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Theory-enriched practical knowledge in mathematics teacher education

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2 Theory and practice in teacher education

2.1 Introduction

Over the last decades, the problem of theory versus practice in teacher education has increasingly become of interest. Before, the topic was highlighted in particular by Dewey (1933), who distinguished ‘reflective action’ and ‘routine action.’ In the 1980s, there was renewed interest for this topic through the work of Donald Schön (1983). His ideas and conceptions – not primarily concerned with teachers – are among those that have contributed to researchers and teacher educators becoming aware that professionals rarely simply ‘apply’ theory in their practice. A teacher decides on the basis of all kinds of situation-related components. Theoretical knowledge and insight do play a part, but they do not unambiguously determine the behavior of the teacher (Schön 1983, 1987).

Schön mentions the ‘reflective practitioner’ as someone who is able to consider his practice reflectively, not only before and after, but also *during* the performance of that practice (reflection in action). There is an extensive literature relevant to Schön’s ideas, gradually also followed by critical response (e.g., Gilroy, 1993; Eraut, 1995a). Other shifts of accents in the last few years have influenced the theory versus practice discussion. The focus on the (prospective) teacher’s thinking process and beliefs characterizes the changes in research on teaching. This focus originates from the idea that the behavior of the teacher can only be understood well, if the cognitions and conceptions that guide this behavior are also taken into consideration. Along with content knowledge, pedagogical content knowledge and general pedagogical knowledge, practical knowledge is seen as an important component of the knowledge base that underlies all actions by teachers (Elbaz, 1983; Carter, 1990; Verloop, 1992).

Teacher training colleges have come to realize that prescriptive transfer of theory is not enough (Brouwer, 1989). At the same time it has become clear that the content itself failed to meet expectations; theory was insufficiently in step with reality and with the complexity of action in practice (Cohen, 1998; Coonen, 1987, p. 243; Corporaal, 1988, p.13; Drever & Cope, 1999; Verloop, 2003, p. 203). Furthermore, student teachers are confronted with different types of ‘theory’ in their practice schools – through their supervisors’ exemplary role (Zanting, 2001). The extent to which the activities of students match the goals of training will partly depend on the level and type of cooperation between training institute and practice school (Emans, 1983; Watts, 1987; Wubbels, Korthagen & Brekelmans, 1997).

It is clear that practical training of student teachers is a factor in the tension between theory and practice. On the one hand both teacher educators and student teachers consider practical training to be an effective way to acquire (practical) knowledge, on

the other hand it is claimed that the realization of teacher education goals – also in terms of integrating theory and practice – is occasionally impeded by the conformist and conservative influence that practical training can have on student teachers (Zeichner et al., 1987). That influence can be a disadvantage for strongly practice-oriented teacher training. There is still another disadvantage to the practice-directed approach. The one-sided focus on school practice leads to insufficient depth in the reflective competence of student teachers (Coonen, 1987).

In the course of the next sections we go from a more general analysis of the concepts of theory and practice in teacher education to a more specific focus on these concepts within the context of mathematics teacher education.

2.2 Orientations in teacher education programs

Over the last few years, research into the relationship between theory and practice in teacher training has focused on the question of how student teachers can integrate theory and practice and in which sense the design of the learning environment can contribute to that integration. However, no unambiguous conception of theory exists, nor of practice or the relationship between the two. In the context of the discussion about relating practical and propositional knowledge, Thiessen (2000) distinguishes three orientations that have been emphasized in teacher education over the last 40 years:

- ‘impactful behaviors,’ leading to the training of prospective teachers in behaviors that appeared to be effective in process-product research;
- ‘reflective practices’ and,
- ‘development of professional knowledge.’

The three orientations should not be seen as mutually exclusive, all are more or less recognizable in current programs. The ‘impactful behaviors’ orientation dominated in the 1970s. Particularly according to the initial teaching preparation programs, this orientation appeared to be unsuccessful in linking student teachers’ theoretical and practical knowledge. Gradually the awareness grew that in order to understand the behavior of the teacher, cognitions have to be considered as well (Clark & Peterson, 1986). The ‘reflective practices’ orientation emerged in the 1980s, after increasing criticism on the empirical base underlying the ‘impactful behaviors’ orientation. According to Thiessen, the reflective practices orientation concentrates on skills which help beginning teachers think through what they have done, are doing or are about to do (Thiessen, 2000, p. 520). In his view, while there are numerous published reports on program innovations in support of the reflective practices orientation, the conceptual rigor and empirical foundation of this work are uneven and less developed. Zeichner (1994) presented an analysis of the different conceptions within this orientation. He distinguished five traditions of reflective practice for teaching and teacher education: the Academic, the Social Efficiency, the

Developmental, the Social Reconstructionist, and the Generic Reflection Tradition. Though intended for the U.S., Zeichner recognizes his framework of traditions of reflective practice in other countries as well. Reflective practice is still an important orientation in many teacher education programs, although this approach is also criticized by different authors. For example, Eraut (1995a) posits that a (prospective) teacher is often faced with lack of time to reflect in action, because of the necessity to react immediately (cf. Dolk, 1997). Furthermore, a danger is that reflections remain superficial through lack of – subtly ‘fed’ – adequate theoretical knowledge (Kennedy, 1992; Oonk, 2001). Another problem is the (tacit) interpretation of the different concepts. Terms such as reflective practice and reflection in action encompass some notion of reflection in the process of professional development, but at the same time disguise conceptual variations that have implications for the design and organization for teacher education courses (Calderhead, 1989; Boerst & Oonk, 2005).

The third orientation – the ‘development of professional knowledge’ – that Thiessen (2000) mentioned, is the most recent one. He claims that this orientation is the most promising for teacher education. In his view – considering the image of teaching as ‘knowledge work’ – the emphasis on concurrent use of practical and propositional knowledge distinguishes this orientation from the impactful behaviors and the reflective practices orientations. He argues that student teachers should experience “the concurrent use of knowledge in each pedagogical phase and context – on campus through strategies which focus on practically relevant propositional knowledge and in schools through strategies which focus on purposeful, defensible practice (i.e. propositionally interpreted practical knowledge)” (Thiessen, 2000, p. 529). What he contends in this way about relating theory and practice, is to some extent in accordance with ideas of Eraut (1995b) and Leinhardt et al. (1995). Verloop et al. argue that, although the importance of integrating formal theoretical knowledge and teacher knowledge is evident, it is necessary to come to a balanced view of both theory and practice before the relationship between those two components of the knowledge base of teaching can be studied adequately (Verloop et al., 2001, p. 445).

In fact, the central question here is which training method will prevent a gap arising between theory and practice. Another, related question, focusing on the development of student teachers, is how integrating several elements of the knowledge base of (prospective) teachers can be realized and how this integration can be stimulated.

As yet, little is known about how student teachers construct knowledge or of the way in which students link theoretical knowledge and practical situations, both vital components of learning to teach. In the next section we will elaborate further on the concepts of theory and practice as well as on the relationship between those concepts.

2.3 The concepts of theory and practice in teacher education

2.3.1 Theory in teacher education

Research literature shows a large variation of definitions and opinions concerning the meaning of the concept of theory. The roots of that concept date back to ancient times, in particular to the Greek philosopher Aristotle. Over the last decades, researchers have rediscovered and deepened his ideas within the context of recent developments in education (Fenstermacher, 1994; Korthagen & Kessels, 1999), in ethics (Nussbaum, 1986), in the theory of knowledge (Toulmin, 1990) and in social science (Van Beugen, 1988). Particularly Aristotle's opinions on the manifestations of knowledge are frequently cited.

Aristotle distinguishes philosophical-contemplative knowledge (the *nous*), knowledge that is related to the surrounding world (*epistème*), knowledge of practical-ethical action (the *phronèsis*) and 'practical' knowledge, skills (the *technè*). Aristotle considers the first two forms of knowledge superior to the last two. The relationship between the *nous* or the *epistème* and reality remain limited to a mental connection, by which, in Aristotelian terms, those two forms of knowing distinguish themselves sharply from knowledge that is aimed at practical action. In present terminology, for *nous* and *epistème*, and to a smaller degree *phronèsis*, we might speak of knowledge which originates from considering phenomena. Such a consideration involves reflection on reality by taking distance from that reality. According to Van Beugen (1988) such a reflective attitude emerges on three levels:

- the reflective attitude that one can adopt in contact with the surrounding reality as an expression of the human ability to know;
- knowledge that rests on generalized experiences;
- knowledge as a system of verifiable judgments according to epistemological rules (scientific theory).

Reflection can lead to 'theory,' according to our view meaning a coherent collection of underpinned judgments or predictions concerning a phenomenon. At the highest level – that of scientific theory – we then end up at the development of a theory that can be expressed in theoretical terms and laws (Koningsveld, 1992). Fenstermacher (1994) demands different requirements of theoretical (formal) knowledge, this is 'justified true belief' for formal knowledge in scientific settings and, 'objectively reasonable belief' as an acceptable form for formal knowledge that is used within the context of the educational practice. In section 2.6 we will elaborate further on the concept of theory by focusing on the characteristics of domain-specific instructional theory (Treffers, 1987).

Also in the field of research on teacher training, we find a jumble of almost equal, related or overlapping elaborations of the concept of theory. Thus, there is a distinction

between objective and subjective theory (Corporaal, 1988), public and personal theory (Eraut, 1995b) academic and reflective theory (Smith, 1992) or academic and practical theory (Even, 1999). Considered in extremes, the distinction concerns the difference between scientifically oriented conceptualization and personal, situational perception of educational phenomena. Between these extremes there exists a range of ideas and conceptions concerning the meaning of the concept of theory, for example characterized by the concepts of abstract or concrete, universal or specific, generalizable or situational, true or not proven, objective or subjective, formal or informal, justified or plausible. Eraut's description of what he defines as theory reflects the common (broad) interpretation of researchers: "Educational theory comprises concepts, frameworks, ideas, and principles that may be used to interpret, explain, or judge intentions, actions, and experiences in educational or educational-related settings" (Eraut, 1994a, p. 70).

However, in that plurality of conceptions a tendency can be observed. Many researchers who distinguish personal or subjective theory in their descriptions of theory honor the belief that each action of the teacher is also an expression of theory (Schön, 1987; Carr & Kemmis, 1986; Elliot, 1987; Griffiths, 1987). The source of that idea must be sought in Aristotle and Dewey (Van Beugen, 1988) and the recent tradition of critical theory (Griffiths & Tann, 1992).

Little is known as yet of how student teachers construct theoretical knowledge and how that process of acquiring knowledge is influenced by their experiences and beliefs (Branger, 1973; Cooney, 2001a; Eraut, 1994a,b; Kagan, 1992; Corporaal, 1988; Coonen, 1987; Grossman, 1992; Hofer & Pintrich, 1997; Jaworski, 2001; Nettle, 1998; Richardson, 1989). Student teachers are frequently of the opinion that they are not offered the theory they need to prepare for their school practice (Knol & Tillema, 1995) and often appear not to be able to integrate the offered theory with their practice (Kagan, Freeman, Horton & Rountree, 1993; Cohen, 1998; Lampert & Loewenberg Ball, 1998).

A prominent function of theory is providing an orientation base for reflection on practice. Studies into research of professional knowledge for teachers, particularly into views on the knowledge-practice link, describe a range of ideas and tools for teachers that are seen as useful for fruitful recognizing and analyzing matters of practice. For example, Tom and Valli (1990) describe one of four ways to portray knowledge as related to practice: "knowledge as a source of schemata that can alter the perception of practitioners" (p. 384). Grimmett & MacKinnon (1992) analyze in their review study among other topics the research of Kohl (1986, 1988), who "(...) is committed to teachers taking control of their work through the refining of their craft" (p. 419). According to Grimmett and MacKinnon, the essential focus of Kohl's books is developing *teaching sensibility*, which finds its expression in the idea of loving students as learners.

Fenstermacher (1986) and Fenstermacher & Richardson (1993) introduce the idea of practical arguments. Practical argument is the formal elaboration of practical reasoning: laying out a series of reasons that can be viewed as premises, and connecting them to a concluding action. Practical reasoning describes according to Fenstermacher and Richardson (p. 103), the more general and inclusive activities of thinking, forming intentions and acting. The authors contend that the process of eliciting and reconstructing practical arguments allow teachers to take control of their justifications, and therefore take responsibility for their actions. Practical argument seems a usable concept. For student teachers it is a reason to use theory in practice, and so for teacher education it is a reason to 'feed' student teachers' learning environment with relevant theory. Pendlebury (1995) agrees on Fenstermacher's and Richardson's assertion that good teaching depends upon sound practical reasoning, but she doesn't agree with their statement that an improvement in teachers' practical arguments results in better practical reasoning. She thinks that sound practical reasoning requires situational appreciation, a way of seeing which is better nurtured by stories than by formal arguments (Pendlebury, 1995, p. 52). It is a relevant comment on Fenstermacher & Richardson's statements. The learning environment of (student) teachers does in any case need the feeding – both implied and explicit – with a variety of theories and theory laden stories and furthermore, the guidance of an expert in order to level up the student teachers' practical reasoning. Moreover, the expert has to be aware of the importance of learning by interaction (Elbers, 1993) and of 'constructive coaching' (Bakker, Sanders, Beijgaard, Roelofs, Tigelaar & Verloop, 2008)¹.

An important question is to what extent the underlying intentions of theoretical reflecting, namely: understanding, formulating, describing, explaining, and improving practice can be realized for student teachers.

2.3.2 Practice in teacher education

The concept of practice can perhaps be best translated as 'professional situation.' It is a (learning) environment – with materials, tools and actors – in which a profession is practiced. The professional worker in that environment has been trained to act professionally, that is to say to act adequately on the basis of (practical) knowledge. A teacher can also be considered as someone who practices a profession (Verloop, 1995). Practice has many representations, which can be based on a number of views. For example, within the Dutch primary education system there are the views of Montessori, Dalton, Freinet, Jenaplan and the Free School. In the case of teacher education, school practice is an important representation of practice, being a learning practice for prospective teachers. In the report of the visitation for the Teacher training colleges (Pabo) in the Netherlands (Inspectie van het Onderwijs, 1989), seven functions of school practice have been described, for example, the function as a training area for

learning to teach or the function of the practice school as a laboratory to review and improve student teachers' educational designs. The functions illuminate the contribution of school practice to the learning environment of the Pabo. A specific elaboration of a learning practice for primary mathematics student teachers is the Multimedia Interactive Learning Environment MILE, that has been a part of the Pabo learning environment for a number of years (Dolk et al., 1996), in the shape of a digital representation of primary school practice for mathematics (chapter 3).

2.3.3 The knowledge base of the (prospective) teacher

In recent years there has been much attention to two characteristics of professionalism, namely monitoring the level of professional actions by experts or by the teachers' network and, secondly, working from a knowledge base which gives direction to professional actions (Verloop, 1999). We will look at the second characteristic. Since the 1970s, the study of the teacher's professional knowledge base has received new impulses as a result of increased attention to factors that guide the actions of the teacher, such as cognitions, aims and beliefs. Up to that time the emphasis was on process-product research and on studies into effective teaching (Rosenshine & Stevens, 1986; Shulman, 1986a; Creemers, 1991). While *Rosenshine & Stevens in the Handbook of research on teaching* placed a heavy claim on the role and the outcomes of process-product research, in the same book Shulman criticized those studies (p. 13). From the 1970s on, after the 'cognitive shift' (Clark & Peterson, 1986), researchers became more and more aware of the distance between research in academic settings and everyday practice (Schön, 1983, 1987; Richardson, Anders, Tidwell & Lloyd, 1991). That applied in particular to teacher training (Beijaard & Verloop, 1996; Harris & Eggen, 1993; Guyton & McIntyre, 1990). New theoretical conceptions were developed in educational research, such as situated cognition (Leinhardt, 1988; Brown, Collins & Duguid, 1989; Borko & Putnam, 1996; Herrington, A., Herrington, J., Sparrow & Oliver, 1998), constructivism (Piaget, 1937, 1974; Kilpatrick, 1987; Cobb, Yackel & Wood, 1992; Von Glasersfeld, 1995; Gravemeijer, 1995), narrativism (McEwan & Egan, 1995; Oonk, 2000), metacognition (Brown, 1980; Boekaerts & Simons, 1993) and learning styles (Vermunt, 1992).

The value of the new conceptions is not proven so much through (comparative) research in terms of effective education, but the new concepts serve especially as a rich source of inspiration for reform (Verloop, 1999). The source provides a cognitive tool with which teachers can improve the formulation and recognition of the teaching-learning process (Fenstermacher & Richardson, 1993; Tom & Valli, 1990). Also, in recent years an entirely new direction in the study of the professional base of knowledge for teachers has emerged. The study of the so-called practical knowledge or teacher knowledge wants to honor the insights into professional practice developed by the teachers themselves. Moreover, there is an intention to examine teacher cognitions more

in context, for example without taking a priori defined variables and analysis categories of researchers as a starting point.

2.3.4 Teacher practical knowledge

It was particularly Elbaz, with her case study called 'The Teacher's practical knowledge: Report of a Case Study' (1981; 1983), who marked the change from research of teachers' thinking to research of teachers' practical knowledge (Calderhead, 1996). Elbaz came to her study especially through dissatisfaction with what she saw as incoherence in the approach of research into the work of the teacher. She considers practical knowledge particularly as personally colored, situational knowledge. In the Netherlands, Verloop (1991) gave an initial interpretation of the concept of 'practical knowledge' in his inaugural lecture 'Practical knowledge of teachers as part of the educational knowledge base.' He referred to teachers' practical knowledge as a blind spot in educational research, as this type of knowledge had not yet been given a place in descriptions of knowledge that teachers should either have or have to acquire. This is generally implicit knowledge concerning all kinds of aspects of learning and teaching. Theoretical notions can be a part of it, but also images and ideas of experiences, for example from teachers' own educational history. International literature of educational research shows us different names for practical knowledge, such as craft knowledge, wisdom of practice and personal knowledge (Grimmett & MacKinnon, 1992). We follow Verloop, Van Driel & Meijer (2001, p. 446) by using the labels 'teacher knowledge' – or 'teacher practical knowledge' – to indicate the whole of the knowledge and insights that underlie teachers' actions in practice. The concept of 'knowledge' in 'teacher knowledge' is used as an overarching, inclusive concept, summarizing a large variety of cognitions, from conscious and well-balanced opinions to unconscious and unreflected intuitions. We will stress that teacher (practical) knowledge is not opposite to theoretical or scientific knowledge. In fact, knowledge gained from lectures, self-instruction and other sources of teacher education may be absorbed and integrated into (student) teachers' practical knowledge. Because practical knowledge is often not simply discernible in teachers' actions, it needs expertise to make practical knowledge explicit. Elbaz outlined characteristics of that 'tacit knowledge' and made a plea under the motto 'giving voice to the tacit' for research into the possibilities of making that knowledge explicit (Elbaz, 1991). Meanwhile research results of study of practical knowledge have been published; this concerns mainly study of the practical knowledge of (prospective) teachers in secondary education (Leinhardt & Smith, 1985; Peterson, Fennema, Carpenter & Loef, 1989; Wubbels, 1992; Meijer, 1999; Korthagen and Kessels, 1999; Verloop et al., 2001). In that research two important research lines can be distinguished. The first not only aims at conscious knowledge realized by reflection, but also at less conscious knowledge (Wubbels, 1992). The terms 'image' (Calderhead,

1989) and ‘Gestalt’ (Korthagen, 1993) are core concepts in that approach. The second research line concerning study of teachers’ practical knowledge, is the study of domain-related cognitions. This direction has in fact been launched with Shulman’s well-known article (1986b), in which is contended that a fundamental component of the expertise of teachers is a matter of translating content knowledge to knowledge that is suitable to educational situations. He studied the kinds of teacher knowledge that teachers possess and that underlie their actions, and developed an overview of domains and categories of teacher knowledge (Shulman, 1987).

- content knowledge;
- general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter;
- curriculum knowledge, with a particular grasp of the materials and programs that serve as ‘tools of the trades’ for teachers;
- pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding;
- knowledge of learners and their characteristics;
- knowledge of educational contexts, ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures;
- knowledge of educational ends, purposes and values, and their philosophical and historical grounds.

(Shulman, 1987, p. 8).

Since then, much attention has been given in the international research literature to this ‘pedagogical content knowledge’ (e.g., Cochran, De Ruiter & King, 1993; Even, 1990; Even, Tirosh & Markovits, 1996; Lerman, 2001; Grossman, 1990; Gess-Newsome & Lederman, 1999). We follow Van Driel, Verloop & Vos (1998), who consider pedagogical content knowledge as a specific type of practical knowledge. In comparison with experienced teachers, student teachers’ practical knowledge will be different, supposedly more extreme, which means either more theoretical (formal) or more of a ‘practical wisdom’ character (informal). Experienced teachers select (filter) useful knowledge on the basis of their teaching experience; student teachers mainly have to draw from experiences from their own educational history or from knowledge that they acquired in ‘colleges’ (Cohen & Ball, 1990; Stipek, Givvin, Salmon & MacGyvers, 2001).

In section 2.4 we discuss the meaning of the phenomenon ‘theory and practice’ in teacher education, firstly in the more general sense, subsequently aimed at mathematics teacher

education and, finally with respect to the specific situation in the Netherlands (2.5).

In section 2.6 we discuss the characteristics of the knowledge base for the subject area of learning and teaching mathematics at primary teacher training colleges (Pabo), at the center of this study.

2.4 The relationship between theory and practice in teacher education

Teacher training colleges have already struggled for decades with the problem of how to define the theoretical dimension of the training programs (Kennedy, 1987). The simplest approach was: you will learn theory during lectures and will then apply it in practice. Drever & Cope (1999) had to say the following about that: “Theory, in this context, was presented as a kind of pseudo-scientific justification for practitioner action, the implication being that, by using it to generate hypothetical solutions to problems, it could be ‘applied in practice.’ ” Student teachers often indicated that knowledge acquired in teacher training did not enable them to handle the uncertainty, the complexity and the instability of actual practice situations (Coonen, 1987; Corporaal, 1988; Zeichner & Gore, 1990; Harris & Eggen, 1993; Oosterheert, 2001). By now one can state that the training philosophy slogan ‘Learning theory at academy and applying theory in practice’ is outdated. Over the last few years a number of researchers have brought up the problem of the relationship between theory and practice (e.g., Freudenthal, 1987; Bengtsson, 1993; Beattie, 1997; Beijaard & Verloop, 1996; Eraut, 1994a,b; Griffiths & Tann, 1992; Korthagen & Kessels, 1999; Leinhardt et al., 1995; Ruthven, 2001; Jaworski, 2001). Some authors express – often implicitly – the belief that there should be no gap between theory and practice in an appropriate teacher training program. Beattie describes a component of a teacher education program based on the principles of reflective practice and inquiry, where “the theory and practice of teaching and learning to teach are inseparable (...)” (Beattie, 1997, p. 10). Leinhardt et al. stress the important role for teacher education to facilitate the process of linking theory and practice.

Future practitioners should be given the opportunity to construct their own theories from their own practice, and to thoughtfully generate authentic episodes of practice from their own theories. We have proposed that the university should take on the task of helping learners integrate and transform their knowledge by theorizing practice and particularizing theory. We believe that the university can facilitate this process because it can create opportunities for time and pace alteration, reflection on practice, and examination of consequences. Ideally, such episodes of integration and transformation should be systematic and comprehensive rather than arbitrary and piecemeal (Leinhardt et al., 1995, p. 404).

Freudenthal contends in an article (1987) concerning theoretical frameworks (e.g., learning lines, structures) and theoretical tools (e.g., mathematizing, didacticizing, context) that the gap between theory and practice can be avoided.

From the wish to understand practice, theory from its side grows and purifies and improves practice. And if theory has been described efficiently enough to re-occur, it will likewise influence the practice of outsiders who did not directly experience the development process. After all, that is the sense and the aim of theory. The proverbial gap between theory and practice does not occur there – as I just said, perhaps somewhat too proudly and too prematurely. I should have been more cautious and say: the gap should not have to exist (translated from Freudenthal, 1987, p. 14).

Van Eerde notices as a result of an analysis of interviews, that Freudenthal for example interpreted observing learning processes as an intuitive process with a more or less implicit role for theory. His observations have been theory-guided, in the sense that theory is only made explicit afterwards, as a reflection on the mathematics teaching that actually occurred (Van Eerde, 1996, p. 43). In his last work (1991) Freudenthal chiefly viewed the theory-practice relationship as derived from the level theory of Van Hiele (1973, 1986). He formulated his own, more extended interpretation of the level theory, both concerning subject matter and concepts (levels of learning, practice, theory)².

Concerning this thesis we already advanced our conceptions concerning the function of theory in teacher training (section 2.3.1). Our assumption is that reflection of student teachers concerning jointly observed and discussed practical experiences, or reflection as a result of investigations in a (digital) practice, will start a process in which they link theory and practice in a meaningful way. We define ‘linking theory and practice’ as the adequate use of theoretical knowledge when considering a (current) practice situation. The situation is the starting point of that activity. Therefore the learning environment has to be ‘charged’ theoretically. The expectation is that theoretical knowledge – as part of the professional knowledge base – will manifest itself in several qualities and gradations. This study takes place within the context of the formerly outlined problems. There are interesting developments in primary teacher education, which might generate answers to the questions that have mentioned in section 2.2. Digital applications such as multimedia learning environments seem to be able to fulfill a useful function within the area between theory and school practice (Lampert & Loewenberg Ball, 1998; Goffree & Oonk, 2001). An environment such as the Multimedia Interactive Learning Environment (MILE, cf. chapter 3) – developed for primary mathematics teacher education but also usable in the field of general education and language teaching – offers a possibility for student teachers to study intensively the authentic practice within the primary school. Student teachers’ own school practice, where ‘survival’ takes first place, is less appropriate for such activities (Ball & Cohen, 1999; Daniel, 1996). Such a learning environment offers the advantages of both ‘reflective practices orientation’ and the ‘development of professional knowledge orientation’ (section 2.2). Unhindered by everyday concerns, student teachers can reflect on authentic situations, whereas in the

same learning environment all kinds of content-related and organizational components can be created that will ‘feed’ the learning environment with theory. In such a learning environment theory can fulfill the desired function of laying an orientation base for reflection on practice (section 2.3.1). We suspect that teacher education arranged in this way should enable student teachers to acquire ‘theory-enriched practical knowledge’ (EPK; section 2.6.5.5 and 3.9).

2.5 Theory and practice in primary mathematics teacher education in the Netherlands

2.5.1 Introduction

The history of primary mathematics teacher training in the Netherlands shows how the concept of theory has changed and evolved in the course of time from a limited subject matter concept to a more extensive concept that aims at the ongoing development of (prospective) teachers’ professionalism (Goffree, 1979, 2000; Freudenthal, 1984a, 1991; Goffree & Dolk, 1995; Dolk et al., 1996; SLO/VSLPC, 1997; PML, 1998; Dolk & Oonk, 1998; Goffree & Oonk, 1999, 2001; Oonk, 2000, 2005; Dolk, Den Hertog & Gravemeijer, 2002; Van Zanten & Van Gool, 2007). Next, in a historical context, we will describe in brief how integrating theory and practice in teacher education developed, in particular concerning mathematics teacher education. First we describe (section 2.5.2) the characteristics of that development before 1971, the year that the Institute for the Development of Mathematics Education (IOWO, nowadays the Freudenthal Institute for Science and Mathematics Education (see also section 3.2) was established. Section 2.5.3 reports on some developments that are characteristic for the last decades.

2.5.2 History

The first Dutch primary teacher training college was established in 1813 by the government. Before 1800, Dutch primary teachers were not specifically trained for teaching as such. For centuries – until the fourteenth century – the profession of teacher was practiced by conventuals. Through the establishment of ‘city schools’ the convent schools gradually disappeared. However, the teacher’s profession changed little: education was mainly seen as memory training. The teacher’s work regarding mathematics was generally limited to explaining instrumentally; arithmetical procedures were described and then exercised through an impressive quantity of problems. Providing insight was seen as unnecessary and a waste of time. The quality of teachers differed widely (Kool, 1999). In those days the best pupil from the graduating class of a primary school would be chosen to assist the head teacher on a regular basis and, after additional lessons at home from the head teacher and demonstrating a sound understanding of the subjects, he or she was expected to teach. In a later period the

private lessons by the head teacher became more systematic or normalized and came to be seen as normal lessons. The so called ‘Normal Schools’ that evolved from this practice became later – around 1800 – the teacher training colleges.

From 1800 up to the present time, development can be seen in views concerning the relationship between theory and practice in the curriculum for primary teacher education. Van Essen suggests:

Opposite the belief that the prospective teacher had to be an especially ‘smart fellow’ with a lot of general ‘book knowledge’ or a sound theoretical, subject matter stock-in-trade, the belief existed that benefit had to be expected in particular from a direct confrontation with school practice (...) (translated: Van Essen, 2006, p. 15).

Nevertheless, it would not be until the introduction of the New Training College Act in 1952, that real change was realized in teacher training. Up to then, the curricula of these training institutes were essentially the same as those for higher secondary schools, albeit with the addition of pedagogy and teaching methodologies and with half a day a week allocated for working in the practice schools. For example, the contents of primary mathematics teacher education in 1923, were arithmetic, algebra and geometry, with the following components:

The art of arithmetic: Knowledge of the central issues of the art of arithmetic: basic operations with whole numbers and fractions; smallest general multiplicator and largest general divider of numbers; geometric proportionalities; determining square roots.

Knowledge of the central issues of commercial arithmetic.

Maths: Algebra. Knowledge of the central issues of algebra up to and including the equations of the second degree with one unknown variable.

Geometry: Knowledge of the central issues of two and three-dimensional geometry. (Goffree, 1979, p. 19)

As a result of the New Training College Act of 1952, teacher training was changed. The school subjects of secondary education were replaced with the teaching methods for the subjects taught in primary schools. For example, mathematics was replaced with teaching methods for arithmetic (Van Gelder, 1964). However, because the teacher educators remained the same, little changed in practice. The teaching methods for arithmetic were frequently augmented with tough calculations for the student teachers, supplemented with some hints for working in the classroom. Most of these hints were of a general educational nature, they referred to for example teaching with visual aids, and elements of educational psychology such as different levels of thinking (Goffree & Oonk, 1999). Although theory and practice were closer in terms of curriculum development, subjects remained isolated and methods were far from the real practice. Goffree (1979) gives an example of a mathematics method book for teacher education

the title of which, 'Theory and Practice,' raises high expectations. In the explanation by the author J.H. Meijer (1963), it turns out that his concept of 'theory' encompassed the art of arithmetic and that 'practice' implied skills of arithmetic. This belief is characteristic for the lack of vision in teacher education during the 1950s and 1960s. The reorganization of 1968, when teacher-training colleges were to be named by law 'Pedagogical Academies,' did not in general lead to important changes. Because of the idea that the methods of 'all subjects of the primary school' should be taught, the matter of domain-specific instructional theory barely existed. As a result, teacher-training curricula remained fragmentary, with consistency and commonality towards goals lacking. In fact, 'theory' for the student teachers comprised mainly general educational theory. In 1984, the teacher training colleges were reorganized to a four year course, but it was only in the early 1990s that any significant changes occurred. Research by among others Coonen (1987) and Corporaal (1988) indicates that the desired consistency between theory and practice was still absent. While there was a shift towards practice, the need to do so did not necessarily arise from motives and considerations based on teacher training philosophy. Coonen wrote for instance the following about that:

The respondents [teacher educators; w.o.] mentioned that the stronger orientation on practice also originates from the resistance of student teachers to everything that is associated with theory. Student teachers appear to show little interest for knowledge of a more abstract, deepening and explanatory nature. Because of this lack of interest, one fears that student teachers acquire a too naive, and too subjectively colored repertory of action, as a result of which their reflecting capacity is also limited. Many teacher educators experience the gap between theory and practice as a large problem (translated from Coonen, 1987, p. 236).

The cause of the changes in the 1990s lays in a large scale inspection of all Pabos in 1991 – the first inspection of its kind. The judgment of the inspection was scathing. The criticism was mainly directed towards the lack of a good academic background for primary school teachers and of a clear training concept involving teaching methods. In the following years, a variety of publications appeared with recommendations for improving the quality of primary teacher education (Inspectie van het Hoger Onderwijs en Basisonderwijs, 1996; SLO/VSLPC, 1997; PML, 1998). Problem-based learning, self-instruction and thematic education were espoused, and teacher educators from all disciplines were expected to develop their own materials according to these concepts. Again, the colleges were required to leave behind the paradigm of a program dominated by the school subjects and to look for themes, case studies, and problems that would have obvious validity to the study of teaching per se.

2.5.3 New developments

Although until the 1990s little change occurred generally in the training curricula concerning the relationship between theory and practice, it appeared to be different for the subject area of mathematics teacher education. With the establishment of the IOWO in 1971 (see section 3.2), in the Netherlands a bottom-up development in primary and secondary mathematics education and the related teacher education started. For primary mathematics teacher education, a model for learning to teach was developed (Goffree, 1979; Goffree & Oonk, 1999).

The main idea was that mathematics education, both for student teachers and pupils, should take concrete situations and familiar contexts as its starting point. While mathematization of those contexts plays an important part in the learning processes of children, for the student teachers it is a process of both mathematizing and didacticizing (Freudenthal, 1991). Student teachers carry out pupils' mathematical activities at their own level and then reflect on and discuss the results of those activities. These reflective discussions create a foundation for learning how to work with children. Freudenthal saw reflective thought as 'a forceful motor of mathematical invention,' i.e. *guided* reinvention for the pupils and the student teachers. The guide should provoke reflective thinking (1991, p. 100). In his view, the theory of the level structure of learning processes (Van Hiele, 1973) shows what matters in such processes, namely the discontinuities, 'the jumps' in learning (Freudenthal, 1991, p. 96).

The model for learning to teach was elaborated in books for primary mathematics teacher education (Goffree, 1982, 1983, 1984; Goffree, Faes & Oonk, 1989), which were used in more than 80% of the teacher training colleges. Following the Standards for primary mathematics education and the Standards for mathematics evaluation and teaching (NCTM, 1989, 1992), in 1990 a group comprising ten mathematics educators started developing national standards and presented the results to colleagues as a handbook for teacher educators (Goffree & Dolk, 1995). The philosophy of teacher education elaborated in the handbook is founded on three pillars: a teacher education adaptation of the socio-constructivist vision of knowledge acquisition, reflection as the main driving force of the professionalization of teachers and the interpretation of practical knowledge as a way of narrative knowing (see section 3.1 and 3.2). Discussions between the developers and fellow teacher educators of four teacher training colleges from the United States, who were interested in the Dutch MTE-standards, had major consequences for the training of Dutch primary school mathematics teachers. The Dutch teacher educators became acquainted with Magdalena Lampert and Deborah Ball's MATH project (Lampert & Loewenberg Ball, 1998). The student learning environment developed in their project coincided with ideas developed in the Netherlands about the training of student teachers and the way they learn. It was

decided to develop an environment for student teachers in the Netherlands similar to that developed in Lampert and Ball's project. And so the MILE-project in the Netherlands was born (see chapter 3), in which the developers saw a position between theory and practice in teacher education and, by which the statement "Real teaching practice has to be the starting point of teacher education" became emphasized.

2.5.4 Perspectives

The knowledge base of primary mathematics teacher education (Pabo), the area in which this study takes place, can be distinguished in two ways from other fields in teacher training. In the first place the nature of the mathematical knowledge requires a constructive commitment and much effort to become 'owner' of the specific insights and procedures. Furthermore, the developments over the last thirty years in this area at the Dutch Pabos led to an approach of integrating subject matter, pedagogical content matter and school practice (Goffree, 1979; Goffree & Dolk, 1995; Goffree & Oonk, 1999). However, such an approach does not lead naturally to student teachers' integration of theory and practice. That will perhaps happen if student teachers can use a 'rich' learning environment, for example the multimedia interactive learning environment MILE. Further research should show how student teachers link theory and practice if they have such a learning environment at their disposal; further study will also express the quality of their activities.

Next, as mentioned in section 2.3.4, we will elaborate further on the role of theory in mathematics teacher education.

2.6 Characteristics of a domain-specific instructional theory. Implications for the learning environment of mathematics teacher educationⁱ

2.6.1 Introduction

2.6.1.1 *Focal points for a theory enriched learning environment*

To get an image of the focal points that are essential for the function of theory in teacher training, first an attempt is made to map core characteristics of theory. To achieve this, the domain-specific instructional theory for learning and teaching to multiply is analyzed, the theory that is part of the student teachers' learning environment in both the small scale study (chapter 4) and the large scale study (chapter 5). Examining existing

ⁱ Section 2.6 has been published in adapted form as: Oonk, W. (2007). Kenmerken van vakdidactische theorie. Implicaties voor de Pabo. [Characteristics of domain-specific theory. Implications for mathematics teacher education]. *Reken-wiskundeonderwijs: onderzoek, ontwikkeling, praktijk* [Primary mathematics education: research, development, practice], 26 (3), 19-32.

theories can make ideas about the ways prospective teachers use theory become manifest (Oonk, 2002). The underlying thought is that focal points for theory in teacher education can be derived from the characteristics that have been found. As a consequence, these points of interest can be important for developing a ‘theory-enriched’ learning environment for student teachers.

2.6.1.2 Working definition of a domain-specific instructional theory

Earlier (section 2.3.1), the large variation of definitions and beliefs concerning the meaning of the concept of theory was mentioned, as well as the concept’s origin in Greek philosophy. According to Plato and Aristotle the purpose of being lay in the *theoria* or contemplation. In Plato’s opinion the highest purpose is to rise above the low. Nevertheless he finds it necessary to return to the low (Bor, Petersma & Kingma, 1995, p. 49). Aristotle, Plato’s brilliant student, is even more attached to the relationship between knowledge and reality than his teacher. According to him, knowledge also exists to enrich the everyday world. Scientists often refer to these different modes of thought. Korthagen (2001), for example, clarifies the term theory using two concepts of knowledge as developed by Plato and Aristotle, namely the *epistēmē* and the *phronēsis*. *Epistēmē* represents academic, conceptual knowledge, the *phronēsis* stands for perceptual knowledge, and this is practical wisdom that has been based on the perception of a situation and the reflection on that situation. Korthagen thinks that the development of the last type is the most important for teacher training.

The entirety of assumptions, arguments and conclusions – what the ancient Greeks saw as contemplation (*theoria*) – provides theory and new knowledge. A changed view on the original problems affords new perspectives and presuppositions and causes a cycle of self-renewing theories. The development of the theory of gravity – with successively the conceptions and findings of Aristotle, Newton and Einstein – is an example of such a course.

For the benefit of the analysis below it is necessary to provide a ‘working definition’ of a domain-specific instructional theory. It seems possible to formulate such a definition as an extension to what has been discussed in section 2.3.1 and, which is also in line with existing ideas about domain-specific instructional theories (Treffers, 1978, 1987; Freudenthal, 1991; Gravemeijer, 1994). As soon as a collection of descriptive concepts displays consistency, and the coherence has been underpinned, one can speak of theory. Such a system can contain statements and arguments as well as explanations, assumptions, conjectures, predictions and proofs. Very different concepts can be an object of establishing a theory. For example, educational researchers have been looking for years for theoretical constructs which can throw a different light on the development of children. The development of children’s numeracy is one such example³.

2.6.1.3 Selection of the theory to analyze

Considering theories on a continuum from ‘pure,’ formal (e.g., the Boolean Algebra) to empirical (e.g., Gestalt theory), the theory of learning and teaching multiplication seems to provide a ‘rich’ situation for analysis. This theory is characteristic of both a formal, mathematical background theory, evolved by deduction, and a more empirical theory developed by induction. The two types distinguish themselves to a certain degree also by objective and subjective characteristics. The characteristic ‘objectivity’ refers in its most extreme form to scientific theories which borrow their ‘status’ from acceptance within the paradigm of a scientific community. A scientific theory is considered as (tentatively) true on the basis of the methodology chosen within that paradigm (Koningsveld, 1992). In the development of the method for learning multiplication, there are also subjective characteristics, colored by individual beliefs.

The formal and empirical components of learning and teaching multiplication also differ in the relationship that they have with (associated) practice. That relationship is partly determined by the way in which theory is produced or used (Fenstermacher, 1986), for example ‘in action’ or ‘on action’ (Schön, 1983), or by the degree in which theory influences or even directs practice (Eraut, 1994b).

An attempt will be made to identify in reflective analysis features of the theory which are characteristic of the theory, for example the coherence between concepts or the grounding of assertions. This allows to make a distinction between intrinsic characteristics, which are aspects of the internal structure of the theory and the remaining, extrinsic characteristics, which appear in the context in which the theory is used or developed. While analyzing the theory, the aforementioned working definition (section 2.6.1.2) of theory is used as a tentative framework. We will start with a description of the history of the theory, i.e. its genesis and development. On the basis of an analysis of the theoretical characteristics we will come to points of interest for the place and function of theory in teacher education.

2.6.2 The theory of learning and teaching to multiply

2.6.2.1 The origin of the theory in the Netherlands

Learning and teaching to multiply occurs within the world of education, at the level of teaching students and, at the level of the school book authors who provide content and formats to teaching, and at the level of science, where theorists are studying the background of learning multiplication. The ‘birthplace,’ the field where work was being done at these different levels (student/teacher, developer/author, scientist/theorist) and from different points of view (e.g., mathematics and psychology) can therefore be seen as the context in which the theory was initiated. Learning (memorizing) the tables of multiplication, is traditionally the core of learning to multiply. In primary education

before 1970 – before Wiskobas⁴– memorizing the tables was therefore given most of the attention there was for multiplication. Many children already knew table products before they understood the meaning of the operation of multiplication. Teachers and schoolbook authors considered multiplying as no more than repeated addition. Teachers were taught that way in teacher training. They learned from a mathematical (arithmetical) point of view that 3×7 is the same as $7 + 7 + 7$. That was calculated step by step as $7 + 7 = 14$ and $14 + 7 = 21$. This approach became the basis for learning the table products; so, through continuous recitation, students gradually learned more and more answers by heart. The smarter students rapidly realized that 6 times 7 could be calculated by adding up 2×7 and 4×7 . Only those who had to do too much calculating did sometimes lose track and forgot where they were in the list; but without this clever calculating the tables of multiplication became a line of meaningless objects for the students. In terms of educational psychology, this way of learning the tables was based on ideas from the theory of association psychology⁵.

Developer and researcher Hans ter Heege tells us about his own past experiences in this field both as a student and as a teacher. Among other things, working with children brought him face to face with his own conceptions. Those conceptions were colored by his own experiences in primary school, and had been confirmed in teacher training college and were reinforced further by working as a primary school teacher using the ‘mechanistic’ textbook ‘To independent arithmetic.’ Reflecting on working with children, his own conceptions and his own learning process, partly fed by the discussions in the Wiskobas team, stimulated him to develop a theory in which mechanistic characteristics were lacking. Ter Heege (1978, 1985, 1986) developed a new theory of learning and teaching to multiply.

2.6.2.2 The ingredients of the theory

The theory of learning and teaching to multiply contains the following components:

- Concepts, such as multiplication (with associated notation $\times$), multiplication strategy, informal strategy, repeated addition, structured multiplication, formal calculation, memorization, automation, properties, anchor points, reproduction, reconstruction, models, line structure, group structure, rectangle structure, contexts, practice, apply.
- Indications for teaching which are related to phasing of the learning process of students in levels of acting and thinking, and indications for shortening that process with examples of student activities in the field of concept attainment, memorization, practice and application.
- Directives for entwining learning trajectories and for interweaving learning and teaching (Treffers, 1991; Van den Heuvel-Panhuizen, Buys & Treffers, 1998, p. 41).

In explaining this theory we will limit ourselves to the core concept of multiplication; other parts will be brought up in section 2.6.2.4.

2.6.2.3 The concept of multiplication, considered phenomenologically and mathematically

The sum $3 \times 4 =$ is bare (formal) multiplication, and every practitioner will know the answer. This multiplication can be the mathematical representation of a number of everyday situations. The practical value of multiplication is undisputed; we know that the operation of multiplying numbers already occurred in ancient times⁶. It is a means to understand, analyze and communicate situations that have a multiplicative structure.

Situations (manifestations) of the multiplication 3×4 or $2 \times 3 \times 4$ are for example:

- 3 pieces of rope of 4 meters;
- 3 pants and 4 shirts, how many combinations?
- 3 golden stars per decimeter on 4 decimeter long Christmas garlands;
- there are 3 of us; everyone gets 4 sheets of paper;
- a square of 3 by 4 tiles;
- 4, three times enlarged;
- 3 routes from the town of Hilversum to the village of Laren and 4 routes from Laren to Huizen. How many routes from Hilversum to Huizen via Laren?
- on the calculator, 3 times 4 becomes...?
- the size of the pond is 2 by 3 by 4 meters. How many liters of water do we need to fill the pond?

The fact that one in three children at the end of primary school will not see a multiplication in some of these situations (Carpenter, 1981), particularly the situations in which combinations appear, illuminates the hidden character of the operation (multiplying) in those situations.

In 1998, developers at the Freudenthal Institute distinguished three types of structures in multiplicative situations: the line structure (3 strips of 4 meters), the group structure (3 boxes with 4 balls each) and the rectangle structure (3 rows of 4 tiles). It is a phenomenological description of mathematical structures, which means a description of concepts and structures (in this case multiplication) related to the phenomena for which 'they are created and to which they were extended in the learning process of humanity' (Freudenthal, 1984a, p. 9). The fundamental mathematical background of multiplying cannot be interpreted as readily as its phenomenological aspects. The operations of adding and multiplying and – with them other operations with numbers – are mathematically grounded in number theories. Euclid (approx. 300 before Chr.) was the first who developed a theory of natural numbers, with the main idea that a natural number exists of units.

Euclid says about that among other things:

Unit is which to which each thing is called *one*. Number is a collection composed from entities. (Euklides VII, def. 2. in Freudenthal, 1984a, p. 86; Struik, 1990)

Until the beginning of the twentieth century this was the dominant concept. Around 1900 this view was formalized by the Italian mathematician Giuseppe Peano (1858-1932) in his design of an axiom scheme for natural numbers. The fifth and last axiom of that scheme, the so-called axiom of complete induction, stipulates in fact the complete, ordered and infinite collection of natural numbers (Loonstra, 1963, p. 18). Complete induction is seen as one of the tools to define addition and multiplication. The definition of multiplication by complete induction can be seen as repeated addition (table production).

A second definition originates from the cardinal approach to multiplication. The German mathematician Georg Cantor (1845-1918), inventor of the Set Theory, lay the foundations for that theory at the end of the nineteenth century by considering the number as a quantity of something. Considered mathematically according to that theory, number is a property of a finite collection, the so-called cardinal number. Adding up can then be defined as uniting disjunct sets, i.e. sets that have no common element. In non-mathematical language this means as much as merging quantities. Multiplication into the cardinal approach is defined as the cardinal number $v = m \times n$ of the product set $[A, B]$ of number pairs (a, b) with $a \in A$ and $b \in B$, where set A consists of m elements (cardinal number m) and, the set B has the cardinal number n . Concretization in a grid (or rectangle) model, demonstrates that this definition of multiplication – different from that according to complete induction – easily exposes its properties, because the structure of multiplication becomes visible.

As an example, we choose the set of products $[A, B]$ with $A = \{a1, a2, a3, a4\}$ and $B = \{b1, b2, b3\}$.

With $m = 4$ and $n = 3$ we construct then the matrix $(A \times B)$, that visualizes the number 4×3 in a grid structure.

$(a1, b3)$	$(a2, b3)$	$(a3, b3)$	$(a4, b3)$
•	•	•	•
$(a1, b2)$	$(a2, b2)$	$(a3, b2)$	$(a4, b2)$
•	•	•	•
$(a1, b1)$	$(a2, b1)$	$(a3, b1)$	$(a4, b1)$
•	•	•	•

For example by turning the grid over ninety degrees, it becomes visible that $3 \times 4 = 4 \times 3$ (formulated generally: the commutative law applies, $m \times n = n \times m$).

What is notable is the effort necessary – even for a trained mathematician – to recognize the two formal approaches of multiplication in the phenomenology of mathematical multiplicative structures. Freudenthal acknowledged that indeed. In his ‘Didactical Phenomenology of Mathematical Structures’ (1983, 1984a), he writes about the relationship between the two mathematical approaches and about their didactical (methodological) relevance. One of the points he contends is that the set-product, as reflected in the grid model, is didactically (methodologically) inadequate. What he intends to say is that students’ learning processes do not need to be a reflection of those mathematical (re)structuring processes⁷. The mathematical foundation does play a background role, but the phenomenology of mathematical concepts and structures is more appropriate for guiding the teacher towards the learning process of the student. Concerning the phenomenological character of multiplication, Freudenthal talks about the obviousness of the operation of multiplying: “No operation – not even adding or subtracting – presents itself so naturally.” (Freudenthal, 1984a, p. 120). He then talks about the way in which young children construe the concept of multiplying. Before they are actually confronted with the arithmetical operation, there are years of experience in which they are already building a meaningful foundation with ‘multiplicative words’ such as ‘double’ and ‘time.’ Some examples: I have double; you already heard it three times now; you can take three times four beads or four beads and four beads and four beads. We will see that Freudenthal’s views have had much influence on the development of the theory of learning to multiply in the Dutch educational situation. Elsewhere he discussed (Freudenthal, 1984c) acquiring basic knowledge through exercises on the computer. He experienced that determination of outcomes was possible with the computer, but it was impossible to make an analysis of errors or justify an arithmetic procedure; he considered the last to be aiming too high. He could not suspect that three years later Klep (together with Gilissen) would develop a software bundle, called ‘a world around tables’ which let the computer do much more than merely produce answers. In his dissertation Klep (1998) showed subsequently that it is possible to develop software with which the computer can follow the students’ process of meaningful practice.

2.6.2.4 Genesis and development of the theory

The period until 1970: the mechanistic theory

It is only recently that changes have appeared in the vision on learning multiplication. Until the 1970s ‘drill and practice’ were considered obvious for learning the tables of multiplication. In fact, for centuries, learning to multiply had consisted of little more than learning a number of arithmetic rules which had been acquired through ‘demonstrating, imitating and practicing.’ From the research of Kool (1999) into Dutch arithmetic textbooks from the fifteenth and sixteenth century, we know that at that time teaching multiplication mainly involved giving the students a definition of multiplying

and a table with table products. The purpose in laying the foundation for multiplying was memorizing knowledge that could be used for arithmetic algorithms. In fact understanding of the procedures was not required for using algorithms either, as can be seen in the work of Willem Bartjens, who is an example of a Dutch schoolmaster from the seventeenth century. He became famous with his textbook ‘Cijfferinge’ (published in 1604; see Bartjens, 2005). His books reflected the arithmetic habits of that time and would remain influential for centuries to come. The expression ‘according to Bartjens’ typifies the arithmetic of that period; the schoolmaster had to explain how the rule was ‘according to Bartjens,’ this is ‘according to the rules of the art of arithmetic.’ This meant instrumental explanation: the arithmetic procedures were first told and then practiced through doing an impressive amount of sums. Teaching understanding was considered unnecessary, and even a waste of time. Students had to learn to do arithmetic as quickly and as well as possible for daily life. Adults needed to train for professions such as clerk, teacher or surveyor; these involved skill and certainty, rather than the underlying explanations. There was no benefit in understanding arithmetic, that was for scientists. Associative psychology and behaviorism have had long and intense influence on that ‘practice.’ Barely thirty years ago, there were still textbooks in use that were based on a mechanistic foundation. One example of a much-used mechanistic method is ‘Naar zelfstandig rekenen (Towards independent reckoning)’ from the 1950s. Even today’s education still contains elements of that approach. The following page from ‘Naar zelfstandig rekenen’ illustrates that textbook’s approach (fig. 2.1).

2	$1 \times 2 = 2$
$2 + 2$	$2 \times 2 = 4$
$2 + 2 + 2$	$3 \times 2 = 6$
$2 + 2 + 2 + 2$	$4 \times 2 = 8$
$2 + 2 + 2 + 2 + 2$	$5 \times 2 = 10$
$2 + 2 + 2 + 2 + 2 + 2$	$6 \times 2 = 12$
$2 + 2 + 2 + 2 + 2 + 2 + 2$	$7 \times 2 = 14$
$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$	$8 \times 2 = 16$
$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$	$9 \times 2 = 18$
$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$	$10 \times 2 = 20$
Learn the two-times table by heart	

figure 2.1 A page from the textbook

‘Learn the two-times table by heart’ is the only instruction given to the students, while there is no further information on possible didactical approaches for the teacher in the manual either.

In summary, the mechanistic theory of learning to multiply can be characterized as a collection of mathematical symbols, definitions, procedures and views, including:

- the symbols $+$, $\times$ and $=$;
- the definition that the repeated addition $b + b + b + \dots$ (and that a times) can be written as $a \times b$ with the silent assumption that both have the same solution (for instance $2 + 2 + 2 = 3 \times 2$);
- the procedure of learning to multiply through the use of table charts;
- the view that learning to multiply occurs through memorizing table products and applying standard procedures.

That the theory survived for so long is probably linked to the authority of the just as tenacious underlying ideas about learning (associative psychology and behaviorism), possibly in combination with the consensus about the approach among teachers and pedagogues in those days, or the lack of communication about criticism of the approach⁸.

The change from 1970 onwards: a new theory

In fact the theory of learning to multiply only significantly changed in the 1970s. Influenced especially by Piaget, who developed and described the clinical interview as a method, an interest in learning and developmental processes in children arose. Such methods were used in mathematics education by several researchers⁹. Ter Heege – a pioneer in the Netherlands on learning to multiply – used it when he discussed their knowledge of tables with children (Ter Heege, 1978, 1986)¹⁰. Using the title ‘Johan, een afhaker haakt aan,’ (Johan, a dropout drops in) Ter Heege analyzed seven interviews with Johan, a twelve-year old boy in grade 5. He discovered that Johan, who was known as being weak in arithmetic, applied a great amount of flexibility in calculating basic multiplications. Johan used his own arithmetic strategies which had not been taught before, becoming aware that it was allowed to do so. The yield of these interviews was both the reason and the means for the development of a new theory on learning multiplication.

Looking back on the development, it starts as a personal theory based on personal experience and analyses. What the ‘theory’ – phrased in conclusions and a recommendation (Ter Heege, 1986, p. 56) – comes down to, is that children find it much harder to learn the products from the tables of multiplication than is suggested by the traditional view on education. Also, the way that children do learn the products, deviates greatly from what teachers and authors of textbooks assume; children often use their own calculation strategies for multiplication. The recommendation is: “Children should learn the basic multiplications by flexible use of a number of characteristics and calculation strategies, such as the commutative property, the strategy of ‘one time more’ or ‘one time less’ and the strategy based on the factor 10.” Ter Heege finds support for this and other conjectures that have been developed into hypotheses through analyzing of prevailing textbooks and an exploration through the literature on the psychology of memory¹¹. This is the case especially for the fact that children will make use of mental strategies on their

own, and that they construct their own cognitive network for basic multiplication. Reflection on that ‘practice’ of research into memory evokes the next development question in him, namely: “(...) whether it is possible to develop an approach that maximizes the opportunity for children to construct an adequate network for the basic products they have to learn as arithmetical facts and have to be able to apply.” (p. 133). In answer to that question, Ter Heege designs a teaching unit for multiplication. He then tests that unit in the practice setting of primary education. In another reflection on practice, which we can now describe as the practice of curriculum development, he describes the theoretical foundations that the development of the program is based upon. In fact, Ter Heege formulates a provisional final version of the theory of learning to multiply. The most important innovation compared to the previous version is its systematic underpinning. He creates the structure for doing so by placing three ‘fundamental elements’ (p. 171) in a central position: children’s own constructions, children’s own productions, and so-called horizontal and vertical mathematization (Treffers, 1978, 1987). This occurred against the background of the current domain-specific instructional theory for realistic mathematics education in the Netherlands (Treffers & Goffree, 1985).

Ter Heege clearly looks for connections with that theory and for consensus with its basis and principle. To do so, he further grounds the earlier-mentioned three ‘theoretical elements’ from his own research. For instance, his research shows that for the role of ‘own constructions’ in the learning process of students, that children, in their struggle to learn multiplication, look for their own solutions and find their own way. It turns out that the solutions the child itself ‘invents’ are better and more profoundly understood than solutions that are ‘taken’ from the teacher.

In ‘de Proeve van een nationaal programma voor het reken-wiskundeonderwijs op de basisschool’ [Standards for primary mathematics education], part II, chapter 3 (Treffers & De Moor, 1990), we can see how the theory on learning multiplication that Ter Heege developed is adopted and elaborated. To legitimize the ‘Proeve,’ it was submitted to a large number of experts. This testing has been further strengthened by previous publications in specialized magazines and by peer discussions during conferences. After another eight years the TAL brochure¹² is published (Van den Heuvel-Panhuizen, et al., 1998, 2000, 2001), describing intermediate goals for mathematics education in the lower groups in primary school. The intermediate attainment targets for multiplication and their justification (p. 59-63), echo the spirit of the theory of learning to multiply that is discussed above. The goals in fact present ‘the theory in activities,’ partly through the nuanced underpinning of the goals and their illustration with core examples.

The description of the domain-specific instructional theory in the TAL-publication leads towards a learning-teaching trajectory in three stages. In the first stage, most children operate at the level of multiplication by counting, helped by the use of jumps on the

number line. They are taught with situations requiring repeated addition. During the second stage they work at the level of structural multiplication. This should put the children into a position in which they can build up or reconstruct table products for themselves (informal strategies). Multiplication can take on the following appearances (models) in context:

- a *line* structure: chain, strip, number line;
- a *group* structure: groups of varying types (bags, boxes, coins);
- a *rectangular* pattern: grids, weave patterns.

Children are able to recognize, partly with the help of the rectangle model, the two core properties of multiplication, namely the distribution property ($6 \times 8 = 5 \times 8 + 1 \times 8$) and the commutative property ($5 \times 8 = 8 \times 5$) (see also Buijs, 2008, p. 41 and note 14).

In the third stage, formal multiplication is used and the tables are gradually automated and finally memorized. Application of the network of table-knowledge will take place through calculating by heart and through algorithmic calculations, keeping the knowledge current.

We can characterize the most recent theory (in ‘De Proeve’ en ‘TAL’) as follows:

- As in earlier theories, the students’ learning process is seen as a process of reconstruction¹³ and defined in stages. Here, however, the stages are substantiated with more nuance and detail.
- Where Ter Heege’s theory only gives little attention to models, their role has become much stronger in the Proeve, and even more so in the TAL brochure.
- A special focus on levels appears. Other than in earlier theories on learning to multiply, rises in the level of children’s learning process are described that allow for recognizing and utilizing differences between children.
- In general the justification for approaches and choices is sharper than in the earlier described theory.

There are however still clear signs of previous theories¹⁴.

2.6.3 Characteristics of the theory of learning and teaching to multiply

In this section we examine characteristics of the theory of learning and teaching multiplication previously discussed, with the intention to derive focal points for theory in teacher education (section 2.6.4).

We distinguish *intrinsic* characteristics (2.6.3.1), that are aspects of the internal structure of the theory, and *extrinsic* characteristics (2.6.3.2), that appear in the context in which the theory is used or developed.

2.6.3.1 *Intrinsic characteristics*

Grounding

The domain-specific instructional theory is grounded in reflections on practice (observations) that are colored by the theory-developers’ own experiences and opinions.

Along the way the foundation becomes more ‘objective’ and a more systematic approach to development is taken, among other things through the use of the results of other development researchers and existing theory. The systematic approach can be seen in the cycle from reasoning processes on designing curriculum content, analysis of and reflection on the results from tryouts and the subsequent development of new theory. The quality of the foundation is determined to a large extent by the persuasiveness of the reasoning (e.g., ‘multiplicative reasoning’).

The range of application of theory

The cyclical process that was referred to before, indicates a strong, reciprocal relationship between theory and practice. We therefore speak of the domain-specific instructional theory of multiplication and the practice of curriculum development. Theory and practice in a way ‘question’ each other. Practice, through examples and counterexamples, can confirm, clarify or refute the theory, show dilemmas or evoke new connections for the theory. The theory provides understanding of practice by describing, explaining or predicting it, provides solutions for practical problems, is of assistance in justifying choices and more. The concept of multiplication takes its meaning from a number of practices. These are ‘daily practices’¹⁵, which initially – until the fifteenth century – involved a limited number of professionals such as merchants and the clergy, the practice of mathematicians for whom the concept of multiplication has a fundamental mathematical meaning and the practice of didacticians, those involved in learning and teaching multiplication. For the latter practice, there is a small distance between practice and theory.

The ‘truth’ of theory

A typical characteristic in the development of this theory is the designers’ search for consensus. It is not just a case of keeping on trying, but also of negotiation. There can be two types of consensus, one relating to finding common ground with other theories, the other relating to the desire to find common ground in the discourse with fellow-scientists or with professionals in the relevant field. Here, development of theory is a subtle combination of individual and collective effort. Not only do such attempts at integration of the newly developed theory with existing theories strengthen that theory, they also facilitate the theory’s ongoing development. The threshold of access to the existing paradigm¹⁶ is lowered, along with the opportunity of extending the theory and widening consensus. That wider consensus means that the theory gains in strength and validity.

2.6.3.2 *Extrinsic characteristics*

Theory in action

The realistic theory in the Netherlands started as a response to mechanistic theory and to the 'New Math' movement in the 1960s. The various prototypes of the realistic theory are a result of adapting, modifying and expanding the previous versions. The designer uses and develops theory when he is working on designing and testing his teaching package. He develops theory as a reflection on his practice¹⁷ and makes use of the latest developments in domain-specific instructional theory. In Schön's terminology (1983) we can speak of interaction between 'theory on action' and 'theory in action.' This is a cyclical process. The process of curriculum development, as we can see it in Ter Heege's work, but also with developers following him, can be named a 'theory-led bricolage.' The term has been used by Gravemeijer (1994, p. 110) to describe such a process, namely as the creative work of a kind of 'bricoleur' (handyman) who brainstorms, invents, improves, adjusts and adapts continuously. He uses appropriate tools and materials, whatever is available, to realize a specific function. While such a process is theory-led, theory is also developed in it. In all facets of the process the views of the designers – including their views on the concept of theory itself – help to determine the dynamics of (the development of) the theory.

The developmental process of the theory shows that it has been a process of trial and error, and that it involves continuous development, the theory is never done. Ter Heege (2005) makes that point yet again when he analyzes the developments in teaching multiplication. He describes how Freudenthal's view on memorization processes for learning arithmetic and mathematics developed. This refers to views that are still recognizable, at various levels and in different educational settings, views that have had a lasting influence on the development of theory in this field.

Adoption of the theory

The theory is influenced by a number of factors, which can make or break the adoption of theory. To begin with, there is the influence from other theories. This domain-specific instructional theory contains elements from mathematical theories and from theories of learning. What is unique is the way in which elements from domain-specific instructional theory, from didactical phenomenology and from learning theories are integrated.

Second, there is the influence of the designer-developer with his knowledge, understanding, values, standards and opinions. For example, earlier we discussed the confrontation with his own opinions that designer-developer Ter Heege experienced. His position as learner, teacher researcher and developer all color his theory. Finally, there is the user who influences (the development of) theory, teachers and students provide important impulses for change and are in the end the determining factor in what

becomes of a theory. These three factors reflect the open character and the dynamism of the theory as well as the complexity¹⁸ of the educational setting in which the theory is to be adopted.

2.6.3.3 Summary of the characteristics

As described before, the history of the development of the theory of learning and teaching multiplication shows a drastic evolution. The content of the theory varies from a collection of simple concepts, rules and views, dominated by associative psychology and behaviorism, to a refined, content knowledge based system of concepts, suggestions and guidelines. In all cases there is a strong connection between the theory and ‘the practice of education.’ The relationship with that practice becomes a solid and specific one only after the 1970s. Freudenthal’s mental legacy, specifically his didactic phenomenology of mathematical structures, stimulates the experts involved (developers, researchers, supervisors, teachers) to change their views. Thinking in terms of increasing complexity of the material offered to students changes to thinking about progress in students’ learning processes. Development of theory on that largely occurs as reflection on practical experiences during developmental research. In that respect the theory can be seen as beginning with reflection on practice, which then evolves into a theoretical foundation and justification of that practice. The reciprocal relationship between theory and practice is determined to a large extent by a theory-led, cyclical process of brainstorming, designing, trying, analyzing and reflecting (from the theory); the designers themselves, as an exalted kind of ‘bricoleurs’ keep this process going, while positioning themselves as learners.

The description in the TAL brochure, even more than for Ter Heege and in ‘De Proeve,’ is characterized as a practical theory; that character is partly determined by the practice-oriented goals, as represented in activities, and the immediately included theoretical justification (called ‘theory in activities’). It is a characteristic that is associated with the description Boersma and Looy (1997) give of practical theory¹⁹.

The evolution of the theory for learning to multiply is influenced by its relationship with practice from another point of view as well, as debate and looking for consensus among those involved play an important part in the relationship between theory and practice. In the course of the process of the development of the theory there has been an increase in the consensus about the theory being developed among experts. From the 1970s onwards a close network of experts and interaction with practice develops, which leads to theoretical statements becoming gradually more profound and better understood and accepted in an ever-wider circle. That process – which as a ‘democratic development of a curriculum’ is of great importance for innovation – leads to a more and more balanced and widely-accepted theory.

The *cohesion* of the theory of learning and teaching to multiply is largely determined by views and goals, for example by learning-teaching principles (Treffers, 1991; see also section 3.2.1).

Looking back to the developmental process of the theory, we notice that its cohesion played an important part in the acceptance of the theory.

2.6.4 Focal points for theory in mathematics teacher education

We can deduce focal points for the use of theory in teacher education from the above-mentioned intrinsic and extrinsic characteristics of theory (see figure 2.2). The relevance of the focal points for this study is that they guided the thinking and designing of the student teachers' learning environment (chapters 3, 4 and 5). To illustrate this, some of these focal points will be discussed here.

The focal point for theory '*Underpinning (intended) actions, seeing patterns in student behavior*' (ad. 1 in figure 2.2), derived from the intrinsic characteristic 'Grounding: reasoning, arguing; patterns,' is a first example. In training colleges where interaction is considered of paramount importance, (learning) reasoning for students and their pupils in their practice schools is a core activity. Considered at a higher level the essence of this focal point for the student is especially learning to motivate (intended) actions and seeing patterns in their students' acting and thinking, and learning to capitalize on that. Lampert, in an article called 'Knowing, Doing and Teaching Multiplication,' already points this out in 1986:

In the lessons [grade 4] my role was to bring students' ideas about how to solve or analyze problems into the public forum of the classroom, to referee arguments about whether those ideas were reasonable and to sanction students intuitive use of mathematical principles as legitimate (Lampert, 1986, p. 339).

Furthermore, she contends that the teacher needs a comprehensive repertoire of content and pedagogical content knowledge to be able to provide that style of teaching. Her colleagues recently brought notice to that necessity again (Ball, Hill & Bass, 2005) by pointing out the specific character of the mathematical knowledge required for teaching. Another focal point of theory we found, derived from the intrinsic characteristic 'the truth of theory,' is '*Cogency and consensus*' (ad. 8; figure 2.2). Whether something is 'true' or not in pure, formal multiplication is determined by 'the answer,' and in the case of the teaching method of multiplication based on arguments from experts. In teacher education it is an important part of the discourse that students debate, for example on a didactical principle directed by their teacher educator, and try to *convince* each other and come to a *deliberated consensus*.

Characteristics of the theory	Focal points for theory in mathematics teacher education
<i>Intrinsic characteristics</i> 1. Grounding: reasoning, arguing; patterns. 2. The nature of theory. 3. Value of reality. 4. A relationship to other theories. 5. The beauty of theory. 6. Level of formalization. 7. Subjectivity. 8. The ‘truth’ of theory. 9. The presence of theory-charged examples. 10. The range of application of theory.	- ad. 1. Underpinning (intended) actions, patterns in student behavior. - ad. 2. The nature of the theory that is brought into play (practical wisdom, the ‘common sense caliber,’ perceptual, conceptual, prescriptive, formal). - ad. 3. Reality as a source for the development of theory (in the sense of didactical phenomenology). - ad. 4a. The context for the development of theory (construction of knowledge). - ad. 4b. Personal theoretical network. - ad. 5. Beautiful reasoning and constructs. - ad. 6. Levels of thought and action (concrete-abstract, material-mental, use of models). - ad. 7. Subjective concepts and theories. Personal preferences and beliefs. - ad. 8. Cogency and consensus, e.g., in the discourse. - ad. 9. Narratives as a means to acquire practical knowledge. - ad. 10. (General) validity of theory, generalizing the knowledge and actions of students and teacher.
<i>Extrinsic characteristics</i> 1. The genesis and dynamics of theory. 2. Theory-in-action; theory-on-action. 3. The discourse in the ‘school’ of scientists. 4. Adoption of theory. 5. Judging the merit of theory. 6. Justification of theory. 7. Development of theory on the basis of research.	- ad. 1a. ‘The Spark,’ intuition and creativity. - ad. 1b. Effort and success stories. - ad. 1c. A theory’s history as an object of study. - ad. 2. Theory as a basis for pedagogical reflection on practice (describing, interpreting, clarifying, predicting, capitalizing on situations). - ad. 3. The discourse (deliberation) as the motor of constructing theoretical knowledge. - ad. 4a. Appreciation of theory. - ad. 4b. Becoming aware of one’s own beliefs. - ad. 4c. The tendency to apply theory (developing sensitivity for the use of theory). - ad. 4d. ‘Appropriating’ theory. - ad. 4e. Confidence through applying theory. - ad. 5a. Applicability of theory. - ad. 5b. Explanatory and predictive value of theory. - ad. 6. Justification: empirically-grounded norms of how the theory works. - ad. 7. Acquiring theory through (re)searching.

figure 2.2 Characteristics of the theory and focal points for mathematics teacher education

An *extrinsic* characteristic that came to the fore in the theory of learning to teach multiplication is *the genesis of the theory* (ad. 1; figure 2.2). The history of the development of multiplication makes it abundantly clear that theory is a human invention, meant to make situations (phenomena) transparent and describe them (conveniently). Core ideas for an innovative, realistic didactic of multiplication arose partly because of *coincidental observations or experiences of success* (ad. 1b; figure 2.2) from a developer, albeit that he created a well thought-out ‘design environment.’²⁰ The stories about the genesis of multiplication and its didactics provide us with a number of focal points for the use of theory in teacher training. They make it clear that especially an inspiring (learning) environment may lead to *the ‘spark,’ a discovery or an aha-experience* (ad. 1a; figure 2.2). Experiences of success in their turn may lead to a chain reaction of targeted activities (Janssen, De Hullu & Tigelaar, 2008), increasing the chance that students will use and further develop theory as a matter of course. Finally there is *the history of (learning) multiplication itself* (ad. 1c; figure 2.2) that deserves attention as an object of study within teacher training courses. Ultimately it is knowledge of history that gives students an insight into the foundations of theory, and, by extension, of what they themselves and their students must be able to do and know to become ‘competent’ in this field (Fauvel & Van Maanen, 2000). History does not only show the knowledge, skills and insights that people gradually acquired, it also shows how important motivation and (work) attitudes are for the process of (learning) the acquisition of theory.

Finally, another (extrinsic) focal point has been derived from the characteristic ‘*theory-in-action, theory-on-action*’ (ad. 2; figure 2.2), two expressions that are often cited in literature, and that have been coined by Schön (1983) to distinguish (theoretical) reflection during and outside practical activities. Theory serves a *clarifying and explanatory function* in nearly all stages of the learning and teaching process. We have seen how the designer uses and develops theory while he works on designing, trying and evaluating his teaching package.

For teacher education we can consider (theoretical) reflection – before, during and after practical activities – as an important derived focal point from the above-mentioned characteristic of theory. A good (future) teacher can be recognized partly from the ability to ‘look ahead through looking back.’ In training *theory mainly functions as a basis for orientation for reflection on practice*. Through theoretical knowledge *practice can be understood, explained, predicted or even improved*; in reverse, practice can also shed new light on theory.

For a complete overview of the remaining characteristics of theory and the focal points for teacher education that have been derived from that, we refer to the outline of characteristics and focal points that has been provided earlier (figure 2.2).

2.6.5 Conclusion

2.6.5.1 *Development of theory*

Theories often owe their origin to the creative discovery of one individual. Development of theory can begin where there is a need for explaining or predicting phenomena, for elaborating ideas or for disproving the findings of others. Often, these findings are ‘spin-offs’ of search processes with other goals. Statements that are made have the character of logical arguments, which may be more or less influenced by intuition or opinion. The theory of learning and teaching multiplication shows what might be called ‘signs of an evolution’ namely the growth of a collection of simple rules and beliefs into a sophisticated system of concepts, suggestions and guidelines.

2.6.5.2 *The relationship between theory and practice*

We characterized the theory of learning and teaching multiplication as a practical theory, on the basis of its practice oriented goals and the manner in which the developers founded their theory and justified their views. What this means is that a reciprocal relationship between theory and (teaching) practice is maintained. The history of the theory of learning to multiply shows continuous development, the theory is never finished. Field and thought experiments provide new concepts, principles and guidelines, from which a new version is developed and tested, in a process that shows similarity to the empirical cycle as described by Koningsveld (1992, p. 27). This is of course different from a pure, formal mathematical theory. This is a closed system of concepts, relations, axioms and theorems. The confirmation of hypotheses and the ‘truth’ of statements are derived from mathematical proofs. In learning to multiply the theory is lent persuasiveness through theoretical reflection on the outcome of experiments; the developers – at least those from later than 1970 – attempt to strengthen that persuasiveness even further by the continuous pursuit of consensus between one’s own practical experience, the experience of professionals in the field and existing theories, with crucial justification coming from the theory’s effects on students’ learning processes. Each theory, to a certain point, arises from a practice situation and maintains a reciprocal (reflexive) relationship with that practice. Theory makes it simpler and more efficient to perform in practice, while practice in its turn, as an application in reality, clarifies thought about the theory.

‘Production and use’ of theory (Fenstermacher, 1986) are not as strongly interwoven with each other in every theory as they are in the (development of) theory of learning and teaching multiplication, where the professional practice of teaching provides the source for the development of theory and where – in reverse – theory provides direction to practice. We see here a parallel between the development of curricula in primary education and for training teachers in primary education (Goffree, 1979).

An important factor that determines differences in the relationship between theory and practice is the way in which theory is used. Theory can be used for instance to test

practice (and the other way around) or to anticipate on practice; in the latter case, it is a matter of ‘theory in action’ or ‘theory on action’ (Schön, 1983). If the practitioner concludes a ‘transaction with the situation’ (Schön, 1983), it is a matter of working and researching in practice as a ‘reflective practitioner’ with no division between knowing and doing. Schön distinguishes that method of working from a technical-rational approach, where activities take place on the basis of more external considerations that have been derived from scientific study.

Other factors that determine the relationship between theory and practice are the extent to which theory influences or even guides practice (Eraut, 1994b) and the extent to which professional characteristics of the practitioner, such as knowledge, insights, skills, attitude and beliefs (Hofer & Pintrich, 1997; Lampert, 2001; Thiessen, 2000; Verloop et al., 2001) encourage or inhibit the integration of practice and theory.

2.6.5.3 *Legitimizing theory*

As well as by its relationship to practice, the character of a theory is determined by the way in which statements are underpinned. That foundation in fact determines the ‘strictness’ of the theory. A (formal) mathematical theory is in that sense a strict theory that all statements can be verified (proven) based on the given concepts, relations and axioms. Statements in a purely empirical theory – for instance Gestalt theory – are often founded on and tested through field experiments. The validity however is of a different caliber than that of a formal theory, where statements can for instance be labeled with one of two values: true or untrue. Statements in an empirical theory have a degree of truth that can at best be expressed in a degree of probability (smaller than 1).

According to the empirical part of the theory of learning and teaching multiplication, statements are mainly empirically justified by experiments and achieving consensus within a paradigm, against the background of learning-teaching principles and goals. Within such a practical theory ‘clarifying and predicting’ as well as ‘explaining and understanding’ have therefore often the function of theoretical justification of practice-oriented goals and activities. The yield for ‘users’ of the theory is high; teachers who have such a theory, can ‘see’ more in similar teaching situations and can therefore think and talk about them in a more differentiated manner (Tom & Valli, 1990; Fenstermacher & Richardson, 1993).

The consensus and validity together with the relationship to practice – especially the relevant causality process (Maxwell, 2004) – , provide the scientific basis for the theory of learning and teaching to multiply.

2.6.5.4 *Revision of the working definition*

In the introduction to this chapter (section 2.6.1.2) we formulated the definition of theory as a collection of cohesive, descriptive concepts. Such a system of grounded coherence can contain statements and argumentation, as well as explanations,

assumptions, conjectures, predictions and proofs. While this working definition contains the main earlier-mentioned intrinsic characteristic ('grounding'), it lacks – certainly as a domain specific instruction theory – important elements such as the relationship to practice, and characteristics we described as extrinsic. Also, in this definition the concept of cohesion is linked to one-sided underpinning, which is a rather meager interpretation of the concept of grounding as described earlier. In the case of the theory of learning to multiply cohesion derives from the underlying concepts of the operation multiplication and from consensus concerning the view on learning (to multiply). In our view, which is colored by our affinity with teacher training, theory arises from reflection on reality. That can be translated as reflection on practice, with reality being seen as a 'collection of practices.' That reflection can represent itself in many ways and on many levels, from a personal practical theory full of individual views, to a scientific theory that can be expressed in theoretical concepts and laws.

In summary we can describe the revised definition of a domain-specific theory – which will never be 'definitive' – as a collection of descriptive concepts that show cohesion, with that cohesion being supported by reflection on 'practice.' The character of the theory is determined by the extent to which intrinsic and extrinsic characteristics manifest themselves.

2.6.5.5 *Practice as a starting point, theory as the basis for orientation for reflection on that practice*

Personal experience (as teacher, teacher educator and researcher) teaches that, particularly in the domain-specific instructional theory of multiplication, there is a parallel between the activities of developers and users. Presumably, this is partly due to the practice relevance and the phenomenological character of the theory, and the specific approach of the developers' activities. Possibly that parallel is a measure for the practical value of an educational theory. It is a characteristic of activities that is to some degree related to that of teacher educators in this specific professional field. In our view on teacher training, practice is the starting point for the professional development of students and the theory of realistic mathematics education is the basis for orientation for reflection on that practice. Students create their own (practical) knowledge, that will generally have a narrative character (Goffree & Dolk, 1995). The underlying thought is that, through reflection on 'theory-laden' practical situations, students will integrate theory into practical knowledge, and will so acquire 'theory-enriched practical knowledge' (EPK; Oonk, Goffree & Verloop, 2004).

2.6.6 Perspective

For the study of the way in which students deal with theory, it is essential to probe the characteristics of theory and their meaning for teacher training. For that reason, for the theory of learning and teaching to multiply we have considered the characteristics we

found from the perspective of theory in training. We have not (yet) considered the question of how teachers (in training) can deal with theory or theoretical knowledge. As a prelude to that exploration, there are some suitable statements from Freudenthal, who was not only a designer of groundwork for pure mathematics, but also a developer of didactics. He tried to bring attention to essential elements of theory through the use of stories from practice. He believed that you cannot comprehend theories from hypotheses and theorems, but from concrete examples²¹. He thought the selection of those examples was of eminent significance, as can be seen from his statement: “It is much less difficult to overwhelm the learner with a shower of countless examples, than to look for that one – paradigmatic – example that works.” (Freudenthal, 1984b, p. 102). Those who want to familiarize themselves with theory can learn from this in so far that practice itself, or typical stories and other representations of practice, form a rich source and a useful starting point. Translating this to teacher training, that starting point can be elaborated into an approach in which students develop a repertoire of ‘theory-enriched practical knowledge.’

The focal points for the use of theory in training serve as benchmarks for designing and improving the learning environment of the student teachers in the different stages of the research process.

2.7 The learning environment

2.7.1 Orientations for designing learning environments

The concept of a learning environment can be interpreted differently, but is frequently described as a context that has to yield, to guide, and to keep going learners’ required learning processes in order to reach the desired learning results. A learning environment is part of the curriculum (Lowyck & Terwel, 2003). Already at the beginning of the previous century, scientists wrote about the context in which learning should take place. For example, Dewey expounds in 1916 his ideas about the characteristics of an educational (learning) environment. (Dewey, 1916; Hansen, 2002). Vergnaud (1983) describes the context in which students learn in terms of ‘mastering situations’ and calls a mastered collection of situations a ‘conceptual field.’ The learner (e.g., math student) ‘masters’ a conceptual field if he or she masters several concepts of a different nature. The learner has to invent how different formulations and symbols should be used for concepts and subjects that appear in different contexts. Lampert (2001) applies the idea of ‘conceptual fields’ not just to learning but also to teaching. Other than in curricula where concepts are offered linearly or spiral shaped, she considers her approach of teaching as facing students with conceptual fields. In that way the frequently capricious learners’ learning – and also student teachers’ learning, is given chances to develop by using a conceptual field that anticipates on the learning process. The conceptual field as such can be considered as a component of the learning environment.

In the Netherlands sometimes the word ‘learning landscape’ is used as a synonym for learning environment. It is a concept that became fashionable at Pabos and elsewhere in the 1990s; it means little more than a material interpretation of a part of the student teachers’ learning environment. At some stage the word has been used as a substitute of the term ‘werkplaats’ (workshop), a name for a study space where student teachers can work freely with textbooks and teaching materials. Fosnot & Dolk (2001) give another interpretation to the term learning landscape, one that shows affinity with the idea of conceptual fields. They consider a learning landscape for primary mathematics education as formed by big ideas, strategies and models; those can help teachers with developing hypothetical learning trajectories. This idea was developed by Simon (1995), who considers it as a ‘learning and teaching framework.’ The framework is hypothetical, because you can never be sure about students’ doing or thinking.

Gradually, as a teacher you have to adapt to the learning trajectory that students show. The big ideas, strategies and models give the teacher guidance in the landscape; sometimes they become means to choose the environment or the context. For students they provide another view and the progress in their mathematical development. According to that view, the learning landscape for student teachers can be considered as an ordered collection of important moments in the student teachers’ development. Thus, at the level of teacher education, the concept of learning landscape can be seen – as in primary education – as a learning and teaching model and so as a component of the learning environment Pabo.

Verschaffel distinguishes five basic principles for designing powerful²² learning environments:

- many types of content matter, cognitive and meta-cognitive knowledge and skills have to be acquired coherently, because they play a meaningful, complementary role;
- acquiring knowledge and skills actively and constructive by the learners have to be guided by adapted forms of aid or support by the educator;
- among other things, the idea that learning is embedded in a context implicates for the development of the learning environment the need for a process of decontextualizing;
- not only the role of the educator is of consequence for acquiring knowledge and skills actively and constructive, but also the interaction and cooperation with fellow-students;
- there should also be systematic attention on the dynamic-affective aspects of the teaching-learning process (Verschaffel 1995, p. 181).

Lowyck & Terwel subscribe to Verschaffel’s view that designing learning environments concerns not only content matter knowledge, but also strategic knowledge and meta-

cognitive knowledge. That can be achieved through orientation, through supporting the construction of knowledge, through making the learner aware of the performed cognitive activities and through further support on self-regulation (Lowyck & Terwel, 2003, p. 296). The present study will take the aforementioned orientations into consideration, in particular the intention to create a balance between:

- the individual and social aspects of knowledge and knowing (Tough, 1971; Kieran, Forman & Sfard, 2001);
- subject content, pedagogical content, general content education and competences (Klep & Paus, 2006);
- self-regulation by student teachers and aid and support from their teacher educators (Vermunt & Verloop, 1999);
- the school practice of the prospective teachers and the desired professional practice of the teacher training institutes (Richardson, 1992).

Thinking in terms of learning environments respects the individual student teachers' own identity and own development. According to this framework, we interpret the student teachers' learning as a cognitive process of individual construction in a social process of participation in a group and of interaction with the learning environment. In section 2.3.1 we stated that the learning environment of (student) teachers will in any case need the feeding – both implied and explicit – with a variety of theories and theory laden stories, and furthermore also requires the guidance of an expert in order to level up the student teachers' 'practical reasoning' (Fenstermacher, 1986). Also, we mentioned the important role of the expert, who has to be aware of the importance of learning through interaction (Elbers, 1993) and of 'constructive coaching' (Bakker et al., 2008).

To move students to practical reasoning, it is important to look for content and ways of working that naturally inspire them. These may be for example questions that are confrontational or that evoke discussion, leading to 'didactical conflicts' that require cognitive flexibility (Spiro, Coulson, Feltovich & Anderson, 1988). A challenging or even confusing situation may arise, leading to a natural need to find a solution, a phenomenon that Piaget describes (1974, p. 264) as a process of reaching equilibration from perturbation. According to Ruthven (2001, p. 167) an 'ideal' level of reasoning is reached when students have achieved the ability of 'practical theorising.' In his description of that ability he refers to Alexander (1984, p. 146) who believes that "students should be encouraged to approach their own practice with the intention of testing hypothetical principles drawn from the consideration of different types of knowledge."

2.7.2 Design research

The described orientations in section 2.7.1 – combined with the other implications mentioned in this chapter – served to inspire the design of the learning environment in both the way it was shaped and its use by pre-service teachers. The learning environment

– necessary to develop in favor of elaborating and answering the research questions – was gradually refined in accordance with the approach to developmental research or educational design research (Gravemeijer, 1994; Cobb, 2000; Gravemeijer & Cobb, 2006). Educational design research is the systematic study of designing, developing and evaluating educational programs, processes and products. The process can yield considerable insight into how to optimize educational interventions and better understand teaching and learning. In the whole development process of designing the learning environment(s) – cyclic in nature – four stages are distinguished.

In the first stage of designing the learning environments for the series of the four research projects in this study ('The first exploratory research'; see section 3.5), the learning environment of the student teachers was composed of ten lessons on CD-roms, information about the lessons (Oonk, 1999) and the first version of a computer search engine. The study was mainly focused on knowledge construction. The actors were two student teachers and their participating teacher educator-researcher. The learning environment in the second stage ('The second exploratory research'; see section 3.8) comprised the new MILE course 'The Foundation' for second year student teachers (Dolk, Goffree, Den Hertog & Oonk, 2000). The teacher educator gave his students a list of 150 key theoretical concepts from previous courses, to serve as a theoretical framework to help student teachers to value their theoretical knowledge. Ten two-hour meetings were held. Following the method of triangulation (Maso & Smaling, 1998), four pairs of student teachers within two classes of 25 student teachers were observed and interviewed, and a participating study of the group work with two student teachers was conducted. The third stage was the small scale research (see chapter 4). The learning environment for the 14 participating student teachers – a group of 6 respectively 8 – was composed of a variant of MILE, including one CD-rom 'The Guide' (Goffree, Markusse, Munk & Olofsen, 2003; see section 4.2.2.2). The improvement to the learning environment in comparison to the previous one was first of all a change with respect to the student teachers' possibilities to relate theory to practice, a change to a 'theory-enriched' environment. The development of this third stage of designing the learning environment was preceded by a try-out of components, in fact an extra 'cycle' in terms of design research.

For the fourth and last stage of designing – conducting the large scale study (see chapter 5) – the learning environment was furthermore optimized to enable the research concerning the refined search questions.

In each of the four stages one can recognize the three broad phases in the process of conducting a design project, as described by Cobb & Gravemeijer (2008), namely: preparing, experimenting in class or group, and conducting a retrospective analysis. For example, components of preparing were: clarifying goals for student teachers and their teacher educators, designing concepts lists and 'initial assessments' as an orientation

basis for both student teachers and teacher educators and not the least: deliberating on and choosing the theoretical foundation of the related research. An example in the stage of conducting a retrospective analysis is the development of a reflection-analysis tool to analyze student teachers' reflective notes.

This component of the cycles of design and analysis started with deliberating and gathering notions of possibilities to gauge student teachers' reflections in the exploratory researches and, ended in the design of a reliable reflection-analysis tool.

The next chapters will provide more details of the process of designing the learning environment.