On the renormalization of random network models

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Summary

This thesis studies the renormalization of random network models, with a focus on the topological and algebraic structures that emerge when networks are coarse-grained or partially observed. The work is divided into two parts.

The first part analyses the Multi-Scale Model (MSM), a random graph model that arises as the fixed point of a renormalization procedure based on aggregating nodes into supernodes while preserving the functional form of the connection probability. The MSM requires that node fitnesses follow a heavy-tailed stable distribution with infinite mean, and this unusual distributional property endows the model with a number of remarkable features.

Chapter 2 characterises the spectrum of the MSM adjacency matrix. The leading eigenvalues are shown to grow as the square root of the network size, to alternate in sign, and to lie at the intersections of the real axis with a logarithmic spiral in the complex plane, characterised analytically in terms of the Gamma function. The corresponding eigenvectors display log-periodic oscillations in the node fitness, a hallmark of discrete scale invariance. The bulk of the spectrum extends to the same scale as the outlier eigenvalues, in stark contrast with the behaviour of finite-mean models.

Chapter 3 proves that the MSM naturally generates a non-trivial clustering function without any appeal to an underlying geometric space. In the infinite-mean regime, all nodes except the highest-degree hubs have local clustering coefficients close to one, and the network becomes asymptotically fully clustered in a precise sense. The analysis also reveals a lack of self-averaging in the average clustering coefficient, a consequence of the heavy-tailed fitness distribution.

The second part develops a renormalization theory for random network models in which edges are correlated. The central observation of Chapter 4 is that marginalising a correlated network model over unobserved variables is equivalent to performing a decimation step in the renormalization group sense. This equivalence is formalised within the framework of exponential random graph models, and is used to derive exact RG flow equations, to identify universality classes, and to develop network inference methods that incorporate uncertainty about missing data in a principled way.

Chapter 5 solves the RG equations exactly for one-dimensional chain topologies, and extends the analysis to models with sparse and uniform disorder. The exact solutions are interpreted in terms of several network-relevant phenomena, including peer effects, preferential attachment, susceptibility, and reinforcement dynamics.

Chapter 6 analyses the class of second-order random network models, to which most correlated models flow under renormalization when higher-order interactions are irrelevant. It is shown that second-order models can be solved in full generality by reducing the computation of the partition function to the diagonalization of a link-adjacency matrix. This approach reveals a rich phase structure, establishes a connection with algebraic graph theory, and provides a unified framework for solving a wide family of edge-correlated network models.