Complex pathways in multistable matter
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Summary

Chapter one

Materials are often thought of as passive objects that simply deform when pressed or pulled. Yet certain complex materials behave in far more surprising ways: they not only react to applied forces but can also *remember* the history of those forces. Such memory effects set them apart from ordinary matter and make them fundamentally different. Even familiar materials like rubber or certain types of metals can display such history-dependent behavior, hinting at a much broader and richer class of physics.

Chapter two

Certain complex materials do not just react to being pushed or squeezed, but remember the order and location of those pushes. In chapter two, we explore happens when such materials are pressed at multiple locations in different sequences. Using a chain-like metamaterial as a model system, we show that the response depends not only on where and how much strain is applied, but also on the precise sequence of actions. For example, pressing two spots in one order can lead to a completely different final shape than pressing them in the reverse order (a “non-Abelian” response). Or the material respond smoothly in one sequence but suddenly snaps along another (“mixed-mode” response). To explain this, we identify basic building blocks of these responses, called *ancestor–descendant–sibling* triplets, and use graph-based maps to capture and explain these path-dependent memory effects. In addition, we introduce a method to simplify and tame the overwhelming complexity that arises when many inputs are applied, making the system easier to interpret, classify, and potentially design.

Beyond physics, we show that this path-dependent behavior can act as a form of natural computation. By treating pushes as “inputs” and the resulting states as “outputs”, the material can perform basic logic operations and store information, much like a computer circuit. Remarkably, a single sample can host multiple possible "circuits" that can be switched by adjusting the strength or location of the inputs. This suggests that complex materials could be used for new forms of computing directly inside matter (*in-materia computing*) while

also shedding light on how the nonlinear response of complex materials.

Chapter three

When materials are repeatedly and cyclically deformed, they typically settle into a simple repeating pattern known as a *limit cycle*. Yet some disordered or frustrated materials exhibit more exotic behaviors, such as long *transients* (in which many deformation cycles occur before a limit cycle is reached) or *multiperiodic* responses (where the repeating pattern itself spans multiple deformation cycles). These behaviors are poorly understand, remain difficult to model, and are challenging to reproduce in designed materials. To address this, we introduce *togglerons*–a fundamental unit that inherently produces multiperiodic responses with period two. We show that ensambles of interacting togglerons can produce long transients and large periods including odd ones. We argue that togglerons belong to a broader hierarchy of building blocks, alongside spins and hysterons, which together provide a framework for understanding, modeling, and designing these exotic behaviors. This hierarchical approach opens new strategies for creating smart materials, with potential applications in soft robotics, adaptive structures, and mechanical computing.

Chapter four

Snap-through mechanisms are fundamental in nature, technology, and even for the togglerons. For buckled beams, snapping is often triggered by a single transverse pusher that must cross the beam's longitudinal axis, limiting how and when the instability occurs. In this chapter, we introduce a dual-tip transverse pusher that enables *early-onset snapping*, where snap-through is initiated before the pusher reaches the longitudinal axis. We employ a modal expansion approach to capture and analyze this new class of snap-through behavior that is crucial for the togglerons of chapter three. Finally, we demonstrate how to maximize the distance between the pusher and the beam's centerline at the moment of snapping. Together, our work opens new avenues for qualitative and quantitative control over the snapping of buckled beams, with potential applications in mechanical sensors, actuators, and metamaterials.

Chapter five

Massively multistable materials can store vast amounts of information and are often represented as collections of bistable elements, or *material bits*. Independent control over these bits is crucial, yet current methods rely on either local, individual access to each bit or slow, multi-step global protocols. In this chapter, we introduce *dynamic control*, a strategy that achieves independent bit manipulation through a single global drive, using one overall input and the rate at which that input changes. We demonstrate this on a rotating mechanical platform, where five material bits are sufficient to encode all 26 letters of the alphabet through simple single-cycle protocols. Beyond mechanics, this approach extends to fluids, electronics, and soft robotics, paving the way for smart, remotely programmable devices.