

## Weak localization coexisting with a magnetic field in a normal-metal–superconductor microbridge

P. W. Brouwer and C. W. J. Beenakker

*Instituut-Lorentz, University of Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands*

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A random-matrix theory is presented which shows that breaking time-reversal symmetry by itself does *not* suppress the weak-localization correction to the conductance of a disordered metal wire attached to a superconductor. Suppression of weak localization requires applying a magnetic field as well as raising the voltage, to break both time-reversal symmetry and electron-hole degeneracy. A magnetic-field-dependent contact resistance obscured this anomaly in previous numerical simulations.

Weak localization is a quantum correction of order  $e^2/h$  to the classical conductance of a metal.<sup>1</sup> The word “localization” refers to the negative sign of the correction,<sup>2</sup> while the adjective “weak” indicates its smallness. In a wire geometry the weak-localization correction takes on the universal value<sup>3</sup>  $\delta G = -\frac{2}{3}e^2/h$  at zero temperature, independent of the wire length  $L$  or mean free path  $\ell$ .<sup>4</sup> The classical (Drude) conductance  $G_0 = (N\ell/L)e^2/h$  is much greater than  $\delta G$  in the metallic regime, where the number of scattering channels  $N \gg L/\ell$ . Theoretically, the weak-localization correction is the term of order  $N^0$  in an expansion of the average conductance  $\langle G \rangle = G_0 + \delta G + \mathcal{O}(N^{-1})$  in powers of  $N$ . Experimentally,  $\delta G$  is measured by application of a weak magnetic field  $B$ , which suppresses the weak-localization correction but leaves the classical conductance unaffected.<sup>5</sup> The suppression occurs because weak localization requires time-reversal symmetry ( $\mathcal{T}$ ). In the absence of  $\mathcal{T}$ , quantum corrections to  $G_0$  are of order  $N^{-1}$  and not of order  $N^0$ . As a consequence, the magnetoconductance has a dip around  $B = 0$  of magnitude  $\delta G$  and width of order  $B_c$  (being the field at which one flux quantum penetrates the conductor).

What happens to weak localization if the normal-metal wire is attached at one end to a superconductor? This problem has been the subject of active research.<sup>6–12</sup> The term  $G_0$  of order  $N$  is unaffected by the presence of the superconductor.<sup>6</sup> The  $\mathcal{O}(N^0)$  correction  $\delta G$  is increased but remains universal,<sup>9,10</sup>

$$\delta G = -(2 - 8\pi^{-2})e^2/h \approx -1.19e^2/h. \quad (1)$$

In all previous analytical work zero magnetic field was assumed. It was surmised, either implicitly or explicitly,<sup>7</sup> that  $\delta G = 0$  in the absence of  $\mathcal{T}$  — but this was never actually calculated analytically. We have now succeeded in doing this calculation and would like to report the result, which was entirely unexpected.

We find that a magnetic field by itself is not sufficient to suppress the weak-localization correction, but only reduces  $\delta G$  by about a factor of 2. To achieve  $\delta G = 0$  requires in addition the application of a sufficiently large voltage  $V$  to break the degeneracy in energy between the electrons (at energy  $eV$  above the Fermi energy  $E_F$ ) and the Andreev-reflected holes (at energy  $eV$  below  $E_F$ ). The electron-hole degeneracy ( $\mathcal{S}$ ) is effectively broken when  $eV$  exceeds the

Thouless energy  $E_c = \hbar v_F \ell / L^2$  (with  $v_F$  the Fermi velocity). Weak localization coexists with a magnetic field as long as  $eV \ll E_c$ . Our analytical results are summarized in Table I. These results disagree with the conclusions drawn in Ref. 7 on the basis of numerical simulations. We have found that the numerical data on the weak-localization effect were misinterpreted due to the presence of a magnetic-field-dependent contact resistance, which was not understood at that time.

The starting point of our calculation is the general relation between the differential conductance  $G = dI/dV$  of the normal-metal–superconductor (NS) junction and the transmission and reflection matrices of the normal region,<sup>6</sup>

$$G = (4e^2/h) \text{tr } m(eV)m^\dagger(eV), \quad (2a)$$

$$m(\varepsilon) = t'(\varepsilon)[1 - \alpha(\varepsilon)r^*(-\varepsilon)r(\varepsilon)]^{-1}t^*(-\varepsilon), \quad (2b)$$

where  $\alpha(\varepsilon) \equiv \exp[-2i \arccos(\varepsilon/\Delta)]$ . Equation (2) holds for subgap voltages  $V \leq \Delta/e$ , and requires also  $\Delta \ll E_F$  ( $\Delta$  being the excitation gap in  $S$ ). We assume that the length  $L$  of the disordered normal region is much greater than the superconducting coherence length  $\xi = (\hbar v_F \ell / \Delta)^{1/2}$ . This implies that the Thouless energy  $E_c \ll \Delta$ . In the voltage range  $V \leq E_c/e$  we may therefore assume that  $eV \ll \Delta$ , hence  $\alpha = -1$ . The  $N \times N$  transmission and reflection matrices  $t$ ,  $t'$ ,  $r$ , and  $r'$  form the scattering matrix  $S(\varepsilon)$  of the disordered normal region ( $N$  being the number of propagating modes at the Fermi level, which corresponds to  $\varepsilon = 0$ ). It is convenient to use the polar decomposition

$$\begin{pmatrix} r' & t' \\ t & r \end{pmatrix} = \begin{pmatrix} v_1 & 0 \\ 0 & w_1 \end{pmatrix} \begin{pmatrix} i\sqrt{R} & \sqrt{T} \\ \sqrt{T} & i\sqrt{R} \end{pmatrix} \begin{pmatrix} v_2 & 0 \\ 0 & w_2 \end{pmatrix}.$$

TABLE I. Dependence of the weak-localization correction  $\delta G$  of a normal-metal wire attached to a superconductor on the presence or absence of time-reversal symmetry ( $\mathcal{T}$ ) and electron hole-degeneracy ( $\mathcal{S}$ ). The entry in the upper-left corner was computed in Refs. 9 and 10.

$-\delta G[e^2/h]$	$\mathcal{T}$	no $\mathcal{T}$
$\mathcal{S}$	$2 - 8/\pi^2$	$2/3$
no $\mathcal{S}$	$4/3$	$0$

Here  $v_1$ ,  $v_2$ ,  $w_1$ , and  $w_2$  are  $N \times N$  unitary matrices,  $T$  is a diagonal matrix with the  $N$  transmission eigenvalues  $T_i \in [0, 1]$  on the diagonal, and  $R = 1 - T$ . Using this decomposition, and substituting  $\alpha = -1$ , Eq. (2b) can be replaced by

$$m(\varepsilon) = \sqrt{T(\varepsilon)} [1 + u(\varepsilon) \sqrt{R(-\varepsilon)} u^*(-\varepsilon) \sqrt{R(\varepsilon)}]^{-1} \times u(\varepsilon) \sqrt{T(-\varepsilon)}, \quad u(\varepsilon) \equiv w_2(\varepsilon) w_1^*(-\varepsilon). \quad (2c)$$

We perform our calculations in the general framework of random-matrix theory. The only assumption about the distribution of the scattering matrix that we make is that it is isotropic, i.e., that it depends only on the transmission eigenvalues.<sup>13</sup> In the presence of  $\mathcal{T}$  (for  $B \ll B_c$ ),  $S = S^T$ , hence  $w_1 = w_2^T$ . (The superscript  $T$  denotes the transpose of a matrix.) If  $\mathcal{T}$  is broken,  $w_1$  and  $w_2$  are independent. In the presence of  $\mathcal{D}$  (for  $eV \ll E_c$ ), the difference between  $S(eV)$  and  $S(-eV)$  may be neglected. If  $\mathcal{D}$  is broken,  $S(eV)$  and  $S(-eV)$  are independent. Of the four entries in Table I, the case that both  $\mathcal{T}$  and  $\mathcal{D}$  are present is the easiest, because then  $u = 1$  and Eq. (2a) simplifies to<sup>6</sup>

$$G = (4e^2/h) \sum_n T_n^2 (2 - T_n)^{-2}. \quad (3)$$

The conductance is of the form  $G = \sum_n f(T_n)$ , known as a linear statistic on the transmission eigenvalues. General formulas<sup>9,10</sup> for the weak-localization correction to the average of a linear statistic lead directly to Eq. (1). The three other entries in Table I, where either  $\mathcal{T}$  or  $\mathcal{D}$  (or both) are broken, are more difficult because  $G$  is no longer a linear statistic. We consider these three cases in separate paragraphs.

(1)  $\mathcal{D}$ , no  $\mathcal{T}$ . Because of the isotropy assumption,  $w_1$  and  $w_2$ , and hence  $u$ , are uniformly distributed in the unitary group  $\mathcal{U}(N)$ . We may perform the average  $\langle \dots \rangle$  over the ensemble of scattering matrices in two steps:  $\langle \dots \rangle = \langle \langle \dots \rangle_u \rangle_T$ , where  $\langle \dots \rangle_u$  and  $\langle \dots \rangle_T$  are, respectively, the average over the unitary matrix  $u$  and over the transmission eigenvalues  $T_i$ . We compute  $\langle \dots \rangle_u$  by an expansion in powers of  $N^{-1}$ . To integrate the rational function (2) of  $u$  over  $\mathcal{U}(N)$ , we first expand it into a geometric series and then use the general rules for the integration of polynomials of  $u$ .<sup>14,15</sup> The polynomials we need are

$$\langle G \rangle_u = \frac{4e^2}{h} \sum_{p,q=0}^{\infty} M_{pq}, \quad (4a)$$

$$M_{pq} = \langle \text{tr } T(u \sqrt{R} u^* \sqrt{R})^p u T u^\dagger (\sqrt{R} u^T \sqrt{R} u^\dagger)^q \rangle_u. \quad (4b)$$

Neglecting terms of order  $N^{-1}$ , we find

$$M_{pq} = \begin{cases} N \tau_1^2 (1 - \tau_1)^{2p} & \text{if } p = q, \\ \tau_1 (\tau_1^2 + \tau_1 - 2\tau_2) (1 - \tau_1)^{p+q-1} - 2 \min(p, q) \\ \times \tau_1^2 (\tau_1^2 - \tau_2) (1 - \tau_1)^{p+q-2} & \text{if } |p - q| \text{ odd,} \\ 0 & \text{else,} \end{cases}$$

where we have defined the moment  $\tau_k = N^{-1} \sum_i T_i^k$ . The summation over  $p$  and  $q$  leads to

$$\frac{h}{4e^2} \langle G \rangle_u = \frac{N \tau_1}{2 - \tau_1} - \frac{4\tau_1 - 2\tau_1^2 + 2\tau_1^3 - 4\tau_2}{\tau_1(2 - \tau_1)^3}. \quad (5)$$

It remains to average over the transmission eigenvalues. Since  $\tau_k$  is a linear statistic, we know that its sample-to-sample fluctuations  $\delta \tau_k \equiv \tau_k - \langle \tau_k \rangle$  are an order  $1/N$  smaller than the average.<sup>13</sup> Hence

$$\langle f(\tau_k) \rangle_T = f(\langle \tau_k \rangle) [1 + \mathcal{O}(N^{-2})], \quad (6)$$

which implies that we may replace the average of the rational function (5) of the  $\tau_k$ 's by the rational function of the average  $\langle \tau_k \rangle$ . This average has the  $1/N$  expansion

$$\langle \tau_k \rangle = \langle \tau_k \rangle_0 + \mathcal{O}(N^{-2}), \quad (7)$$

where  $\langle \tau_k \rangle_0$  is  $\mathcal{O}(N^0)$ . There is no term of order  $N^{-1}$  in the absence of  $\mathcal{T}$ . From Eqs. (5)–(7) we obtain the  $1/N$  expansion of the average conductance,

$$\frac{h}{4e^2} \langle G \rangle = \frac{N \langle \tau_1 \rangle_0}{2 - \langle \tau_1 \rangle_0} - \frac{4 \langle \tau_1 \rangle_0 - 2 \langle \tau_1 \rangle_0^2 + 2 \langle \tau_1 \rangle_0^3 - 4 \langle \tau_2 \rangle_0}{\langle \tau_1 \rangle_0 (2 - \langle \tau_1 \rangle_0)^3} + \mathcal{O}(N^{-1}). \quad (8)$$

Equation (8) is generally valid for any isotropic distribution of the scattering matrix. We apply it to the case of a disordered wire in the limit  $N \rightarrow \infty$ ,  $\ell/L \rightarrow 0$  at constant  $N\ell/L$ . The moments  $\langle \tau_k \rangle_0$  are given by<sup>3</sup>

$$\langle \tau_1 \rangle_0 = \ell/L, \quad \langle \tau_2 \rangle_0 = \frac{2}{3} \ell/L. \quad (9)$$

Substitution into Eq. (8) yields the weak-localization correction  $\delta G = -\frac{2}{3} e^2/h$ , cf. Table I.

(2)  $\mathcal{T}$ , no  $\mathcal{D}$ . In this case one has  $u^\dagger(-eV) = u(eV)$  and  $u(eV)$  is uniformly distributed in  $\mathcal{U}(N)$ . A calculation similar to that in the previous paragraph yields for the average over  $u$ :

$$\frac{h}{4e^2} \langle G \rangle_u = N \tau_+ \tau_- (\tau_+ + \tau_- - \tau_+ \tau_-)^{-1} + (\tau_+ + \tau_- - \tau_+ \tau_-)^{-3} [2\tau_+^2 + \tau_-^2 - \tau_+^3 + \tau_-^3 - \tau_+^2 \tau_- - \tau_-^2 \tau_+ + \tau_+ \tau_-^3 + \tau_-^3 \tau_+], \quad (10)$$

where we have abbreviated  $\tau_{k\pm} = \tau_k(\pm eV)$ . The next step is the average over the transmission eigenvalues. We may still use Eq. (6), and we note that  $\langle \tau_k(\varepsilon) \rangle \equiv \langle \tau_k \rangle$  is independent of  $\varepsilon$ . [The energy scale for variations in  $\langle \tau_k(\varepsilon) \rangle$  is  $E_F$ , which is much greater than the energy scale of interest  $E_c$ .] Instead of Eq. (7) we now have the  $1/N$  expansion

$$\langle \tau_k \rangle = \langle \tau_k \rangle_0 + N^{-1} \delta \tau_k + \mathcal{O}(N^{-2}), \quad (11)$$

which contains also a term of order  $N^{-1}$  because of the presence of  $\mathcal{T}$ . The  $1/N$  expansion of  $\langle G \rangle$  becomes

$$\frac{h}{4e^2} \langle G \rangle = \frac{N \langle \tau_1 \rangle_0}{2 - \langle \tau_1 \rangle_0} + \frac{2 \delta \tau_1}{(2 - \langle \tau_1 \rangle_0)^2} + \frac{2 \langle \tau_1 \rangle_0^2 - 2 \langle \tau_1 \rangle_0^3 - 2 \langle \tau_2 \rangle_0 + 2 \langle \tau_1 \rangle_0 \langle \tau_2 \rangle_0}{\langle \tau_1 \rangle_0 (2 - \langle \tau_1 \rangle_0)^3} + \mathcal{O}(N^{-1}). \quad (12)$$

For the application to a disordered wire we use again Eq. (9) for the moments  $\langle \tau_k \rangle_0$ , which do not depend on whether  $\mathcal{T}$  is broken or not. We also need  $\delta\tau_1$ , which in the presence of  $\mathcal{T}$  is given by<sup>3</sup>  $\delta\tau_1 = -\frac{1}{3}$ . Substitution into Eq. (12) yields  $\delta G = -\frac{4}{3}e^2/h$ , cf. Table I.

(3) *No  $\mathcal{T}$ , no  $\mathcal{L}$ .* Now  $u(eV)$  and  $u(-eV)$  are independent, each with a uniform distribution in  $\mathcal{N}(N)$ . Carrying out the two averages over  $\mathcal{N}(N)$  we find

$$\frac{h}{4e^2} \langle G \rangle_u = \frac{N\tau_1 + \tau_1 -}{\tau_1 + + \tau_1 - - \tau_1 + \tau_1 -}. \quad (13)$$

The average over the transmission eigenvalues becomes

$$\frac{h}{4e^2} \langle G \rangle = \frac{N\langle \tau_1 \rangle_0}{2 - \langle \tau_1 \rangle_0} + \mathcal{O}(N^{-1}), \quad (14)$$

where we have used that  $\delta\tau_1 = 0$  because of the absence of  $\mathcal{T}$ . We conclude that  $\delta G = 0$  in this case, as indicated in Table I.

This completes the calculation of the weak-localization correction to the average conductance. Our results, summarized in Table I, imply a universal  $B$  and  $V$  dependence of the conductance of an NS microbridge. Raising first  $B$  and then  $V$  leads to two subsequent increases of the conductance, while raising first  $V$  and then  $B$  leads first to a decrease and then to an increase.

So far we have only considered the  $\mathcal{O}(N^0)$  correction  $\delta G$  to  $\langle G \rangle = G_0 + \delta G$ . What about the  $\mathcal{O}(N)$  term  $G_0$ ? From Eqs. (8), (12), and (14) we see that if either  $\mathcal{T}$  or  $\mathcal{L}$  (or both) are broken,

$$G_0 = \frac{4e^2}{h} \frac{N\langle \tau_1 \rangle_0}{2 - \langle \tau_1 \rangle_0} = (2e^2/h)N(\frac{1}{2} + L/\ell)^{-1}. \quad (15)$$

In the second equality we substituted<sup>3</sup>  $\langle \tau_1 \rangle_0 = (1 + L/\ell)^{-1}$ , which in the limit  $\ell/L \rightarrow 0$  reduces to Eq. (9). If both  $\mathcal{T}$  and  $\mathcal{L}$  are unbroken, then we have instead the result<sup>16</sup>

$$G_0 = (2e^2/h)N[1 + L/\ell + \ell(\ell/L)]^{-1}. \quad (16)$$

The difference between Eqs. (15) and (16) is a contact resistance, which equals  $h/4Ne^2$  in Eq. (15) but is twice as large in Eq. (16). In contrast, in a normal-metal wire the contact resistance is  $h/2Ne^2$ , independent of  $B$  or  $V$ . The  $B$ - and  $V$ -dependent contact resistance in an NS junction is superimposed on the  $B$ - and  $V$ -dependent weak-localization correction. Since the contribution to  $\langle G \rangle$  from the contact resistance is of order  $(e^2/h)N(\ell/L)^2$ , while the weak-localization correction is of order  $e^2/h$ , the former can only be ignored if  $N(\ell/L)^2 \ll 1$ . This is an effective restriction to the diffusive metallic regime, where  $\ell/L \ll 1$  and  $N\ell/L \gg 1$ . To measure the weak-localization effect without contamination from the contact resistance if  $N(\ell/L)^2$  is not  $\ll 1$ , one has two options: (1) measure the  $B$  dependence at fixed  $V \gg E_c/e$ ; (2) measure the  $V$  dependence at fixed  $B \gg B_c$ . In both cases we predict an increase of the conductance, by an amount  $\frac{4}{3}e^2/h$  and  $\frac{2}{3}e^2/h$ , respectively. In contrast, in the normal-state weak localization leads to a  $B$  dependence, but not to a  $V$  dependence.

We performed numerical simulations similar to those of Ref. 7 in order to test the analytical predictions. The disordered normal region was modeled by a tight-binding

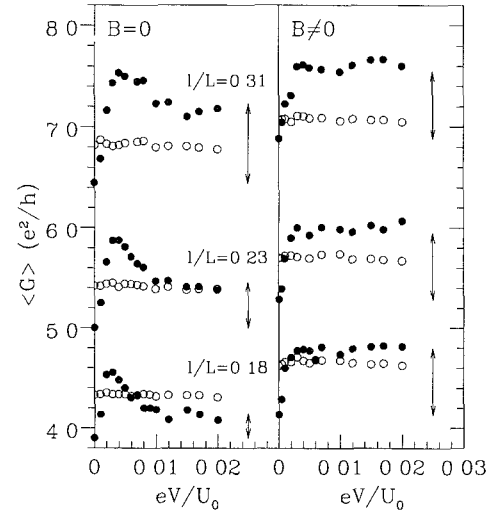


FIG. 1. Numerical simulation of the voltage dependence of the average differential conductance for  $B=0$  (left panel) and for a flux  $6 h/e$  through the disordered normal region (right panel). The filled circles are for an NS junction; the open circles represent the  $V$ -independent conductance in the normal state. The three sets of data points correspond, from top to bottom, to  $\ell/L = 0.31$ ,  $0.23$ , and  $0.18$ , respectively. The arrows indicate the theoretically predicted net increase of  $\langle G \rangle$  between  $V=0$  and  $V \gg E_c/e$ .

Hamiltonian on a square lattice (lattice constant  $a$ ), with a random impurity potential at each site (uniformly distributed between  $\pm \frac{1}{2}U_d$ ). The Fermi energy was chosen at  $E_F = 1.57U_0$  from the band bottom ( $U_0 = \hbar^2/2ma^2$ ). The length  $L$  and width  $W$  of the disordered region are  $L = 167a$ ,  $W = 35a$ , corresponding to  $N = 15$  propagating modes at  $E_F$ . The mean free path is obtained from the conductance  $G = (2e^2/h)N(1 + L/\ell)^{-1}$  of the normal region in the absence of  $\mathcal{T}$ . The scattering matrix of the normal region was computed numerically at  $\varepsilon = \pm eV$ , and then substituted into Eq. (2a) to obtain the differential conductance.

In Fig. 1 we show the  $V$  dependence of  $G$  (averaged over some  $10^3$  impurity configurations) for three values of  $\ell$ . The left panel is for  $B=0$  and the right panel for a flux of  $6 h/e$  through the disordered region. The  $V$  dependence for  $B=0$  is mainly due to the contact resistance effect of order  $N(\ell/L)^2$ , and indeed one sees that the amount by which  $G$  increases depends significantly on  $\ell$ .<sup>17</sup> The  $V$  dependence in a  $\mathcal{T}$ -violating magnetic field is entirely due to the weak-localization effect, which should be insensitive to  $\ell$  (as long as  $\ell/L \ll 1 \ll N\ell/L$ ). This is indeed observed in the simulation. Quantitatively, we would expect that application of a voltage increases  $\langle G \rangle$  by an amount  $\frac{2}{3}e^2/h$  for the three

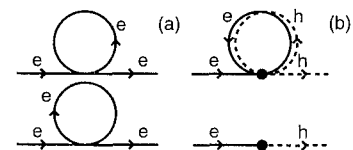


FIG. 2. (a) Two Feynman paths interfering constructively in the presence of  $\mathcal{T}$ . (b) Two paths involving Andreev reflection (solid dot), which interfere destructively both in the presence and absence of  $\mathcal{T}$ .

curves in the right panel, which agrees very well with what is observed. In the absence of a magnetic field the analytical calculation predicts a net increase in  $\langle G \rangle$  by 0.79, 0.46, and  $0.25 \times e^2/h$  (from top to bottom), which is again in good agreement with the simulation.

In normal metals, the weak-localization correction  $\delta G$  is explained in terms of constructive interference of pairs of time-reversed Feynman paths [Fig. 2(a)].<sup>5</sup> This interference is destroyed by a magnetic field. One might wonder what kind of interfering paths are responsible for  $\delta G$  in an NS junction without  $\mathcal{T}$ . Although our theory is not formulated in terms of Feynman paths, an interpretation of the quantity  $M_{pq}$  in Eq. (4b) using Feynman paths is possible. The two

simplest interfering paths are shown in Fig. 2(b). Regardless of whether  $\mathcal{T}$  is broken or not, there is an exact cancellation of the phase shifts accumulated by the electron and the hole which traverse the loop in the same direction. What remains is a phase shift of  $\pi$  due to the double Andreev reflection. As a consequence, the path with the double loop interferes destructively with the path without a loop, giving rise to a negative  $\delta G$ .

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<sup>17</sup>Our analytical calculation refers to the net increase of  $\langle G \rangle$  between  $V=0$  and  $V \gg E_c/e$ . We cannot describe the nonmonotonic  $V$  dependence at intermediate  $V$ , observed in the simulation for  $B=0$ .