on-mathematical-statistics-and-probability/Proceedings-of-the-Fourth-Berkeley-Symposium-on-Mathematical-Statistics-and/chapter/Optimal-Gambling-Systems-for-Favorable-Games/bsmsp/1200512159`. Publisher: University of California Press.

S. Bubeck, N. R. Devanur, Z. Huang, and R. Niazadeh. Multi-scale Online Learning: Theory and Applications to Online Auctions and Pricing. *Journal of Machine Learning Research*, 20(62):1–37, 2019. ISSN 1533-7928. URL `http://jmlr.org/papers/v20/17-498.html`.

L. Bégin, P. Germain, F. Laviolette, and J.-F. Roy. PAC-Bayesian Bounds based on the Rényi Divergence. In *Proceedings of the 19th International Conference on Artificial Intelligence and Statistics*, pages 435–444. PMLR, May 2016. URL `https://proceedings.mlr.press/v51/begin16.html`. ISSN: 1938-7228.

O. Catoni. Pac-Bayesian Supervised Classification: The Thermodynamics of Statistical Learning. *IMS Lecture Notes Monograph Series*, 56:1–163, 2007. ISSN 0749-2170. doi: 10.1214/074921707000000391. URL `http://arxiv.org/abs/0712.0248`. arXiv: 0712.0248.

N. Cesa-Bianchi and G. Lugosi. *Prediction, learning, and games.* Cambridge university press, 2006.

N. Cesa-Bianchi, P. Gaillard, C. Gentile, and S. Gerchinovitz. Algorithmic Chaining and the Role of Partial Feedback in Online Nonparametric Learning. In *Conference on Learning Theory*, pages 465–481, June 2017. URL `http://proceedings.mlr.press/v65/cesa-bianchi17a.html`.

J. T. Chang and D. Pollard. Conditioning as disintegration. *Statistica Neerlandica*, 51(3):287–317, 1997. ISSN 1467-9574. doi: 10.1111/1467-9574.00056. URL `https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-9574.00056`.

Bibliography

L. Chen, H. Luo, and C.-Y. Wei. Impossible Tuning Made Possible: A New Expert Algorithm and Its Applications. In *Proceedings of Thirty Fourth Conference on Learning Theory*, pages 1216–1259. PMLR, July 2021. URL https://proceedings.mlr.press/v134/chen21f.html. ISSN: 2640-3498.

Y. J. Choe and A. Ramdas. Comparing Sequential Forecasters, Dec. 2022. URL http://arxiv.org/abs/2110.00115. arXiv:2110.00115 [cs, math, stat].

B. Chugg, H. Wang, and A. Ramdas. A unified recipe for deriving (time-uniform) PAC-Bayes bounds, Feb. 2023. URL http://arxiv.org/abs/2302.03421. arXiv:2302.03421 [cs, math, stat].

T. M. Cover and J. A. Thomas. *Elements of Information Theory*. Wiley Series in Telecommunications and Signal Processing. Wiley-Interscience, New York, NY, USA, 2006. ISBN 978-0-471-24195-9.

D. R. Cox. Sequential tests for composite hypotheses. *Mathematical Proceedings of the Cambridge Philosophical Society*, 48(2):290–299, Apr. 1952. ISSN 1469-8064, 0305-0041. doi: 10.1017/S030500410002764X. URL https://www.cambridge.org/core/journals/mathematical-proceedings-of-the-cambridge-philosophical-society/article/abs/sequential-tests-for-composite-hypotheses/3424D357C438E368282290BE6D28A99A. Publisher: Cambridge University Press.

D. R. Cox. Regression Models and Life-Tables. *Journal of the Royal Statistical Society. Series B (Methodological)*, 34(2):187–220, 1972. ISSN 0035-9246. URL https://www.jstor.org/stable/2985181. Publisher: [Royal Statistical Society, Wiley].

D. R. Cox. Partial Likelihood. *Biometrika*, 62(2):269–276, 1975. ISSN 0006-3444. doi: 10.2307/2335362. URL https://www.jstor.org/stable/2335362. Publisher: [Oxford University Press, Biometrika Trust].

A. Cutkosky and F. Orabona. Black-Box Reductions for Parameter-free Online Learning in Banach Spaces. In *Conference On Learning Theory*, pages 1493–1529, July 2018. URL http://proceedings.mlr.press/v75/cutkosky18a.html.

D. A. Darling and H. Robbins. Confidence sequences for mean, variance, and median. *Proceedings of the National Academy of Sciences*, 58(1):66–68, July 1967. doi: 10.1073/pnas.58.1.66. URL https://www.pnas.org/doi/abs/10.1073/pnas.58.1.66. Publisher: Proceedings of the National Academy of Sciences.

D. A. Darling and H. Robbins. Some further remarks on inequalities for sample sums*. *Proceedings of the National Academy of Sciences*, 60(4):1175–1182, Aug. 1968a. doi: 10.1073/pnas.60.4.1175. URL https://www.pnas.org/doi/abs/10.1073/pnas.60.4.1175. Publisher: Proceedings of the National Academy of Sciences.

D. A. Darling and H. Robbins. Some nonparametric sequential tests with power one*. *Proceedings of the National Academy of Sciences*, 61(3):804–809, Nov. 1968b. doi: 10.1073/pnas.61.3.804. URL `https://www.pnas.org/doi/abs/10.1073/pnas.61.3.804`. Publisher: Proceedings of the National Academy of Sciences.

A. P. Dawid. Present Position and Potential Developments: Some Personal Views: Statistical Theory: The Prequential Approach. *Journal of the Royal Statistical Society. Series A (General)*, 147(2):278–292, 1984. ISSN 0035-9238. doi: 10.2307/2981683. URL `https://www.jstor.org/stable/2981683`. Publisher: [Royal Statistical Society, Wiley].

A. P. Dawid, M. Stone, and J. V. Zidek. Marginalization Paradoxes in Bayesian and Structural Inference. *Journal of the Royal Statistical Society. Series B (Methodological)*, 35(2):189–233, 1973. ISSN 0035-9246. URL `https://www.jstor.org/stable/2984907`. Publisher: [Royal Statistical Society, Wiley].

R. de Heide and P. D. Grünwald. Why optional stopping can be a problem for Bayesians. *Psychonomic Bulletin & Review*, 28(3):795–812, June 2021. ISSN 1531-5320. doi: 10.3758/s13423-020-01803-x. URL `https://doi.org/10.3758/s13423-020-01803-x`.

S. de Rooij, T. van Erven, P. D. Grünwald, and W. M. Koolen. Follow the Leader If You Can, Hedge If You Must. *Journal of Machine Learning Research*, 15(37):1281–1316, 2014. ISSN 1533-7928. URL `http://jmlr.org/papers/v15/rooij14a.html`.

D. Dubhashi and D. Ranjan. Balls and bins: A study in negative dependence. *Random Structures & Algorithms*, 13(2):99–124, 1998.

J. Duchi, E. Hazan, and Y. Singer. Adaptive Subgradient Methods for Online Learning and Stochastic Optimization. *Journal of Machine Learning Research*, 12 (61):2121–2159, 2011. ISSN 1533-7928. URL `http://jmlr.org/papers/v12/duchi11a.html`.

R. Durrett. *Probability: Theory and Examples*. Number 49 in Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, 5 edition, Apr. 2019. ISBN 978-1-108-47368-2. Google-Books-ID: b22MDwAAQBAJ.

M. L. Eaton. Group Invariance Applications in Statistics. *Regional Conference Series in Probability and Statistics*, 1:i–133, 1989. ISSN 1935-5912. URL `https://www.jstor.org/stable/4153172`.

M. L. Eaton and W. D. Sudderth. Consistency and Strong Inconsistency of Group-Invariant Predictive Inferences. *Bernoulli*, 5(5):833–854, 1999. ISSN 1350-7265. doi: 10.2307/3318446. URL `https://www.jstor.org/stable/3318446`. Publisher: International Statistical Institute (ISI) and Bernoulli Society for Mathematical Statistics and Probability.

M. L. Eaton and W. D. Sudderth. Group invariant inference and right Haar measure. *Journal of Statistical Planning and Inference*, 103(1):87–99, Apr. 2002. ISSN 0378-3758. doi: 10.1016/S0378-3758(01)00199-9. URL `http://www.sciencedirect.com/science/article/pii/S0378375801001999`.

B. Efron. The Efficiency of Cox's Likelihood Function for Censored Data. *Journal of the American Statistical Association*, 72(359):557–565, Sept. 1977. ISSN 0162-1459. doi: 10.1080/01621459.1977.10480613. URL `https://www.tandfonline.com/doi/abs/10.1080/01621459.1977.10480613`. Publisher: Taylor & Francis _eprint: https://www.tandfonline.com/doi/pdf/10.1080/01621459.1977.10480613.

R. Fisher. Statistical Methods and Scientific Induction. *Journal of the Royal Statistical Society. Series B (Methodological)*, 17(1):69–78, 1955. ISSN 0035-9246. URL `https://www.jstor.org/stable/2983785`. Publisher: [Royal Statistical Society, Wiley].

T. R. Fleming and D. P. Harrington. *Counting Processes and Survival Analysis*. John Wiley & Sons, Sept. 2011. ISBN 978-1-118-15066-5.

D. J. Foster, S. Kale, M. Mohri, and K. Sridharan. Parameter-Free Online Learning via Model Selection. In I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett, editors, *Advances in Neural Information Processing Systems 30*, pages 6020–6030. Curran Associates, Inc., 2017. URL `http://papers.nips.cc/paper/7183-parameter-free-online-learning-via-model-selection.pdf`.

Y. Freund. Open Problem: Second order regret bounds based on scaling time. In *Conference on Learning Theory*, pages 1651–1654, 2016.

Y. Freund and R. E. Schapire. A Decision-Theoretic Generalization of On-Line Learning and an Application to Boosting. *Journal of Computer and System Sciences*, 55 (1):119–139, Aug. 1997. ISSN 0022-0000. doi: 10.1006/jcss.1997.1504. URL `https://www.sciencedirect.com/science/article/pii/S002200009791504X`.

P. Gaillard and S. Gerchinovitz. A Chaining Algorithm for Online Nonparametric Regression. In *Conference on Learning Theory*, pages 764–796, June 2015. URL `http://proceedings.mlr.press/v40/Gaillard15.html`.

P. Gaillard, G. Stoltz, and T. van Erven. A second-order bound with excess losses. In *Proceedings of The 27th Conference on Learning Theory*, pages 176–196. PMLR, May 2014. URL `https://proceedings.mlr.press/v35/gaillard14.html`. ISSN: 1938-7228.

J. W. Gibbs. *Elementary Principles in Statistical Mechanics*. Charles Scribner's Sons, New York, 1902.

N. Giri, J. Kiefer, and C. Stein. Minimax Character of Hotelling's T^2 Test in the Simplest Case. *The Annals of Mathematical Statistics*, 34(4):1524–1535, Dec. 1963. ISSN 0003-4851, 2168-8990. doi:

10.1214/aoms/1177703884. URL `https://projecteuclid.org/journals/annals-of-mathematical-statistics/volume-34/issue-4/Minimax-Character-of-Hotellings-T2-Test-in-the-Simplest-Case/10.1214/aoms/1177703884.full`. Publisher: Institute of Mathematical Statistics.

S. Greenland, S. J. Senn, K. J. Rothman, J. B. Carlin, C. Poole, S. N. Goodman, and D. G. Altman. Statistical tests, P values, confidence intervals, and power: a guide to misinterpretations. *European Journal of Epidemiology*, 31:337–350, 2016. ISSN 0393-2990. doi: 10.1007/s10654-016-0149-3. URL `https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4877414/`.

P. Grünwald. The Safe Bayesian. In N. H. Bshouty, G. Stoltz, N. Vayatis, and T. Zeugmann, editors, *Algorithmic Learning Theory*, Lecture Notes in Computer Science, pages 169–183. Springer Berlin Heidelberg, 2012. ISBN 978-3-642-34106-9.

P. Grünwald. Beyond Neyman-Pearson, Feb. 2023. URL `http://arxiv.org/abs/2205.00901`. arXiv:2205.00901 [stat].

P. Grünwald, R. de Heide, and W. Koolen. Safe Testing. *arXiv:1906.07801 [cs, math, stat]*, June 2020. URL `http://arxiv.org/abs/1906.07801`. Accepted for publication as a read paper in the Journal of the Royal Statistical Society, Series B. 2023.

P. D. Grünwald. Viewing all models as "probabilistic". In *Proceedings of the twelfth annual conference on Computational learning theory*, COLT '99, pages 171–182, New York, NY, USA, July 1999. Association for Computing Machinery. ISBN 978-1-58113-167-3. doi: 10.1145/307400.307436. URL `https://dl.acm.org/doi/10.1145/307400.307436`.

P. D. Grünwald. *The Minimum Description Length Principle*. Adaptive Computation and Machine Learning series. Mar. 2007. ISBN 978-0-262-52963-1.

P. D. Grünwald and N. A. Mehta. A tight excess risk bound via a unified PAC-Bayesian–Rademacher–Shtarkov–MDL complexity. In *Proceedings of the 30th International Conference on Algorithmic Learning Theory*, pages 433–465. PMLR, Mar. 2019. URL `https://proceedings.mlr.press/v98/grunwald19a.html`. ISSN: 2640-3498.

P. D. Grünwald and N. A. Mehta. Fast Rates for General Unbounded Loss Functions: From ERM to Generalized Bayes. *Journal of Machine Learning Research*, 21(56):1–80, 2020. ISSN 1533-7928. URL `http://jmlr.org/papers/v21/18-488.html`.

P. D. Grünwald and T. Roos. Minimum description length revisited. *International Journal of Mathematics for Industry*, 11(01):1930001, Dec. 2019. ISSN 2661-3352. doi: 10.1142/S2661335219300018. URL `https://www.worldscientific.com/doi/10.1142/S2661335219300018`. Publisher: World Scientific Publishing Co.

P. D. Grünwald and T. van Ommen. Inconsistency of Bayesian Inference for Misspecified Linear Models, and a Proposal for Repairing It. *Bayesian Analysis*, 12(4): 1069–1103, Dec. 2017. ISSN 1936-0975, 1931-6690. doi: 10.1214/17-BA1085. URL `https://projecteuclid.org/euclid.ba/1510974325`.

P. D. Grünwald, T. Steinke, and L. Zakynthinou. PAC-Bayes, MAC-Bayes and Conditional Mutual Information: Fast rate bounds that handle general VC classes. In *Proceedings of Thirty Fourth Conference on Learning Theory*, pages 2217–2247. PMLR, July 2021. URL `https://proceedings.mlr.press/v134/grunwald21a.html`. ISSN: 2640-3498.

P. D. Grünwald, M. F. Pérez-Ortiz, and Z. Mhammedi. Exponential Stochastic Inequality, Apr. 2023. URL `http://arxiv.org/abs/2304.14217`. arXiv: 2304.14217 [math, stat].

B. Guedj. A Primer on PAC-Bayesian Learning, May 2019. URL `http://arxiv.org/abs/1901.05353`. arXiv:1901.05353 [cs, stat].

W. J. Hall, R. A. Wijsman, and J. K. Ghosh. The Relationship Between Sufficiency and Invariance with Applications in Sequential Analysis. *The Annals of Mathematical Statistics*, 36(2):575–614, 1965. ISSN 0003-4851. URL `https://www.jstor.org/stable/2238164`. Publisher: Institute of Mathematical Statistics.

W. J. Hall, R. A. Wijsman, and J. K. Ghosh. Correction: The Relationship Between Sufficiency and Invariance with Applications in Sequential Analysis. *The Annals of Statistics*, 23(2):705–705, 1995. ISSN 0090-5364. URL `https://www.jstor.org/stable/2242359`. Publisher: Institute of Mathematical Statistics.

E. Hazan. Introduction to Online Convex Optimization, Dec. 2021. URL `http://arxiv.org/abs/1909.05207`. arXiv:1909.05207 [cs, math, stat].

F. Hellström and G. Durisi. A New Family of Generalization Bounds Using Samplewise Evaluated CMI. *Advances in Neural Information Processing Systems*, 35:10108–10121, Dec. 2022. URL `https://proceedings.neurips.cc/paper_files/paper/2022/hash/41b6674c28a9b93ec8d22a53ca25bc3b-Abstract-Conference.html`.

A. Henzi and J. F. Ziegel. Valid sequential inference on probability forecast performance. *Biometrika*, page asab047, Sept. 2021. ISSN 0006-3444. doi: 10.1093/biomet/asab047. URL `https://doi.org/10.1093/biomet/asab047`.

S. R. Howard, A. Ramdas, J. McAuliffe, and J. Sekhon. Exponential line-crossing inequalities. *arXiv:1808.03204 [math]*, Aug. 2018a. URL `http://arxiv.org/abs/1808.03204`. arXiv: 1808.03204.

S. R. Howard, A. Ramdas, J. McAuliffe, and J. Sekhon. Uniform, nonparametric, non-asymptotic confidence sequences. *arXiv:1810.08240 [math, stat]*, Oct. 2018b. URL `http://arxiv.org/abs/1810.08240`. arXiv: 1810.08240.

Y.-G. Hsieh, K. Antonakopoulos, and P. Mertikopoulos. Adaptive Learning in Continuous Games: Optimal Regret Bounds and Convergence to Nash Equilibrium. In *Proceedings of Thirty Fourth Conference on Learning Theory*, pages 2388–2422. PMLR, July 2021. URL `https://proceedings.mlr.press/v134/hsieh21a.html`. ISSN: 2640-3498.

K. Jang, K.-S. Jun, I. Kuzborskij, and F. Orabona. Tighter PAC-Bayes Bounds Through Coin-Betting, Feb. 2023. URL `http://arxiv.org/abs/2302.05829`. arXiv:2302.05829 [cs, stat].

S. H. Jeffreys and S. H. Jeffreys. *The Theory of Probability*. Oxford Classic Texts in the Physical Sciences. Oxford University Press, Oxford, New York, third edition, third edition edition, Aug. 1998. ISBN 978-0-19-850368-2.

K. V. Jespersen, V. Pando-Naude, J. Koenig, P. Jennum, and P. Vuust. Listening to music for insomnia in adults. *The Cochrane Database of Systematic Reviews*, 8(8): CD010459, Aug. 2022. ISSN 1469-493X. doi: 10.1002/14651858.CD010459.pub3.

K. Joag-Dev and F. Proschan. Negative Association of Random Variables with Applications. *The Annals of Statistics*, 11(1):286–295, 1983. ISSN 0090-5364. URL `https://www.jstor.org/stable/2240482`.

J. D. Kalbfleisch and R. L. Prentice. Marginal Likelihoods Based on Cox's Regression and Life Model. *Biometrika*, 60(2):267–278, 1973. ISSN 0006-3444. doi: 10.2307/2334538. URL `https://www.jstor.org/stable/2334538`. Publisher: [Oxford University Press, Biometrika Trust].

T. Kariya. Locally Robust Tests for Serial Correlation in Least Squares Regression. *The Annals of Statistics*, 8(5):1065–1070, Sept. 1980. ISSN 0090-5364, 2168-8966. doi: 10.1214/aos/1176345143. URL `https://projecteuclid.org/journals/annals-of-statistics/volume-8/issue-5/Locally-Robust-Tests-for-Serial-Correlation-in-Least-Squares-Regression/10.1214/aos/1176345143.full`. Publisher: Institute of Mathematical Statistics.

J. L. Kelly Jr. A New Interpretation of Information Rate. *Bell System Technical Journal*, 35(4):917–926, 1956. ISSN 1538-7305. doi: 10.1002/j.1538-7305.1956.tb03809.x. URL `https://onlinelibrary.wiley.com/doi/abs/10.1002/j.1538-7305.1956.tb03809.x`. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/j.1538-7305.1956.tb03809.x.

K. Kim and D. L. DeMets. Confidence Intervals Following Group Sequential Tests in Clinical Trials. *Biometrics*, 43(4):857–864, 1987. ISSN 0006-341X. doi: 10.2307/2531539. URL `https://www.jstor.org/stable/2531539`. Publisher: [Wiley, International Biometric Society].

J. P. Klein and M. L. Moeschberger. *Survival Analysis: Techniques for Censored and Truncated Data*. Statistics for Biology and Health. Springer, New York, NY,

2003. ISBN 978-0-387-95399-1 978-0-387-21645-4. doi: 10.1007/b97377. URL `http://link.springer.com/10.1007/b97377`.

W. M. Koolen and T. van Erven. Second-order Quantile Methods for Experts and Combinatorial Games. In *Proceedings of The 28th Conference on Learning Theory*, pages 1155–1175. PMLR, June 2015. URL `https://proceedings.mlr.press/v40/Koolen15a.html`. ISSN: 1938-7228.

W. M. Koolen, P. D. Grünwald, and T. van Erven. Combining Adversarial Guarantees and Stochastic Fast Rates in Online Learning. In *Advances in Neural Information Processing Systems*, volume 29. Curran Associates, Inc., 2016. URL `https://proceedings.neurips.cc/paper_files/paper/2016/hash/db116b39f7a3ac5366079b1d9fe249a5-Abstract.html`.

I. Kuzborskij and N. Cesa-Bianchi. Locally-Adaptive Nonparametric Online Learning. In *Advances in Neural Information Processing Systems*, volume 33, pages 1679–1689. Curran Associates, Inc., 2020. URL `https://proceedings.neurips.cc/paper_files/paper/2020/hash/12780ea688a71dabc284b064add459a4-Abstract.html`.

T. L. Lai. On Confidence Sequences. *The Annals of Statistics*, 4(2):265–280, Mar. 1976. ISSN 0090-5364, 2168-8966. doi: 10.1214/aos/1176343406. URL `https://projecteuclid.org/journals/annals-of-statistics/volume-4/issue-2/On-Confidence-Sequences/10.1214/aos/1176343406.full`. Publisher: Institute of Mathematical Statistics.

T. L. Lai. Sequential Analysis: Some Classical Problems and New Challenges. *Statistica Sinica*, 11(2):303–351, 2001. ISSN 1017-0405. URL `https://www.jstor.org/stable/24306854`. Publisher: Institute of Statistical Science, Academia Sinica.

J. Langford and J. Shawe-Taylor. PAC-Bayes & Margins. In *Advances in Neural Information Processing Systems*, volume 15. MIT Press, 2002. URL `https://proceedings.neurips.cc/paper_files/paper/2002/hash/68d309812548887400e375eaa036d2f1-Abstract.html`.

E. L. Lehmann and J. P. Romano. *Testing Statistical Hypotheses*. Springer Texts in Statistics. Springer-Verlag, New York, 3 edition, 2005. ISBN 978-0-387-98864-1. doi: 10.1007/0-387-27605-X. URL `https://www.springer.com/gp/book/9780387988641`.

J. Li and A. Barron. Mixture Density Estimation. In *Advances in Neural Information Processing Systems*, volume 12. MIT Press, 1999. URL `https://papers.nips.cc/paper/1999/hash/a0f3601dc682036423013a5d965db9aa-Abstract.html`.

Q. J. Li. *Estimation of Mixture Models*. PhD Thesis, Yale University, New Haven, CT, USA, 1999.

F. Liang and A. Barron. Exact minimax strategies for predictive density estimation, data compression, and model selection. *IEEE Transactions on Information Theory*, 50(11):2708–2726, Nov. 2004. ISSN 1557-9654. doi: 10.1109/TIT.2004.836922. Conference Name: IEEE Transactions on Information Theory.

M. Lindon and A. Malek. Anytime-Valid Inference for Multinomial Count Data, May 2022. URL http://arxiv.org/abs/2011.03567. arXiv:2011.03567 [math, stat].

N. Littlestone and M. K. Warmuth. The Weighted Majority Algorithm. *Information and Computation*, 108(2):212–261, Feb. 1994. ISSN 0890-5401. doi: 10.1006/inco.1994.1009. URL https://www.sciencedirect.com/science/article/pii/S0890540184710091.

G. Lugosi and G. Neu. Generalization Bounds via Convex Analysis. In *Proceedings of Thirty Fifth Conference on Learning Theory*, pages 3524–3546. PMLR, June 2022. URL https://proceedings.mlr.press/v178/lugosi22a.html. ISSN: 2640-3498.

O.-A. Maillard. *Mathematics of Statistical Sequential Decision Making*. Habilitation à diriger des recherches, Université de Lille, Sciences et Technologies, Feb. 2019. URL https://hal.archives-ouvertes.fr/tel-02162189.

N. Mantel. Evaluation of survival data and two new rank order statistics arising in its consideration. *Cancer Chemotherapy Reports*, 50(3):163–170, Mar. 1966. ISSN 0069-0112.

T. V. Marinov and J. Zimmert. The Pareto Frontier of model selection for general Contextual Bandits. In *Advances in Neural Information Processing Systems*, volume 34, pages 17956–17967. Curran Associates, Inc., 2021. URL https://proceedings.neurips.cc/paper/2021/hash/9570efef719d705326f0ff817ef084e6-Abstract.html.

A. Maurer. A Note on the PAC Bayesian Theorem, Nov. 2004. URL http://arxiv.org/abs/cs/0411099. arXiv:cs/0411099.

D. A. McAllester. Some PAC-Bayesian Theorems. In *Proceedings of the Eleventh Annual Conference on Computational Learning Theory*, COLT' 98, pages 230–234, New York, NY, USA, 1998. ACM. ISBN 978-1-58113-057-7. doi: 10.1145/279943.279989. URL http://doi.acm.org/10.1145/279943.279989.

D. V. Mehrotra and A. J. Roth. Relative risk estimation and inference using a generalized logrank statistic. *Statistics in Medicine*, 20 (14):2099–2113, 2001. ISSN 1097-0258. doi: 10.1002/sim.854. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/sim.854. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/sim.854.

Z. Mhammedi, P. D. Grunwald, and B. Guedj. PAC-Bayes Un-Expected Bernstein Inequality. *arXiv:1905.13367 [cs, stat]*, May 2019. URL `http://arxiv.org/abs/1905.13367`. arXiv: 1905.13367.

J. Mourtada and S. Gaïffas. On the optimality of the Hedge algorithm in the stochastic regime. *Journal of Machine Learning Research*, 20(83):1–28, 2019. ISSN 1533-7928. URL `http://jmlr.org/papers/v20/18-869.html`.

A. G. Nogales and J. A. Oyola. Some Remarks on Sufficiency, Invariance and Conditional Independence. *The Annals of Statistics*, 24(2):906–909, 1996. ISSN 0090-5364. URL `https://www.jstor.org/stable/2242682`. Publisher: Institute of Mathematical Statistics.

P. C. O'Brien and T. R. Fleming. A Multiple Testing Procedure for Clinical Trials. *Biometrics*, 35(3):549–556, 1979. ISSN 0006-341X. doi: 10.2307/2530245. URL `https://www.jstor.org/stable/2530245`. Publisher: [Wiley, International Biometric Society].

Office of Scientific Research and Development. Summary Technical Report of the Applied Mathematics Panel, NDRC. Volume 3. Probability and Statistical Studies in Warfare Analysis. Part 1: Bombing Studies. Part 2: Miscellaneous Studies. Technical report, Washington DC, 1946. URL `https://apps.dtic.mil/sti/citations/ADB809137`. Section: Technical Reports.

A. L. Paterson. *Amenability*. Number 29. American Mathematical Soc., 2000.

R. Peto. Discussion of: Regression models and life tables, by DR Cox. *Journal of the Royal Statistical Society, Series B*, 26:205–207, 1972.

R. Peto and J. Peto. Asymptotically Efficient Rank Invariant Test Procedures. *Journal of the Royal Statistical Society. Series A (General)*, 135(2):185–207, 1972. ISSN 0035-9238. doi: 10.2307/2344317. URL `https://www.jstor.org/stable/2344317`. Publisher: [Royal Statistical Society, Wiley].

S. J. Pocock. Group sequential methods in the design and analysis of clinical trials. *Biometrika*, 64(2):191–199, Aug. 1977. ISSN 0006-3444. doi: 10.1093/biomet/64.2.191. URL `https://doi.org/10.1093/biomet/64.2.191`.

S. J. Pocock. Current controversies in data monitoring for clinical trials. *Clinical Trials*, 3(6):513–521, Dec. 2006. ISSN 1740-7745. doi: 10.1177/1740774506073467. URL `https://doi.org/10.1177/1740774506073467`. Publisher: SAGE Publications.

M. A. Proschan, K. K. G. Lan, and J. T. Wittes. *Statistical Monitoring of Clinical Trials: A Unified Approach*. Springer Science & Business Media, Dec. 2006. ISBN 978-0-387-44970-8. Google-Books-ID: BCu8c8NwXxcC.

M. F. Pérez-Ortiz and W. M. Koolen. Luckiness in Multiscale Online Learning. *Advances in Neural Information Processing Systems*, 35:25160–25170, Dec. 2022. URL `https://papers.nips.cc/paper_files/paper/2022/hash/a0d2345b43e66fa946155c98899dc03b-Abstract-Conference.html`.

M. F. Pérez-Ortiz, T. Lardy, R. de Heide, and P. Grünwald. E-Statistics, Group Invariance and Anytime Valid Testing, Aug. 2022. URL `http://arxiv.org/abs/2208.07610`. arXiv:2208.07610 [math, stat], under submission.

A. Rakhlin and K. Sridharan. On Equivalence of Martingale Tail Bounds and Deterministic Regret Inequalities. In *Proceedings of the 2017 Conference on Learning Theory*, pages 1704–1722. PMLR, June 2017. URL `https://proceedings.mlr.press/v65/rakhlin17a.html`. ISSN: 2640-3498.

S. Rakhlin and K. Sridharan. Optimization, Learning, and Games with Predictable Sequences. In *Advances in Neural Information Processing Systems*, volume 26. Curran Associates, Inc., 2013. URL `https://proceedings.neurips.cc/paper_files/paper/2013/hash/f0dd4a99fba6075a9494772b58f95280-Abstract.html`.

A. Ramdas, J. Ruf, M. Larsson, and W. Koolen. Admissible anytime-valid sequential inference must rely on nonnegative martingales. *arXiv:2009.03167 [math, stat]*, Sept. 2020. URL `http://arxiv.org/abs/2009.03167`. arXiv: 2009.03167.

A. Ramdas, P. D. Grünwald, V. Vovk, and G. Shafer. Game-theoretic statistics and safe anytime-valid inference, Oct. 2022a. URL `http://arxiv.org/abs/2210.01948`. arXiv:2210.01948 [cs, math, stat].

A. Ramdas, J. Ruf, M. Larsson, and W. M. Koolen. Testing exchangeability: Fork-convexity, supermartingales and e-processes. *International Journal of Approximate Reasoning*, 141:83–109, Feb. 2022b. ISSN 0888-613X. doi: 10.1016/j.ijar.2021.06.017. URL `https://www.sciencedirect.com/science/article/pii/S0888613X21000980`.

H. Reiter and J. D. Stegeman. *Classical Harmonic Analysis and Locally Compact Groups*. London Mathematical Society Monographs. Oxford University Press, Oxford, New York, second edition edition, Nov. 2000. ISBN 978-0-19-851189-2.

Z. Ren and R. F. Barber. Derandomized knockoffs: leveraging e-values for false discovery rate control, Feb. 2023. URL `http://arxiv.org/abs/2205.15461`. arXiv:2205.15461 [stat].

H. Robbins and D. Siegmund. Boundary Crossing Probabilities for the Wiener Process and Sample Sums. *The Annals of Mathematical Statistics*, 41(5):1410–1429, Oct. 1970. ISSN 0003-4851, 2168-8990. doi: 10.1214/aoms/1177696787. URL `https://projecteuclid.org/journals/annals-of-mathematical-statistics/volume-41/issue-5/Boundary-Crossing-Probabilities-for-the-Wiener-Process-and-`

Sample-Sums/10.1214/aoms/1177696787.full. Publisher: Institute of Mathematical Statistics.

J. N. Rouder, P. L. Speckman, D. Sun, R. D. Morey, and G. Iverson. Bayesian t tests for accepting and rejecting the null hypothesis. *Psychonomic Bulletin & Review*, 16(2):225–237, Apr. 2009. ISSN 1531-5320. doi: 10.3758/PBR.16.2.225. URL https://doi.org/10.3758/PBR.16.2.225.

S. N. Roy and R. E. Bargmann. Tests of Multiple Independence and the Associated Confidence Bounds. *The Annals of Mathematical Statistics*, 29(2):491–503, June 1958. ISSN 0003-4851, 2168-8990. doi: 10.1214/aoms/1177706624. URL https://projecteuclid.org/journals/annals-of-mathematical-statistics/volume-29/issue-2/Tests-of-Multiple-Independence-and-the-Associated-Confidence-Bounds/10.1214/aoms/1177706624.full. Publisher: Institute of Mathematical Statistics.

R. Royall. *Statistical Evidence: A Likelihood Paradigm*, volume 71. Chapman and Hall/CRC, New York, 1 edition, 1997. ISBN 978-1-03-247800-5. URL https://www.routledge.com/Statistical-Evidence-A-Likelihood-Paradigm/Royall/p/book/9781032478005.

S. Rushton. On a Two-Sided Sequential t-Test. *Biometrika*, 39(3/4):302–308, 1952. ISSN 0006-3444. doi: 10.2307/2334026. URL https://www.jstor.org/stable/2334026. Publisher: [Oxford University Press, Biometrika Trust].

I. R. Savage. Contributions to the Theory of Rank Order Statistics-the Two-Sample Case. *The Annals of Mathematical Statistics*, 27(3):590–615, Sept. 1956. ISSN 0003-4851, 2168-8990. doi: 10.1214/aoms/1177728170. URL https://projecteuclid.org/journals/annals-of-mathematical-statistics/volume-27/issue-3/Contributions-to-the-Theory-of-Rank-Order-Statistics-the-Two/10.1214/aoms/1177728170.full. Publisher: Institute of Mathematical Statistics.

D. Schoenfeld. The Asymptotic Properties of Nonparametric Tests for Comparing Survival Distributions. *Biometrika*, 68(1):316–319, 1981. ISSN 0006-3444. doi: 10.2307/2335833. URL https://www.jstor.org/stable/2335833. Publisher: [Oxford University Press, Biometrika Trust].

M. Seeger. PAC-Bayesian Generalisation Error Bounds for Gaussian Process Classification. *Journal of Machine Learning Research*, 3(Oct):233–269, 2002. ISSN ISSN 1533-7928. URL https://www.jmlr.org/papers/v3/seeger02a.html.

Y. Seldin, F. Laviolette, N. Cesa-Bianchi, J. Shawe-Taylor, and P. Auer. PAC-Bayesian Inequalities for Martingales. *IEEE Transactions on Information Theory*, 58(12):7086–7093, Dec. 2012. ISSN 1557-9654. doi: 10.1109/TIT.2012.2211334. Conference Name: IEEE Transactions on Information Theory.

T. Sellke and D. Siegmund. Sequential analysis of the proportional hazards model. *Biometrika*, 70(2):315–326, 1983. Publisher: Oxford University Press.

G. Shafer. The Language of Betting as a Strategy for Statistical and Scientific Communication. *arXiv:1903.06991 [math, stat]*, Oct. 2019. URL http://arxiv.org/abs/1903.06991. arXiv: 1903.06991.

G. Shafer. Testing by betting: A strategy for statistical and scientific communication. *Journal of the Royal Statistical Society Series A*, 184(2):407–431, 2021. Publisher: Royal Statistical Society.

G. Shafer and V. Vovk. *Probability and Finance: It's Only a Game!* New York, 2001. ISBN 978-0-471-40226-8.

G. Shafer, A. Shen, N. Vereshchagin, and V. Vovk. Test Martingales, Bayes Factors and p-Values. *Statistical Science*, 26(1):84–101, Feb. 2011. ISSN 0883-4237, 2168-8745. doi: 10.1214/10-STS347. URL https://projecteuclid.org/euclid.ss/1307626567. Publisher: Institute of Mathematical Statistics.

O. V. Shalaevskii. Minimax Character of Hotelling's T2 Test. I. In V. M. Kalinin and O. V. Shalaevskii, editors, *Investigations in Classical Problems of Probability Theory and Mathematical Statistics: Part I*, pages 74–101. Springer US, Boston, MA, 1971. ISBN 978-1-4684-8211-9. doi: 10.1007/978-1-4684-8211-9_2. URL https://doi.org/10.1007/978-1-4684-8211-9_2.

S. Shalev-Shwartz and S. Ben-David. *Understanding Machine Learning: From Theory to Algorithms*. Cambridge University Press, Cambridge, 2014. ISBN 978-1-107-05713-5. doi: 10.1017/CBO9781107298019. URL https://www.cambridge.org/core/books/understanding-machine-learning/3059695661405D25673058E43C8BE2A6.

C. E. Shannon. A Mathematical Theory of Communication. *Bell System Technical Journal*, 27(3):379–423, 1948. ISSN 1538-7305. doi: 10.1002/j.1538-7305.1948.tb01338.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/j.1538-7305.1948.tb01338.x. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/j.1538-7305.1948.tb01338.x.

D. Siegmund. *Sequential Analysis*. Springer Series in Statistics. Springer, New York, NY, 1985. ISBN 978-1-4419-3075-0 978-1-4757-1862-1. doi: 10.1007/978-1-4757-1862-1. URL http://link.springer.com/10.1007/978-1-4757-1862-1.

A. V. Skorokhod. *Random Processes with Independent Increments*, volume 47 of *Mathematics and its Applications*. Springer Dordrecht, 1 edition, 2012. ISBN 978-94-010-5650-2. URL https://link.springer.com/book/9780792303404. Originally published in Russian.

E. V. Slud. Sequential Linear Rank Tests for Two-Sample Censored Survival Data. *Annals of Statistics*, 12(2):551–571, June 1984. ISSN 0090-5364, 2168-8966. doi:

10.1214/aos/1176346505. URL `https://projecteuclid.org/euclid.aos/1176346505`. Publisher: Institute of Mathematical Statistics.

E. V. Slud. Partial Likelihood for Continuous-Time Stochastic Processes. *Scandinavian Journal of Statistics*, 19(2):97–109, 1992. ISSN 0303-6898. URL `https://www.jstor.org/stable/4616231`. Publisher: [Board of the Foundation of the Scandinavian Journal of Statistics, Wiley].

P. Subbaiah and G. S. Mudholkar. A Comparison of Two Tests for the Significance of a Mean Vector. *Journal of the American Statistical Association*, 73(362):414–418, June 1978. ISSN 0162-1459. doi: 10.1080/01621459.1978.10481592. URL `https://www.tandfonline.com/doi/abs/10.1080/01621459.1978.10481592`. Publisher: Taylor & Francis _eprint: https://www.tandfonline.com/doi/pdf/10.1080/01621459.1978.10481592.

D. Sun and J. O. Berger. Objective Bayesian analysis for the multivariate normal model. *Bayesian Statistics*, 8:525–562, 2007.

V. Syrgkanis, A. Agarwal, H. Luo, and R. E. Schapire. Fast Convergence of Regularized Learning in Games. In *Advances in Neural Information Processing Systems*, volume 28. Curran Associates, Inc., 2015. URL `https://proceedings.neurips.cc/paper_files/paper/2015/hash/7fea637fd6d02b8f0adf6f7dc36aed93-Abstract.html`.

M. Talagrand. *Upper and Lower Bounds for Stochastic Processes: Modern Methods and Classical Problems*. Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge / A Series of Modern Surveys in Mathematics. Springer-Verlag, Berlin Heidelberg, 2014. ISBN 978-3-642-54074-5. URL `//www.springer.com/la/book/9783642540745`.

A. Tartakovsky, I. Nikiforov, and M. Basseville. *Sequential Analysis: Hypothesis Testing and Changepoint Detection*. Chapman and Hall/CRC, New York, Aug. 2014. ISBN 978-0-429-15234-4. doi: 10.1201/b17279.

J. B. Taylor. An Interview with Milton Friedman. *Macroeconomic Dynamics*, 5(1):101–131, Feb. 2001. ISSN 1469-8056, 1365-1005. doi: 10.1017/S1365100501018053. URL `https://www.cambridge.org/core/journals/macroeconomic-dynamics/article/abs/an-interview-with-milton-friedman/4F9B7590919E146F06B80BFAD022080B`. Publisher: Cambridge University Press.

J. ter Schure and P. Grünwald. ALL-IN meta-analysis: breathing life into living systematic reviews. Technical Report 11:549, F1000Research, May 2022. URL `https://f1000research.com/articles/11-549`. Type: article.

J. ter Schure and P. D. Grünwald. Accumulation Bias in meta-analysis: the need to consider *time* in error control. Technical Report 8:962, F1000Research, June 2019. URL `https://f1000research.com/articles/8-962`. Type: article.

J. ter Schure, M. F. Pérez-Ortiz, A. Ly, and P. Grünwald. The Safe Logrank Test: Error Control under Continuous Monitoring with Unlimited Horizon, July 2021. URL `http://arxiv.org/abs/2011.06931`. arXiv:2011.06931 [math, stat], under submission.

I. O. Tolstikhin and Y. Seldin. PAC-Bayes-Empirical-Bernstein Inequality. In *Advances in Neural Information Processing Systems*, volume 26. Curran Associates, Inc., 2013. URL `https://papers.nips.cc/paper_files/paper/2013/hash/a97da629b098b75c294dffdc3e463904-Abstract.html`.

A. A. Tsiatis. A Large Sample Study of Cox's Regression Model. *The Annals of Statistics*, 9(1):93–108, Jan. 1981. ISSN 0090-5364, 2168-8966. doi: 10.1214/aos/1176345335. URL `https://projecteuclid.org/journals/annals-of-statistics/volume-9/issue-1/A-Large-Sample-Study-of-Coxs-Regression-Model/10.1214/aos/1176345335.full`. Publisher: Institute of Mathematical Statistics.

A. A. Tsiatis. *Group sequential methods for survival analysis with staggered entry*. Institute of Mathematical Statistics, 1982. ISBN 978-0-940600-02-7. doi: 10.1214/lnms/1215464854. URL `https://projecteuclid.org/euclid.lnms/1215464854`. Pages: 257-268 Publication Title: Survival Analysis.

J. W. Tukey. Review of Sequential Analysis of Statistical Data: Applications. *The Annals of Mathematical Statistics*, 18(1):142–144, 1947. ISSN 0003-4851. URL `https://www.jstor.org/stable/2236115`. Publisher: Institute of Mathematical Statistics.

R. Turner, A. Ly, and P. Grünwald. Two-Sample Tests that are Safe under Optional Stopping, with an Application to Contingency Tables. *arXiv:2106.02693 [cs, math, stat]*, Oct. 2021. URL `http://arxiv.org/abs/2106.02693`. arXiv: 2106.02693.

R. Turner, A. Ly, M. F. Perez-Ortiz, J. ter Schure, and P. D. Grunwald. safestats: Safe Anytime-Valid Inference, Nov. 2022. URL `https://CRAN.R-project.org/package=safestats`.

S. Urban, R. Sreenivasan, and V. Kannan. It's All A/Bout Testing, Nov. 2021. URL `https://netflixtechblog.com/its-all-a-bout-testing-the-netflix-experimentation-platform-4e1ca458c15`.

J. V. Uspensky. Introduction to mathematical probability. 1937.

S. van de Geer and J. Lederer. The Bernstein–Orlicz norm and deviation inequalities. *Probability Theory and Related Fields*, 157(1):225–250, Oct. 2013. ISSN 1432-2064. doi: 10.1007/s00440-012-0455-y. URL `https://doi.org/10.1007/s00440-012-0455-y`.

A. W. van der Vaart and J. Wellner. *Weak Convergence and Empirical Processes: With Applications to Statistics.* Springer Series in Statistics. Springer-Verlag, New York, 1996. ISBN 978-0-387-94640-5. URL `//www.springer.com/la/book/9780387946405`.

T. van Erven. PAC-Bayes Mini-tutorial: A Continuous Union Bound. *arXiv:1405.1580 [stat]*, May 2014. URL `http://arxiv.org/abs/1405.1580`. arXiv: 1405.1580.

T. van Erven and P. Harremoes. Rényi Divergence and Kullback-Leibler Divergence. *IEEE Transactions on Information Theory*, 60(7):3797–3820, July 2014. ISSN 0018-9448. doi: 10.1109/TIT.2014.2320500.

T. Van Erven, P. D. Grünwald, N. A. Mehta, M. D. Reid, and R. C. Williamson. Fast rates in statistical and online learning. *Journal of Machine Learning Research*, 16: 1793–1861, 2015.

V. Vapnik. *Statistical learning theory. 1998*, volume 3. Wiley, New York, 1998.

J. Ville. *Étude Critique de la Notion de Collectif.* Number 218 in Thèses de l'entre-deux-guerres. Paris, 1939. URL `http://archive.numdam.org/item/THESE_1939__218__1_0/`.

V. Vovk. A Game of Prediction with Expert Advice. *Journal of Computer and System Sciences*, 56(2):153–173, Apr. 1998. ISSN 0022-0000. doi: 10.1006/jcss.1997.1556. URL `http://www.sciencedirect.com/science/article/pii/S0022000097915567`.

V. Vovk and R. Wang. E-values: Calibration, combination and applications. *The Annals of Statistics*, 49(3):1736–1754, June 2021. ISSN 0090-5364, 2168-8966. doi: 10.1214/20-AOS2020. URL `https://projecteuclid.org/journals/annals-of-statistics/volume-49/issue-3/E-values-Calibration-combination-and-applications/10.1214/20-AOS2020.full`. Publisher: Institute of Mathematical Statistics.

V. G. Vovk. Aggregating Strategies. In *Proceedings of the Third Annual Workshop on Computational Learning Theory*, COLT '90, pages 371–386, San Francisco, CA, USA, 1990. Morgan Kaufmann Publishers Inc. ISBN 978-1-55860-146-8. URL `http://dl.acm.org/citation.cfm?id=92571.92672`.

E.-J. Wagenmakers. A practical solution to the pervasive problems ofp values. *Psychonomic Bulletin & Review*, 14(5):779–804, Oct. 2007. ISSN 1531-5320. doi: 10.3758/BF03194105. URL `https://doi.org/10.3758/BF03194105`.

A. Wald. Sequential Tests of Statistical Hypotheses. *The Annals of Mathematical Statistics*, 16(2):117–186, June 1945. ISSN 0003-4851, 2168-8990. doi: 10.1214/aoms/1177731118. URL `https://projecteuclid.org/journals/annals-of-mathematical-statistics/volume-16/issue-2/Sequential-Tests-of-Statistical-Hypotheses/10.1214/aoms/1177731118.full`. Publisher: Institute of Mathematical Statistics.

A. Wald. *Sequential Analysis.* 1947. URL `http://archive.org/details/in.ernet.dli.2015.510091`.

A. Wald and J. Wolfowitz. Optimum Character of the Sequential Probability Ratio Test. *The Annals of Mathematical Statistics*, 19(3):326–339, Sept. 1948. ISSN 0003-4851, 2168-8990. doi: 10.1214/aoms/1177730197. URL `https://projecteuclid.org/journals/annals-of-mathematical-statistics/volume-19/issue-3/Optimum-Character-of-the-Sequential-Probability-Ratio-Test/10.1214/aoms/1177730197.full`. Publisher: Institute of Mathematical Statistics.

W. A. Wallis. The Statistical Research Group, 1942-1945. *Journal of the American Statistical Association*, 75(370):320–330, 1980. ISSN 0162-1459. doi: 10.2307/2287451. URL `https://www.jstor.org/stable/2287451`. Publisher: [American Statistical Association, Taylor & Francis, Ltd.].

Q. Wang, R. Wang, and J. Ziegel. E-backtesting, Sept. 2022. URL `https://papers.ssrn.com/abstract=4206997`.

R. Wang and A. Ramdas. False discovery rate control with e-values. *arXiv:2009.02824 [math, stat]*, Nov. 2020. URL `http://arxiv.org/abs/2009.02824`. arXiv: 2009.02824.

R. Wang and A. Ramdas. False Discovery Rate Control with E-values. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 84(3):822–852, July 2022. ISSN 1369-7412. doi: 10.1111/rssb.12489. URL `https://doi.org/10.1111/rssb.12489`.

R. A. Wijsman. Cross-sections of orbits and their application to densities of maximal invariants. *Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability, Volume 1: Statistics*, 5.1:389–401, Jan. 1967. URL `https://projecteuclid.org/ebooks/berkeley-symposium-on-mathematical-statistics-and-probability/Proceedings-of-the-Fifth-Berkeley-Symposium-on-Mathematical-Statistics-and/chapter/Cross-sections-of-orbits-and-their-application-to-densities-of/bsmsp/1200512999`. Publisher: University of California Press.

R. A. Wijsman. Proper Action in Steps, with Application to Density Ratios of Maximal Invariants. *The Annals of Statistics*, 13(1):395–402, Mar. 1985. ISSN 0090-5364, 2168-8966. doi: 10.1214/aos/1176346600. URL `https://projecteuclid.org/journals/annals-of-statistics/volume-13/issue-1/Proper-Action-in-Steps-with-Application-to-Density-Ratios-of/10.1214/aos/1176346600.full`. Publisher: Institute of Mathematical Statistics.

J. Wolfowitz. Abraham Wald, 1902-1950. *The Annals of Mathematical Statistics*, 23(1):1–13, 1952. ISSN 0003-4851. URL `https://www.jstor.org/stable/2236396`. Publisher: Institute of Mathematical Statistics.

J. Wu and X. Xiong. Group Sequential Survival Trial Design and Monitoring Using the Log-Rank Test. *Statistics in Biopharmaceutical Research*, 9(1):35–43, Jan. 2017. ISSN null. doi: 10.1080/19466315.2016.1189355. URL `https://doi.org/10.1080/19466315.2016.1189355`. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/19466315.2016.1189355.

Y.-S. Wu and Y. Seldin. Split-kl and PAC-Bayes-split-kl Inequalities for Ternary Random Variables. *Advances in Neural Information Processing Systems*, 35:11369–11381, Dec. 2022. URL `https://proceedings.neurips.cc/paper_files/paper/2022/hash/49ffa271264808cf500ea528ed8ec9b3-Abstract-Conference.html`.

T. Zhang. From Epsilon-Entropy to KL-Entropy: Analysis of Minimum Information Complexity Density Estimation. *The Annals of Statistics*, 34(5):2180–2210, 2006a. ISSN 0090-5364. URL `https://www.jstor.org/stable/25463505`. Publisher: Institute of Mathematical Statistics.

T. Zhang. Information-theoretic upper and lower bounds for statistical estimation. *IEEE Transactions on Information Theory*, 52(4):1307–1321, Apr. 2006b. ISSN 0018-9448. doi: 10.1109/TIT.2005.864439.