Boethius and the Importance of Basic Logic and Mathematics for Philosophy

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I. Introduction

Anicius Manlius Severinus Boethius (480-525 A.D.) is regarded as a great thinker of the early Middle Ages. Indeed, Boethius is a man of immense erudition and his works touch upon many philosophical fields, including mathematics, logic and theology. In Boethius' classification system, mathematics and theology belong to speculative philosophy, and logic is both a part of philosophy and an instrument of philosophy.[1] In his short life[2], he wrote some monographs and commentaries, which can be classified into four main groups: his works on mathematical disciplines including arithmetic, music, geometry, and astronomy[3]; his translations, commentaries, and monographs on Aristotelian logic and areas of rhetoric closely related to logic[4]; his five short theological treatises; and his last but most famous work, *Consolatio Philosophiae*[5].

Despite writing these works, Boethius is not regarded as an original thinker. Instead, his mathematical works and most of his logical works are regarded as translations, and even his monographs on logic are also considered to be little more than translation. His theological tractates and *Consolatio Philosophiae* are thought to patch up Neoplatonic materials. It seems that Boethius was nothing less than a transmitter of ancient thoughts to the Middle Ages. However, I will argue a different perspective on the issue of Boethius as an original thinker.

There are many translations and commentaries of Boethius' works

[1] Boethius discusses the division of philosophy both in his theological treatise on the Trinity and his commentary on Porphyry's *Isagoge*. Cf. *De trinitate*, II.5-21 and *2InIsag.*, 74C-D.

[2] On Boethius' life, cf. Coster (1968); Matthews (1981), "Anicius Manlius Severinus Boethius," and Kirkby (1981), "The Scholar and His Public," in Gibson (ed.), pp. 15-43 and pp. 44-69; Kaylor (2012), "Introduction: The Times, Life, and Work of Boethius," in Kaylor and Phillips (eds.), pp. 1-46; Marenbon (2003), pp. 7-16; Moorhead (2009), "Boethius' Life and the World of Late Antique Philosophy," in Marenbon (ed.), pp. 13-33; Morton (1981); Patch (1947); Stewart (1974), pp. 15-54.

[3] Cf. Section I.2.2.

[4] Cf. Section I.2.3.

[5] Cf. Section I.2.4.

(especially *Consolatio Philosophiae*[6]) in different languages, as well as several studies on Boethius. Nonetheless, Boethius could not be studied as a whole[7]. Boethius is learned, grasping knowledge of various subjects. However, scholars from different fields, such as historians, philosophers, theologians, and literary scholars, tend to focus only on the part of Boethius' thoughts that relate to their own disciplines.[8] The result is a lack of understanding Boethius as a whole. However, there are few scholars researching the relationship between Boethius' mathematics and logic or applications of them in his theology and *Consolatio Philosophiae*.[9] My dissertation will focus on such analysis.

In my dissertation, I will reassess Boethius' mathematics and logic, and their roles in philosophy, and on the basis of these, I will explore the

[6] Godden and Irvine (2009) edit two volumes of the old English versions of *Consolatio*. On translations and commentaries of *Consolatio*, cf. Brancato (2012), "Readers and Interpreters of the *Consolatio* in Italy, 1300–1500," and Johnson (2012), "Making the *Consolatio* in Middle English," in Kaylor and Phillips (eds.), pp. 357-412 and pp. 413-446; Cally (1996); Jefferson (1917); Kaylor (1992 and 2007). And Donato (2013a) reinterprets *Consolatio* in a striking new way. Donato investigates how the study of *Consolatio* can profit from the knowledge of Boethius' cultural, political and social background that is available today.

[7] Cf. Marenbon (2009), "Introduction: Reading Boethius Whole," in Marenbon (ed.). In that introduction, Marenbon gives two main reasons why Boethius is usually not read as a whole. The first one has nothing to do with Boethius, but points to the limitations of academic specialization; the second one is that Boethius is seen as an almost entirely unoriginal thinker, or regarded mainly as a sort of a conduit to the Middle Ages.

[8] On Boethius' logic, cf. De Rijk (1964a, 1964b, and 1988); Dürr (1951); Green-Pedersen (1984); Magee (1989, 1998, and 2010); Shiel (1957, 1958, 1974, and 1982); Stump (1974, 1978, 1981a, 1981b, 1987, and 1988). On Boethius' music, cf. Bower (1978, 1981, 1984, and 1989). On Boethius' Liberal Arts, cf. Masi (1979, 1981a, and 1981b). On commentaries on Boethius, cf. Häring (1966 and 1971); Scott (1993). On Boethius' literary, cf. Lerer (1985). On a concordance of Boethius, cf. Cooper (1928). On comparisons of Boethius with other people, cf. Arber (1942); Coster and Patch (1948); Ford (1968); Merlan (1968); Shiel (1957 and 1974). And Barrett (1940), Chadwick (1981b), Gibson (1981), and Marenbon (2003) give a full description of Boethius and his thoughts, but they, too, introduce Boethius according to separate fields without discussing the connections between these fields.

[9] Chamberlain wrote an article on music in Boethius' *Consolatio* from the perspective of literature; cf. Chamberlain (1970). Kijewska points out some connections between Boethius' mathematics and theology, especially *De trinitate* and *Consolatio* without detailed explanations; cf. Kijewska (2003), pp. 632-637. Schrade wrote an article on music in Boethius' philosophy; cf. Schrade (1947).

connections between mathematics and logic in Boethius and their applications to theological or philosophical topics. However, I will not focus on the applications of all his mathematical and logical knowledge but only on his basic ideas of arithmetic and music, and basic logic including knowledge of categories, theories of division and definition.[10]

Scholars have investigated the chronology of Boethius' work according to various methods. Although they have reached different conclusions on the chronology[11], they agree on a general order. Boethius first finished works on mathematics among which only works on arithmetic and music are extant.[12] After the study on mathematics, he started studying logic. Then he applied theories or examples of mathematics and theories of basic logic to his investigations into theological issues and in his *Consolatio Philosophiae*. Boethius studied Aristotle's categories, and using categorical logic, he discussed the Nature and Person of God. He usually defined a concept first and then put it to use in theology. He also focused explicitly on the arguments and reasoning that serve his discussions, which is particularly helpful in the interlocution with Lady Philosophy in the *Consolatio Philosophiae*.

My dissertation has three main parts. The first part (Chapter II) focuses on Boethius' elementary disciplines, logic and mathematics. I will give a short introduction to these two disciplines and their roles in Boethius' whole knowledge; then I will point out applications of each one to the other. The second and third parts (Chapter III and Chapter IV) trace applications of mathematics and basic logic in Boethius' theological treatises and in the *Consolatio Philosophiae* respectively. Before the three main chapters, the background of Boethius will be introduced first. In this chapter, I will give a description of the curriculum before Boethius and an introduction to Boethius' curriculum on mathematics, logic, theology, and his *Consolatio Philosophiae*.

[10] Cf. Section II.1.2, Section II.2.1.3, and Section II.2.2.3.
[11] Cf. Suto (2012), Chart 2; De Rijk (1964a and 1964b); McKinlay (1907); Phillips (2012), "Anicius Manlius Severinus Boethius: A Chronology and Selected Annotated Bibliography," in Kaylor and Phillips (eds.), pp. 551-590.
[12] Cf. Section I.2.2.

I.1. The Curriculum Before Boethius

The first work of Boethius focuses on arithmetic, *De Institutione Arithmetica*. The preface of this works is a letter to Symmachus[13] in which Boethius sets forth his plans. Boethius says that it was at Symmachus' request that he took it upon himself to make Latin readers acquainted with the riches of Greek culture, which may refer to Symmachus' long-term plan to bring over Greek writings to the Roman language.[14]

At the beginning of his work on arithmetic, Boethius says that he plans to finish works on arithmetic, music, geometry, and astronomy. Boethius gives the name "quadrivium" ("the four ways" or "a place where four roads meet") to these four mathematical disciplines. The quadrivium is regarded as a part of the study of Liberal Arts, and the other part is the trivium which was given the name in imitation of the quadrivium in the Carolingian era, including grammar, logic, and rhetoric. As a matter of fact, the disciplines in Liberal Arts changed many times before the Middle Ages.

Artes liberales (Liberal Arts) used by the Romans stemmed from the Greek term *enkuklios paideia* or *enkuklia mathemata*. The Greeks used *enkuklios paideia* or *enkuklia mathemata* for the subjects that should be learnt for three years after the age of fourteen (from this age, the education should be changed from primary to secondary).[15] The Liberal Arts were taken over from Greek ideas of education with very little change. Greek ideas of education were referred to those subjects, which in classical antiquity were considered essential for a free citizen to study. The persons who were most interested in the full span of subjects were philosophers. Therefore, the curriculum of Liberal Arts was taught only by philosophers, which is demonstrated in the educational programs of Greek philosophers

[13] Quintus Aurelius Memmius Symmachus (died 526 A.D.) was a 6th-century Roman aristocrat and a historian. He was a patron of secular learning and became the consul for the year 485. He supported Pope Symmachus (cf. the preface to Chapter III in my dissertation) in the schism over the Popes' election and was executed with his son-in-law Boethius after being charged with treason. Cf. McGeachy (1942).
[14] Cf. *Arithmetica*, preface; Reiss (1982), p. 12.
[15] Cf. Clarke (1971), p. 2.

Pythagoras (fl. 530 B.C.), Isocrates (436-338 B.C.), and Plato (429-347 B.C.).

Among Liberal Arts, four mathematical disciplines had first been taught first by the Pythagoreans[16]. Pythagoras found the mathematical proportion between intervals, which he used to establish the mathematical foundation for music, and he classified music as one of mathematical disciplines. It is Pythagoras, Proclus (about 411-485 A.D.) believes, who "transformed mathematical philosophy into a scheme of liberal education."[17] And a Pythagorean mathematician Archytas (428-347 B.C.) concludes, at the beginning of his *Harmonics*, that "indeed concerning geometry and arithmetic and sphaeric they handed down to us a clear set of distinctions and not least also concerning music. For these sciences seem to be akin."[18] Thus, he believed that these four sciences had been derived from mathematicians, or, in other words, that these four sciences were subcategories of mathematics. Strictly speaking, because of Pythagoras, mathematics became an intellectual study and had its place among the Liberal Arts. Therefore, Pythagoras should be regarded as the founder of the quadrivium. The real founder of the trivium in Liberal Arts should be the Sophists. During about 460 B.C. to 380 B.C., Sophists began the study of dialectic, grammar, and the technical study of language, of the etymology and usage of words.

In Antiquity, the importance of mathematics was believed to rest on its usefulness. This view came from Isocrates, who says that "rather it seems to me both that those who hold, that this training is of no use in practical life are right and that those who speak in praise of it have truth on their side."[19] He regarded mathematics as "a gymnastic of the mind and a preparation for philosophy."[20] Isocrates taught mathematics, physical science, history, and

[16] On Pythagoreans, cf. Kahn (2001); O'Meara (1989); Zhmud (2012). On Iamblichus' Platonic curriculum, cf. O'Meara (2003), pp. 62-65.
[17] Proclus, *A Commentary on the First Book of Euclid's Elements*, II.iv.65; Morrow (1970). Cf. Heath (1956), pp. 65-117 on Pythagorean achievements in arithmetic; and pp. 141-169 on Pythagorean geometry.
[18] Archytas, *Harmonics*; Huffman (2005), p. 109. Cf. Huffman (2005), pp. 126-127 are on Archytas and the sciences.
[19] Isocrates, *Antidosis*, 263-264; Norlin (1929).
[20] Isocrates, *Antidosis*, 266; Norlin (1929).

other disciplines, but all of such disciplines are considered as preliminary elementary knowledge, and the final purpose is the study of rhetoric and the participation in ruling the state.[21] Not unlike Isocrates, Plato advises in his *Republic* that the rulers of the city-state must receive the education of physical exercise and poetry when they are young, and after they grow up to be adults, trainee philosopher kings should learn arithmetic, geometry, solid geometry, astronomy, and harmonics to surpass the sensible world and enter the abstract intelligible world.[22] We can see the connection between the Liberal Arts and his program of education.

However, the content of Liberal Arts was not fixed. Later, Marcus Terentius Varro (116-27 B.C.), an ancient Roman scholar and writer, added medicine and architecture which were expounded in his *Disciplinarum Libri IX*, so Liberal Arts included nine disciplines. The earliest one who had a clear idea about seven Liberal Arts is Martianus Capella (fl. 5th century A.D.), who wrote a work, *De Nuptiis Philologiae et Mercurii* (*On the Marriage of Philology and Mercury*)[23]. As a matter of fact, between Varro and Capella, there is no evidence that any handbook of these seven Liberal Arts was written, until Capella's contemporary Augustine of Hippo (354-430 A.D.) started one. Later in Roman history, the seven Liberal Arts took a more permanent shape, and included grammar, dialectic (or logic), rhetoric, arithmetic, music, geometry, and astronomy. Augustine planned to write works on all these seven Liberal Arts, but in the end he only finished the works on grammar, rhetoric, dialectic, some works on geometry, and six books on music. Augustine believed the number seven is related to the Holy Spirit. Thanks to the authority of Augustine and the mystery of the number seven, "seven Liberal Arts" were inherited and developed. "Seven" became the standard number of Liberal Arts. A century later, in the second part of *Institutiones Divinarum et Saecularium Litterarum* (*Institutes of Divine and Secular Learning*)[24], Cassiodorus (about 485-580 A.D.) introduces grammar, rhetoric, dialectic, arithmetic, music, geometry, and astronomy. He believed the Liberal Arts as secular learning

[21] Cf. Isocrates, *Antidosis*, 261-268; Norlin (1929).
[22] Cf. Plato, *Republic*, 521e-531d; Waterfield (1993).
[23] Cf. Stahl, Johnson, and Burge (1971 and 1971-1977); Sharples (1991).
[24] Cf. Cassiodorus, *An Introduction to Divine and Human Readings*; Jones (1946).

were not opposed to divine learning, and that the disciplines of the Liberal Arts were part of the content of the Bible. Therefore, Cassiodorus encouraged studying the seven Liberal Arts. Because of him, the seven Liberal Arts received a sacred and unshakable authoritative position.

So in the Middle Ages, "Liberal Arts" referred to seven arts, i.e. arithmetic, geometry, music, and astronomy, which are called the quadrivium; and Grammar, Rhetoric, and Logic, which are called the trivium. The initial medieval university curriculum was constituted of the trivium which dealt with language, and the quadrivium which dealt with mathematical disciplines. In the Middle Ages, these seven Liberal Arts became the preparatory curriculum and the necessary way to the higher wisdom (philosophy and theology) and were taught in the schools. Before the Renaissance, most Greek works were seldom known by people, and Western Europe could get only limited knowledge about ancient civilization. Hence the seven Liberal Arts were regarded as the sum total of all human secular knowledge.

I.2. Boethius' Curriculum

In Boethius' curriculum[25], the seven Liberal Arts are all involved. For example, he wrote works on four mathematical disciplines, and he also wrote, interpreted, or commented on logic, also in its relation to rhetoric. However, Boethius devoted most of his time on logic and four mathematical disciplines. The intellectual environment in which Boethius lived influenced the subjects on which he focused.

I.2.1. Intellectual Environment in Boethius' Times

During the centuries before Theodoric the Great[26], the number of disciplines in Liberal Arts was fixed at seven, including grammar, rhetoric, dialectic, arithmetic, music, geometry, and astronomy. Among them, the

[25] Cf. Reiss (1982, pp. 9-14) discusses a program of education about Boethius.

[26] Theodoric (454-526 A.D.) was king of the Germanic Ostrogoths (471-526 A.D.), ruler of Italy (493-526 A.D.), regent of the Visigoths (511-526A.D.), and a viceroy of the Eastern Roman Empire.

principal one was rhetoric. In Athens and in Rome, rhetoric was essential to men who wanted to pursue political careers. The needs of law and government dominated largely in the cultural tradition of Rome, and rhetoric could provide men with the skills relevant to speaking in law courts and in political assemblies. Thus, it was necessary for those men who wanted a political career to learn rhetoric. Subordinate to rhetoric was grammar, which was preparatory to rhetoric. Other disciplines in Liberal Arts tended to be pushed out.

Although in the Greek-speaking world the Liberal Arts continued to play their role in education, the situation in the Latin-speaking West was different.[27] The Latin world possessed good guides in grammar and in rhetoric but was less well served in the other Liberal Arts. A typical example is Augustine. There were many books on grammar and rhetoric, but no books on other Liberal Arts. The absence of textbooks suggests an absence of teacher; as a result, Augustine had to acquire his knowledge of other Liberal Arts, such as logic and mathematics without the help of any teacher.[28] The example of Augustine suggests that among these seven Liberal Arts, rhetoric and grammar were paid most attention, and were taught by teachers. It is true that the other Liberal Arts, i.e. logic and mathematics, passed to Rome from the Hellenistic world and were still recognized as part of education. However, they were almost neglected in practice.

As a philosopher, Boethius regarded it as part of his vocation to share the richness of Greek thoughts with the Latin speakers. And the weakness of the mathematical disciplines and logic in the Latin world stimulated Boethius' interest, so he dedicated himself to arithmetic, music, geometry, astronomy and the higher studies of philosophical logic.

There is an order in Boethius' curriculum on mathematics and logic, which is affected by the philosophical situation at his time. In other words, the study order of Boethius is in line with that of the Neoplatonic schools.

Ammonius Saccas (3rd century A.D.) tried to harmonize the doctrines of

[27] Cf. Clarke (1971), p. 7.
[28] Augustine, *Confessions*, IV.16.30; Schaff (1886).

Plato and Aristotle, and from his time, the Platonic school tended to absorb Aristotelian doctrine, and it also absorbed the Pythagoreans. Later, a significant development of philosophy happened with the pioneers, Plotinus (about 205-270 A.D.) and his pupil Porphyry (234-305 A.D.). To modern scholars, this movement was known as Neoplatonism, although people of the time referred to its exponents simply as Platonists. In the Neoplatonic tradition, thinkers focused on reworking Plato's ideas, but they began their curriculum of teaching with explanations of some of Aristotle's works.[29] Influenced by Porphyry[30] and other Neoplatonists, Boethius made a great plan to harmonize Plato and Aristotle. At the beginning of Book 2 of the second commentary on Aristotle's *On Interpretation*, Boethius gives a clear exposition of his purpose.

> "It is my fixed purpose, if the favour of the godhead should approve more firmly, to translate into the Roman style and write commentaries on the whole Aristotelian corpus that has come into my hands. ... And so anything written by Aristotle which results from his subtlety in logic, serious mindedness in moral experience and sharp-wittedness in natural truth, all of this I will translate in proper order and illuminate, too, with what light a commentary affords. And by translating all the dialogues of Plato and also commenting on them I will bring them into a Latin form. When this is achieved I would not shrink from somehow bringing the ideas of Plato and Aristotle into a single harmony and proving that they do not disagree in everything as most think but that they agree in most things and in the most important philosophical issues. ... When this is achieved I would not shrink from somehow bringing the ideas of Plato and Aristotle into a single harmony and proving that they do not disagree in everything as most think but that they agree in most things and in the most important philosophical issues." (*2InInter.*, 58)

Although Boethius did not finish his great plan, he carried on the ideas of

[29] Cf. Moorhead (2009), "Boethius' Life and the World of Late Antique Philosophy," in Marenbon (ed.), p. 25.
[30] Cf. Section I.2.3.

his plan in his works. Boethius chose to follow Porphyry and accepted his approach to the interpretation of Aristotelian logic. In his point of view, the subject matters of Aristotle and Plato are different, and there is no conflict between them.[31] For example, in his theological treatise against Eutyches and Nestorius, his treatment of the word "person" varies according to the degree to which Aristotelian language about primary or individual substance is enfolded within a Platonic metaphysic of universals.[32] In his consolation, he also tries to harmonize Aristotle and Plato, which is demonstrated in his definition of man as the combination of the definitions of Aristotle and Plato.[33]

In Plotinus' point of view, the philosopher "must be given mathematical studies to train him in philosophical thought and accustom him to firm confidence in the existence of the immaterial," which means that the first stage of education should be the mathematical disciplines. And after mathematics, the mind must acquire skill in dialectical methods.[34] Accordingly, Boethius started his project with four mathematical disciplines, i.e. arithmetic, music, geometry, and astronomy, and then the study on logic.

I.2.2. Boethius on Mathematics

Among four works on mathematics, only two works on arithmetic and music are extant,[35] which are *De Institutione Arithmetica* (*Principles of*

[31] Cf. Marenbon (2003), p. 41.
[32] Cf. Section III.2.1.2.
[33] Cf. Section IV.2.2.2.
[34] Cf. Plotinus, *Enneads*, I.3; Armstrong (1966).
[35] According to Cassiodorus, Boethius completed his work on geometry, which is the translation of Euclid' work; cf. Cassiodorus, *An Introduction to Divine and Human Readings*, II.vi.3; Jones (1946). From the letter of "Theoderic to the Illustrious Patrician Boethius", Cassiodorus suggests that Boethius finished his works on quadrivium. "For it is in your translations that Pythagoras the musician and Ptolemy the astronomer are read as Italians; that Nicomachus on arithmetic and Euclid on geometry are heard as Ausonians [Italians]; that Plato debates on metaphysics and Aristotle on logic in the Roman tongue; you have rendered Archimedes the engineer to his native Sicilians in Latin dress. (Cassiodorus, *Variae*, I.45.4; Barnish, 1992/2006)" Cf. Chadwick (1981b), p. 103; Stahl (1962), pp. 196-197. On Boethius' works on quadrivium and their influence, cf. Chadwick (1981b), pp. 102-107; Caldwell (1981), "The *De Institutione Arithmetica* and the *De Institutione Musica*," and Pingree (1981), "Boethius' Geometry and Astronomy," and White (1981), "Boethius in the Medieval Quadrivium," in Gibson (ed.), pp. 135-154, and pp.

Arithmetic)[36] and *De Institutione Musica* (*Principles of Music*)[37]. The principal person related to these two works is Nicomachus of Gerasa (60-120 A.D.).

Nicomachus is regarded as a Pythagorean, and Pythagoreanism to him is not simply a mathematically based philosophy. He believes that there is an apparent harmony in the cosmos, and he synthesizes mathematical theories with this belief, which he considers as binding together human soul and human body.[38] Only two works of Nicomachus are extant, viz. *Introduction to Arithmetic* and *Manual of Harmonics*, which have important influences on Boethius' thoughts.

Introduction to Arithmetic is Nicomachus' most famous work, which is of much use to introducing the discipline of arithmetic, as its name suggests. This work is significant for arithmetic to become an independent discipline. Kline gives a proper evaluation of this work.

> "The *Introductio* had value because it was a systematic, orderly, clear, and comprehensive presentation of the arithmetic of integers and ratios of integers freed of geometry. It was not original as far as ideas were concerned, but was a very useful compilation. It also incorporated speculative, aesthetic, mystical, and moral properties of numbers, but no practical applications. The *Introductio* was the standard text in arithmetic for a thousand years. At Alexandria, from the time of Nicomachus, arithmetic rather than geometry became the favorite study."[39]

155-161, and pp. 162-205; Evans (1975 and 1981); Guillaumin (2012), "Boethius's *De institutione arithmetica* and its Influence on Posterity," and McCluskey (2012), "Boethius's Astronomy and Cosmology," and Moyer (2012), "The Quadrivium and the Decline of Boethian Influence," and Rimple (2012), "The Enduring Legacy of Boethian Harmony," in Kaylor and Phillips (eds.), pp. 135-162, and pp. 47-74, and pp. 479-518, and pp. 447-478; Masi (1979, 1981a, and 1981b); Reiss (1982), pp. 14-27.

[36] In the following chapters, I will use *Arithmetica* as the short name of Boethius' work on arithmetic. The translations of *Arithmetica* I cite in my dissertation are Masi's (1983).

[37] In the following chapters, I will use *Musica* as the short name of Boethius' work on music. The translations of *Musica* I cite in my dissertation are Bower's (1989).

[38] To Nicomachus, this is a synthesis of science and religion which has merit. Cf. Chadwick (1981b), p. 72.

[39] Kline (1972), p. 138.

Manual of Harmonics is a brief work that can be considered as a minimal introduction to Pythagorean musical thoughts. In this work, he promises that when he has leisure time and a rest from his journey, he would compose "a longer and more detailed" musical treatise, *Introduction to Music*.[40] As it is, we do not have the *Introduction to Music*, either because Nicomachus did not finish it, or because it was not preserved.

At the beginning of his *Introduction to Arithmetic*, Nicomachus states the standard Neopythagorean order of four mathematical disciplines — arithmetic, music, geometry, and astronomy — which he called *methodoi,* meaning paths or methods for proceeding upwards in a steady progress towards higher knowledge and wisdom. At the beginning of his *Arithmetica* Boethius gives an exposition about why the first discipline to be learned is arithmetic, next music, then geometry, and last astronomy — following Nicomachus. The word "quadrivium[41]" that Boethius has coined, has a similar meaning to *methodoi*, and throughout the Middle Ages, "quadrivium" was the name for the four mathematical disciplines.

It is easy for people to understand that arithmetic and geometry are mathematical disciplines, which is also the modern idea on mathematics. In Antiquity, arithmetic was a discipline that dealt with the theory of number, which means it studies multitude in itself. Geometry, after the day of Euclid (fl. 300 B.C.), would follow in Euclid's footsteps: it began with definitions, then set out certain postulates and axioms, and finally proceeded to prove a series of theorems; geometry studies immovable magnitude. Unlike arithmetic and geometry, astronomy might seem to be different, but essentially they are similar, for astronomy studies movable magnitude. It is not hard to include these three disciplines in mathematics, but it is a little harder to understand that music belongs to mathematical disciplines. The learning of music included two aspects: theoretical and practical. Aristotle stressed musical education which is concerned with the practical aspect of music. When regarding music as one of Liberal Arts, however, one considers theoretical aspects of music. Music, according to the ancient theorists, had two sides. One side of music is rhythm (including metric),

[40] Nicomachus, *Manual of Harmonics*, I; Levin (1994).
[41] Cf. Section I.1.

which was taken over for teaching aims by grammarians in later antiquity. The second side of music is melody, and in later antiquity, this side was left to the musicians who discussed notes, intervals, scales, and the characteristics of the different modes.[42] And all of these subjects in music were studied together with an introduction in arithmetic, especially the proportional principles which were the essential for understanding musical theories. This relation shows that the study of music in Antiquity depended on number theory, or in other words, music became one of mathematical disciplines.

The mathematical disciplines play a vital role in Boethius' philosophy, which derives from the Platonic tradition on mathematics. Plato believes that pure numbers can elevate the soul to understand its own nature. The four mathematical disciplines which all involve the study of number can train the mind to understand truth and reality and then make the mind transcend the physical world of sense-perception.[43] In this way Plato regards mathematics as preparatory to, or forming part of philosophy.

In Boethius' point of view, the quadrivium is an indispensable preliminary to both the investigation of the physical world and the purification of the human soul that is necessary to comprehend the divine. Therefore, by the quadrivium a superior mind is brought from knowledge of things in the world offered by the senses, to the more certain things of the intellect, and is prepared for abstract reasoning. He also summarizes Nicomachus' quotation from *Republic*, 527d-e[44] that "There are various steps and certain dimensions of progressing by which the mind is able to ascend so that by means of the eye of the mind, which (as Plato says) is composed of many corporeal eyes and is of higher dignity than they, truth

[42] Cf. Clarke (1971), p. 54.

[43] Plato, *Republic*, 524e-531d; Waterfield (1993).

[44] Cf. Nicomachus, *Introduction to Arithmetic*, I.III.7; D'Ooge (1938). The original passage of Plato reads: "You amuse me: You're like someone who's afraid that the majority will think he is prescribing useless subjects. It is no easy task — indeed it is very difficult — to realize that in every soul there is an instrument that is purified and rekindled by such subjects when it has been blinded and destroyed by other ways of life, an instrument that it is more important to preserve than ten thousand eyes, since only with it can the truth be seen. (Plato, *Republic*, 527d-e; Cooper, 1997)" Cf. Theon of Smyrna quotes the same passage in *Mathematics Useful for Understanding Plato*, I.1; Lawlor (1979).

can be investigated and beheld. This eye, I say, submerged and surrounded by the corporal senses, is in turn illuminated by the disciplines of the *quadrivium*." (*Arithmetica*, I.1) Boethius clearly expresses that if anyone wants to reach the highest perfection of the disciplines of philosophy, he must dwell on the nobility of such wisdom in the quadrivium, which will hardly be hidden from those properly respectful of expertise. If someone lacks knowledge of the quadrivium, he cannot find the truth, so on this conception of the quadrivium, truth cannot be rightly known without it. Boethius believes the quadrivium is the path to wisdom; philosophy is the love of wisdom, so he who spurns the quadrivium shows contempt for philosophy (*Arithmetica*, I.1).

An illustration of the importance of mathematics to man comes from Boethius work on division, in which the relationship between man and mathematics is taken as an example to explain what the meaning of "conceptually inseparable" is.[45] Boethius shows that mathematics is not inherent in man and does not belong to the class of differentiae; for it is not the ability to do mathematics, but still it cannot be separated from man. Man can use mathematics, and "if this capacity should be removed from man, then man himself no longer remains" (*Divisio*, 881c). Therefore, we can see how vital mathematics is to man.

Mathematical works constitute the first of Boethius' works. His works on the quadrivium focus on "numbers", "propositions", and "harmony" which are close to the research of philosophy and theology. Of these mathematical works, only the *Arithmetica* and an incomplete *Musica* survive.[46] *Arithmetica* deals with pure mathematics; *Musica* and the works on geometry and astronomy touch upon the applied theory of numbers. By the study on numbers, Boethius learned the intrinsic harmony, which served a purpose in philosophy and theology, that is to say, what he focused on is not the mathematical meaning of numbers, but their symbolical or philosophical meaning.

[45] Cf. Section II.1.1.2.a.

[46] Because only these two survive, in my dissertation when I discuss applications of mathematics, I refer to applications of arithmetic and music, and sometimes I refer to geometrical methods.

I.2.3. Boethius on Logic

As I said in Section I.2.1, Boethius devoted himself to two disciplines in his curriculum: mathematics and logic. Logic, in the Platonic tradition, is originally called "dialectic", and the difference between "logical" and "dialectical" reasoning was not marked by the Neoplatonists of Athens and Alexandria, nor by Boethius.[47] The modern sense of "logic" derives from the Peripatetic Alexander of Aphrodisias (fl. 200 A.D.).[48]

Concerning the question what logic itself should be, there are two main opinions: according to one logic is an instrument of philosophy; according to the other logic is a part of philosophy.[49] The important persons holding the first opinion are the Aristotelians, who follow Aristotle's *Topics,* 163b9-11[50], in regarding logic as a practical instrument for the discovery of fallacies in argument on any subject, and as an indispensable tool for every department of human inquiry. Hence they called logic an instrument (*organon*). Alexander of Aphrodisias denies the view that logic is a part of philosophy by saying that logic differs from theoretical philosophy and practical philosophy in subject-matter, and in its end and purpose. Alexander draws an analogy between logic and a hammer and anvil to show that logic is an instrument.[51] Important representatives of the second opinion are the Stoics. In their point of view, logic was a part and an independent branch of philosophy.[52]

Boethius states these two kinds of view in his second commentary on Porphyry's *Isagoge*. And after listing these two opinions, Boethius comes to

[47] Cf. Chadwick (1981b), p. 108.
[48] Cf. Kneale (1984), p. 7.
[49] Cf. Chadwick (1981b), pp. 108-111; Sorabji (2004b), pp. 32-36. Also cf. Ierodiakonou (1997); Mueller (1969).
[50] Aristotle says that "to take and to have taken in at a glance the results of each of two hypotheses is no mean instrument for the cultivation of knowledge and philosophic wisdom." Cf. Aristotle, *Topica*, 163b9-11; Forster (1938).
[51] Cf. Alexander of Aphrodisias, *On Aristotle Prior Analytics I.1-7*, 1.7-3.29; Barnes [etc.] (1991). Also see in Olympiodorus' commentary on Aristotle's *Categories*: like Alexander, Olympiodorus claims that logic has neither the same subject matter nor the same purpose as philosophy, so logic is not a part of philosophy, but an instrument; cf. Sorabji (2004b), p. 35.
[52] Cf. Sorabji (2004b), p. 32.

his conclusion about the role of logic in philosophy.

> "The same logic form serves at the same time the function of part and of instrument. For since it retains its own end, and this end is considered by philosophy only, it must be asserted to be a part of philosophy, but since that end of logic, which philosophy alone contemplates, promises its aid to the other parts of philosophy, we do not deny that it is the instrument of philosophy; but the end of logic is the discovery and judgment of reasons." (*2InIsag.*, 74C-D)[53]

Boethius regards logic as a significant discipline in his philosophy. In his second commentary of Porphyry's *Isagoge*, Boethius stresses that reason can show the right way according to which the incorrupt truth of reality cannot be found, and logic is the discipline which can discern truth from falsity, therefore, those who reject logic are bound to make mistakes (*2InIsag.*, 73A). This means that logic is so important that it should not be neglected.

Boethius gives priority to logic and tries to write double commentaries on Aristotle's logical works while also writing monographs on logic.[54] The principal source of his logical works is the Neoplatonist Porphyry.[55] The most well-known work of Porphyry is the *Isagoge* which discusses five key terms: genus, species, differentia, property and accident. Porphyry intended it to throw light on Aristotle's categories and on definition, division and demonstration. His *Isagoge* is designed as an introduction to Aristotle's *Categories*, and the five predicables discussed in *Isagoge* are the more general types of predicate. After the introductory work, Porphyry also wrote two commentaries on *Categories.* One of Porphyry's commentaries is a short one, which survives in part, and the other is a long one, which survives only in fragments. The influence of Porphyry penetrates in Boethius' works.

[53] In his commentary on Aristotle's *Prior Analytics*, Ammonius states that Plato uses logic as both a part and an instrument; cf. Sorabji (2004b), p. 35. And Plotinus maintains that dialectic is "the precious part of philosophy." In *The Enneads*, Plotinus says that "We must not think of it as the mere tool of the metaphysician: Dialectic does not consist of bare theories and rules: it deals with verities." Cf. Plotinus, *The Enneads*, I.3.5; MacKenna (1991).

[54] Cf. Barnes (1981), "Boethius and the Study of Logic," in Gibson (ed.), pp. 73-89.

[55] Cf. Chadwick (1981b), pp. 120-127.

The most complete one is Boethius' commentaries on Porphyry's *Isagoge*. In Boethius' other logical works, he refers to Porphyry more than once. At the beginning of his commentary on Aristotle's *Categories*, Boethius points out that he proposes to follow Porphyry's commentary on the *Categories*. And in his second commentary on Aristotle's *On Interpretation*, Boethius again says that "We have arranged our explication of this work in Latin following Porphyry as far as possible, although we have included material from others too. For Porphyry, our guide, seems pre-eminent in intellectual sharpness and ability to marshal in his ideas." (*2InInter.*, 7.5-8)

In the trivium, Boethius focuses on the study of logic. He regards logic as the leader of the procession of the trivium. He believes that logic is more basic than grammar and rhetoric. The untenable things in logic are far from grammar and rhetoric. However, it does not mean that he thinks little of grammar and rhetoric; on the contrary, people generally admit that Boethius' works are more literary than Cicero's. Boethius' logical works can be divided into three parts: the first part consists of the translations of the works of Aristotle and Porphyry, which are not preserved; the second part consists of the commentaries on logical works, including *In Isagogen Porphyrii Commenta* (*editio prima* and *editio secunda*), *In Categorias Aristotelis*, *In Aristotelis De Interpretatione* (*editio prima* and *editio secunda*), and *In Ciceronis Topica*; the third part consists of his independent works in which he adopts and develops the logic of his predecessors. These works include *De Divisione*, *Introductio ad Syllogismos Categorico*, *De Syllogismo Categorico*, *De Syllogismo Hypothetico*, and *De Topicis Differentiis*.[56]

Logic and mathematics are regarded as elementary disciplines for Boethius, which are preparatory for the study of philosophy. Thus, among

[56] The English titles of these logical works are: *Commentaries on Isagoge* (1 and 2) ("*1InIsag.*" and "*2InIsag.*" for short); *Commentary on Airstotle's Categories* ("*In Categorias*" for short); *Commentaries on On Interpretation* (1 and 2) ("*1InInter.*" and "*2InInter.*" for short); *Commentary on Cicero's Topics* ("*C.Topica*" for short); *On Division* ("*Divisio*" for short); *Introduction to Categorical Syllogisms* ("*I.S.Categorico*" for short); *On the Categorical Syllogism* ("*S.Categorico*" for short); *On Hypothetical Syllogisms* ("*S.Hypothetico*" for short); *On Topical Differentiae* ("*TopicisD.*" for short). On the translations of these works I cite in my dissertation, cf. Section II.1.

Boethius' curriculum, I will first introduce his logic and mathematics, which will be discussed in Chapter II.

I.2.4. Boethius' Theological Treatises and *Consolatio Philosophiae*[57]

In addition to works on mathematics and logic, Boethius also wrote five theological treatises, viz. *De trinitate*, *Utrum pater et filius et spiritus sanctus de divinitate substantialiter praedicentur*, *Quomodo substantiae in eo quod sint bonae sint cum non sint substantialia bona*, *De fide catholica*, and *Contra Eutychen et Nestorium*. Nevertheless, he is not a theologian[58], and the reason he devotes himself to Christian doctrine is related to the political situation in his day, which will be discussed in Chapter III. As a philosopher, Boethius composed his theological tractates, like his mathematical and logical works, from the perspective of philosophy. However, since mathematics and logic are regarded as elementary disciplines, we may expect there to be applications of elementary mathematics and basic logic in his theological treatises, into which I will investigate in Chapter III. Applications of elementary mathematics and basic logic can also be found in his last and most famous work, the *Consolatio Philosophiae*, which I will discuss in Chapter IV.

[57] The English titles of these five treatises are: *The Trinity is One God not Three Gods*; *Whether Father, Son, and Holy Spirit may be Substantially Predicated of the Divinity*; *How Substances can be Good in Virtue of Their Existence without Being Absolute Goods*; *On the Catholic Faith*; *A Treatise Against Eutyches and Nestorius*. In later chapters, I will use short Latin names for each work, viz. "*De trinitate*", "*Utrum pater*", "*Quomodo substantiae*", "*De fide*", and "*Contra Eutychen et Nestorium*". And in later chapters, I will use *Consolatio* for short of *Consolatio Philosophiae*. The translations of the five treatises and the *Consolatio* I cite in my dissertation are Stewart's (1926).

[58] On the reason why Boethius is not a theologian, see Section III.5.