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## On the renormalization of random network models

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# On the Renormalization of Random Network Models

Alessio Catanzaro

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# On the Renormalization of Random Network Models

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