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## Spatial populations with seed-bank

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### Citation

Oomen, M. (2021, November 18). *Spatial populations with seed-bank*.  
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# Spatial populations with seed-bank

Margriet Oomen

The research in this thesis was supported by the NWO Gravitation Grant no 024.002.003-NETWORKS of Frank den Hollander

# Spatial populations with seed-bank

Proefschrift

ter verkrijging van  
de graad van doctor aan de Universiteit Leiden,  
op gezag van rector magnificus prof.dr.ir. H. Bijl,  
volgens besluit van het college voor promoties  
te verdedigen op donderdag 18 november 2021  
klokke 12.30 uur

door

Margriet Oomen  
geboren te Roosendaal  
in 1990

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