Contributions to contribution analysis: improving our understanding of contributions to impacts in Life Cycle Assessment
Meide, M.T. van der

Chapter 3

Hypothetical Extraction Method in Life Cycle Assessment: Method and Uses

Authors Marc van der Meide

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Abstract

Hypothetical Extraction Method (HEM) is an approach in Input Output Analysis (IOA) used to assess the effects of *'target'* economic sectors have on other sectors. If we interpret HEM in a Life Cycle Assessment (LCA) context, HEM could be described as 'assessing the contribution of processes (or groups of processes) to the impact of other processes', making this another potential Contribution Analysis (CA) approach. In this study, we introduce HEM in LCA and demonstrate how HEM may provide different insights compared to the already existing CA approaches in two example case-studies. We do this by showing how HEM may be applied in LCA in four steps: 1) defining a scenario for extraction, 2) extracting *target* processes, 3) calculating results for the *original* and *'remaining'* system and 4) calculating results for the *target* processes. We then demonstrate HEM in LCA using two different example case-studies and compare HEM against other CA approaches. We also demonstrate process contributions to the *target* and *remaining* sections, allowing deeper insight into results. With this study we have shown how and when HEM can be a useful approach for CA in cases where current CA approaches may be lacking, providing an additional CA approach to the LCA practitioner in understanding impacts from their systems.

3.1 Introduction

Hypothetical Extraction Method (HEM) is a method in Input Output Analysis (IOA) used to assess the effects economic sectors have on other sectors (Hertwich et al., 2024). To do this, a counterfactual scenario is created where the *'target'* sector (Dente et al., 2018) is extracted from the IOA model and results are re-calculated. The difference between the *original* result and the HEM scenario is considered to be caused by the extracted *target* sector.

HEM can be used for many different use-cases such as assessing the contribution of sectors to carbon footprints of products and consumption baskets (Hertwich, 2021; Rasul & Hertwich, 2023), assessing the 'importance' of an economic sector to other sectors in terms of consumption (Miller & Lahr, 2001) or assessing the effect of disruptive events (Dietzenbacher et al., 2019; Haddad et al., 2021). For an extensive overview of examples we refer to Miller & Lahr (2001) and Hertwich et al. (2024).

Even though IOA and Life Cycle Assessment (LCA) have similar mathematical structures, HEM is not used in LCA, though their similar structures could make this possible.

If we loosely interpret the goal of HEM 'assessing the impact of sectors on other sectors' to an LCA context, we find something like 'assessing the impact from processes (or groups of processes) to other processes'. This could be interpreted as similar to Contribution Analysis (CA) (what is the contribution of x to the total?) (Guinée et al., 2002; Heijungs & Kleijn, 2001). Through the extraction, this would assess not only the contribution of a process' direct contribution (Srocka & Montiel, 2021) but also all consumption of that process, meaning it would include indirect contributions as well (Reinhard et al., 2016; van der Meide et al., 2025). Other CA approaches that assess both direct and indirect contributions are First-Tier CA (van der Meide et al., 2025), assessing contributions of products feeding into the Functional Unit (FU). Life Cycle Stage CA (van der Meide et al., 2025), dividing the system into relevant sections of the supply chain such as 'production' and 'use'. And Path/Sankey CA (Srocka & Montiel, 2021), showing the 'path' contributions take to the FU. While these approaches each have their own unique strengths, they cannot be used to assess large groups of processes, for example industry sectors or regions effectively.

CA approaches could also be applied to systematically assess system-wide changes, applying it at another level than CA. Some examples are: Finding ‘causer’ and ‘connector’ processes in LCI databases (Reinhard et al., 2016), where these processes either have direct (Srocka & Montiel, 2021) or indirect (van der Meide et al., 2025) contributions, respectively. Two other examples are the identification of important capital goods in an LCI database (Frischknecht et al., 2007) and an analysis of abiotic resources in an LCI database (Rørbech et al., 2014).

While there are already various approaches to assessing indirect contributions in LCA, approaches to assess the indirect contribution from large groups of processes, such as sectors or regions do not currently exist. The objective of this study is to introduce HEM in LCA as a new approach to CA and demonstrate how HEM may provide different insights compared to the already existing CA approaches. We formalize the method in LCA and demonstrate with a simplified case study how it may be used and how results may be interpreted. Next, we compare it against other approaches with the simplified case study. After this, we also demonstrate HEM using a much larger example using the ecoinvent LCI database (Wernet et al., 2016) to simulate a more realistic case study. Finally, we discuss how HEM in LCA may help LCA practitioners to find new insights and discuss the nuances between different CA approaches.

3.2 Method

3.2.1 Notation

We demonstrate the approach in vectorized format. We use *italic lower case* for scalar values, ***bold lower case*** for vectors and ***BOLD UPPER CASE*** for matrices. Column vectors are written as $\boldsymbol{v}$, while row vectors are written as $\boldsymbol{v}^T$. All results presented are rounded, we refer to the excel file SI 2 for unrounded results.

3.2.2 Mathematical structure of LCA

We shortly introduce the mathematical structure of LCA based on Heijungs & Suh (2002) and the SI of van der Meide et al. (2025). LCA is generally done as IO-LCA or as process-LCA (Heijungs et al., 2022). In this study the focus is on process-LCA where applying IOA methods has traditionally been challenging or impossible (Heijungs et al., 2022; Suh & Heijungs, 2007), we also shortly discuss their differences in SI 1.1. We refer to Heijungs & Suh (2002) for an extensive introduction to the mathematical background of LCA and van der Meide et al. (2025) for a more extensive treatment of CA in LCA.

We start with the known information; $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{f}$ and $\boldsymbol{q}^T$. Here, $\boldsymbol{A}$ is the technology matrix, $\boldsymbol{B}$ is the Elementary Flow (EF) matrix, $\boldsymbol{f}$ is the functional unit (FU) vector and $\boldsymbol{q}^T$ is the characterization vector.

The technology matrix $\boldsymbol{A}$ has processes as columns, with inputs (consumption) to that process as negative values and the produced product as a positive value for output. Each row in $\boldsymbol{A}$ shows how much of the product/service the given process produces is consumed in other processes. The EF matrix $\boldsymbol{B}$ shows which interventions each process causes as an EF in the environment as columns. It has the same amount of columns as $\boldsymbol{A}$.

First, a scaling vector $\boldsymbol{s}$ is calculated, this represents how many times each process needs to produce its product to fulfil the FU expressed by $\boldsymbol{f}$:

$$\boldsymbol{f} = \boldsymbol{A}\,\boldsymbol{s} \;\rightarrow\; \boldsymbol{s} = \boldsymbol{A}^{-1}\,\boldsymbol{f} \tag{1}$$

Next, $\boldsymbol{B}$ and $\boldsymbol{s}$ can be multiplied to calculate the inventory results vector $\boldsymbol{g}$, which shows all EFs emitted or consumed to fulfill the FU $\boldsymbol{f}$:

$$\boldsymbol{g} = \boldsymbol{B}\,\boldsymbol{s} \tag{2}$$

Finally, the inventory results vector is characterized to a characterized result h by multiplying by the characterization vector $\boldsymbol{q}$:

$$h = \boldsymbol{q}^T\,\boldsymbol{g} \tag{3}$$

Or:

$$h = \boldsymbol{q}^T\,\boldsymbol{B}\,\boldsymbol{A}^{-1}\,\boldsymbol{f} \tag{4}$$

As the results from eq. (2) and (3) only show one characterized result for every FU and every impact category, these don't show any insights into what processes or EFs contribute to the total result. These results can be disaggregated to show which process causes each EF for the given FU, turning the inventory into a contribution matrix $\boldsymbol{g} \rightarrow \boldsymbol{G}$ (Srocka & Montiel, 2021).

To calculate the contribution matrix, the $\boldsymbol{s}$ and $\boldsymbol{q}$ vector are diagonalized. Diagonalizing means putting the values of the respective vector on the diagonal of a square $\mathbf{0}$ matrix of the same size.

This leads to adjustments of eq. (2) and (3) below:

$$\boldsymbol{G} = \boldsymbol{B}\, diag(\boldsymbol{s}) \tag{5}$$

$$\boldsymbol{H} = diag(\boldsymbol{q})\, \boldsymbol{G} \tag{6}$$

Through the above adjustment, $\boldsymbol{G}$ and $\boldsymbol{H}$ become matrices with a value ($\boldsymbol{g}$ for inventory or h for characterized results) for each process as a column and each EF as a row.

3.2.3. Example case

We demonstrate HEM in LCA based on a simple case study example of four processes; coal mining, electricity production, iron ore mining and steel production. We show the system in Figure 3.1 and define the inventory of these processes in Table 3.1. We use this simplified example system to demonstrate the principles of HEM in LCA, show example results and compare similar approaches to HEM using this example system. We will focus primarily on the production of '1kg steel' as FU, it will be make clear in instances where this is not the case. As an impact category, we use climate change (GWP 100) from the IPCC (2022), characterizing CO_2 emissions as 1 $kgCO_{2eq.}$/kg and CH_4 emissions as 29.8 $kgCO_{2eq.}$/kg.

We strongly emphasize that this example is exclusively for the demonstration of HEM and is not meant to represent reality nor to assess the actual impact of processes within this system.

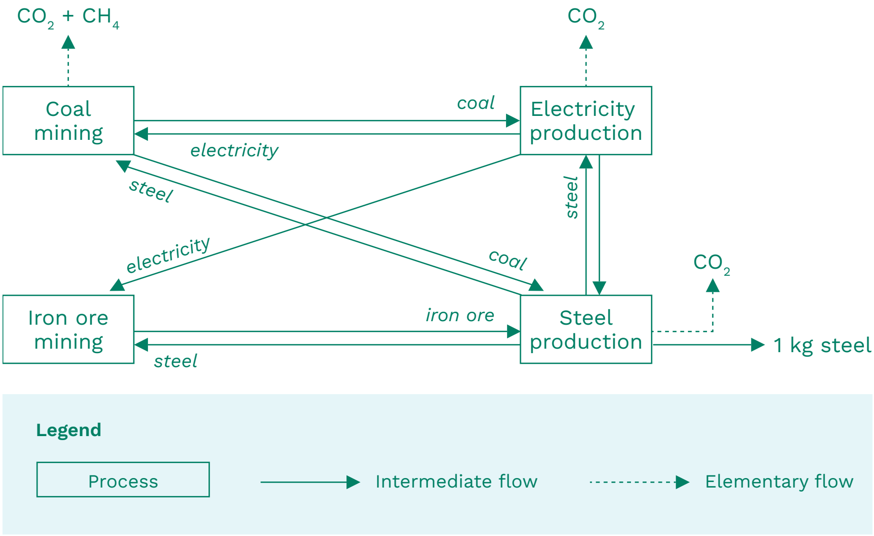

Figure 3.1 - Product system for the example case study.

Process name	Functional flow (Product/Service)	Intermediate flows	Elementary flows
Coal mining	Coal: 1 kg	Electricity: 0.04 kWh Steel: 0.001 kg	CO_2: 0.05 kg CH_4: 0.01 kg
Electricity production	Electricity: 1 kWh	Coal: 0.4 kg Steel: 0.001 kg	CO_2: 0.8 kg
Iron ore mining	Iron ore: 1 kg	Electricity: 0.1 kWh Steel 0.001 kg	
Steel production	Steel: 1 kg	Coal: 0.8 kg Iron ore: 2 kg	CO_2: 1.6 kg

Table 3.1 - Inventory table for the example case study, showing functional flow, intermediate flows and EF per process.

Based on Table 3.1 above, we get the following technology matrix $\boldsymbol{A}$, EF matrix $\boldsymbol{B}$ and characterization vector $\boldsymbol{q}^T$ from the description above:

$$\boldsymbol{A} = \begin{bmatrix} 1 & -0.4 & 0 & -0.8 \\ -0.04 & 1 & -0.1 & 0 \\ 0 & 0 & 1 & -2 \\ -0.001 & -0.001 & -0.001 & 1 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0.05 & 0.8 & 0 & 1.6 \end{bmatrix} \quad \boldsymbol{q}^T = \begin{bmatrix} 29.8 \\ 1 \end{bmatrix}$$

Where the rows/columns from the top/left in technology matrix $\boldsymbol{A}$ are 'Coal mining', 'Electricity Production', 'Iron ore mining' and 'Steel production'. For the EF matrix $\boldsymbol{B}$ and characterization vector $\boldsymbol{q}^T$, we first list CH_4 and then CO_2.

3.2.4. Applying HEM to LCA

In general the steps of applying HEM in IOA is that a counterfactual (hypothetical) scenario is created where one or more target sectors are removed (extracted) and results are re-calculated without for the *'remaining'* sectors. The difference between the *original* result and the *remaining* result is the effect from the extracted *target* sector.

The goal of HEM in IOA is to assess the effect of a sector on other sectors in a final result can be translated to the context of LCA. Here, the goal becomes 'assessing the contribution a process (or group of processes) has on the final result from the FU'. We develop HEM in LCA further with this goal in mind. As the mathematical structure differs, our approach is slightly different as well.

To apply HEM to LCA, we need to take 4 steps:
1. Define an HEM scenario
2. Extract the processes
3. Calculate the results for the *original* and *remaining* system
4. Calculate the results for the *target* processes

In the further text, in all cases that could apply to one or multiple processes, we will refer to this as 'processes' in plural for brevity. In specific instances where only one or the other is meant, it will be made clear.

3

We emphasize that the ‘H’ in HEM stands for Hypothetical, meaning that we do not truly assume that the *target* processes would suddenly ‘not exist’, the extraction is rather a tool to assess the contribution of the *target* and/or *remaining* processes to the total result.

All information to replicate this approach conceptually or in small product-systems can be found in this article and its SI, the code make use of this approach or to replicate it using Brightway2 (Mutel, 2017a) on real-world systems is provided in van der Meide (2025a).

HEM scenario definition

First, the practitioner must choose *target* processes to extract. The *target* processes can be any set of processes of interest for the question the practitioner is trying to answer. This grouping can be based on the process metadata (information describing the process, like name, product, location or an industry sector (e.g. as given in ecoinvent metadata (Wernet et al., 2016)). The processes should have some relationship to each other that makes them relevant to extract together. For example, a practitioner could choose all ecoinvent processes with the Central Product Classification (CPC) (UN, 2015) Division ’14: Metal ores’ and they could assess the contribution of products classified as such to the final result. Another example could be, if the practitioner would choose all processes with the location ‘NL’ in an LCI database, the practitioner could assess the impact processes in the Netherlands have on the final result, mirroring the regional example Miller and Lahr (2001) give.

Extracting processes

After defining an HEM scenario, we calculate the scaling vector $\boldsymbol{s}$ (following eq. (1)) for the *original* system and the *remaining* system. As Miller and Lahr (2001) discuss, there are several options to extract sectors. In process-LCA, the list of options is more limited because we use a different mathematical model. The mathematical format of process-based LCA (Heijungs et al., 2022) makes it impossible to calculate results for any FU if the reference flow of any process is ‘0’ (not producing anything), this reduces the amount of options we can use compared to Miller and Lahr (2001). We discuss this further in SI 1.2. Taking these limitations into account, the following four options in Table 3.2 are possible approaches to HEM in LCA with corresponding case numbers from Miller and Lahr (2001) for comparison.

Case	Description
1	Extracting the processes from the technology matrix, decreasing its size (deleting both the rows and columns).
2a	Extracting both *consumers* (row values) and *consumption* (column values) of the target processes, resulting in processes that are both un-used and 'empty' (replacing all process flow values except reference flow with 0).
3a	Extracting all *consumers* (row values) of the *target* processes in the technology matrix, resulting in processes that are un-used (replacing all process flows *from* this process with 0).
3b	Extracting all *consumption* (column values) of the *target* processes in the technology matrix, resulting in 'empty' processes (replacing all process flows to this process with 0).

Tabel 3.2 - Options to extract processes for HEM in LCA. The numbering of the cases is the same as in Miller and Lahr (2001)

While the first option (case 1) seems most intuitive, this first option could be potentially challenging as it changes the size of matrices, making it harder to keep consistent links between the *original* system and the HEM scenario for comparison later.

The second option (2a) may seem like a good option as it 'isolates' the processes and maintains the matrix sizes. Though making the extracted processes have 'no consumption' is not ideal as the *target* processes would always have '0' impact due to not consuming anything.

The third option (3a), would (similar to case 2a) also 'isolate' the processes. However, if one or more of the extracted processes in question is part of the FU, this would now still show what that process would consume from the *remaining* system (or from other extracted process), thus being a better representation for all processes that could be in the FU.

The last option (3b) would still 'isolate' the *target* processes, and remove their associated impacts to the system, this makes the results equivalent to option 2a in LCA. However, as explained with case 3a, this form of isolation would not be representative in case processes in the FU are part of the extracted processes.

We continue with option 3a as this allows for most interpretability of the system, and as we show later, can still differentiate between direct and indirect impacts from the extracted *target* processes by combining HEM with process CA. We demonstrate all four options further in SI 1.2 and show that all four cases give the same result in LCA if the FU does not contain the extracted process.

While in IOA the interest often lies in the technology matrix, in LCA, the interaction of the processes with the environment is very important to consider as well. This means that with HEM in LCA we also extract the EF matrix $\boldsymbol{B}$.

We show the extraction of 'electricity production' from our example system below. This gives us the *remaining* system $\boldsymbol{A}^*$ and $\boldsymbol{B}^*$, marked with a '*' to show these are changed. We also mark the changed items in yellow. Note here that we mirror the superscript notation for *remaining* and *target* systems (*, o) of Hertwich et al. (2024) for consistency:

Matrix - *Original* system

$$\boldsymbol{A} = \begin{bmatrix} 1 & -0.4 & 0 & -0.8 \\ -0.04 & 1 & -0.1 & 0 \\ 0 & 0 & 1 & -2 \\ -0.001 & -0.001 & -0.001 & 1 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0.05 & 0.8 & 0 & 1.6 \end{bmatrix}$$

Matrix - *Remaining* system case 3a

$$\boldsymbol{A} = \begin{bmatrix} 1 & -0.4 & 0 & -0.8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ -0.001 & -0.001 & -0.001 & 1 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0.05 & 0 & 0 & 1.6 \end{bmatrix}$$

Results for the Original and Remaining system

With the extraction complete, the remaining scaling vector $\boldsymbol{s}^*$ can be calculated (following eq. (1) again) from the $\boldsymbol{A}^*$ technology matrix. Where $\boldsymbol{s}^*$ is a *partial* scaling vector of the FU for the *remaining* system. A partial scaling vector represents a partial fulfilment of the *original* FU, where all partial scaling vectors together fulfil the full FU (van der Meide et al., 2025).

We can calculate the inventory for both the *original* ($\boldsymbol{g}$) and *remaining* system ($\boldsymbol{g}^*$) with eq. (2). We can logically take this a step further to now calculate characterized results with eq. (3) for the *original* ($\boldsymbol{h}$) and *remaining* ($\boldsymbol{h}^*$) systems.

Results for the Target processes

So far, we have calculated results for the *remaining* processes (the hypothetical extraction), we take this a step further to find results for the *target* processes as well, which is the goal of HEM. As we discussed above, the scaling vector $\boldsymbol{s}^*$ is a partial scaling vector, making $\boldsymbol{g}^*$ the partial inventory. As our FU did not change, the difference between the partial inventory and the *original* inventory will caused by the extraction of the *target* processes. We can calculate the *target* inventory by subtracting the partial inventory $\boldsymbol{g}^*$ from the *original* inventory $\boldsymbol{g}$:

$$\boldsymbol{g}^o = \boldsymbol{g} - \boldsymbol{g}^* \tag{7}$$

Note that eq. (7) can also be applied to a diagonalized instance of $\boldsymbol{g}$, as $\boldsymbol{G}$, this is shown in the example later. We can then calculate the characterized results $\boldsymbol{h}^o$ for the *target* as well by again using eq. (3).

In the context of HEM in LCA, the inventory and characterized results of the *target* system should be interpreted as the EFs or impact caused by the products or services provided by the *target* processes. This would include both the direct and indirect EFs or impacts. In other words, all the EFs that are caused to produce all intermediate flows of the *target* throughout the entire system (in the example, the CH_4 emitted to mine the coal to produce the electricity). The inventory and characterized results of the *remaining* system are from everything else. We show how this may be interpreted in the results section.

Following from eq. (7), if we sum the partial *target* and *remaining* inventory vectors, we still have the original inventory vector ($\boldsymbol{g} = \boldsymbol{g}^* + \boldsymbol{g}^o$). While this may seem trivial, it is important to note, as this does guarantee that this approach is not double (or under) counting. By splitting the inventory into two partial inventories, we can assess the contribution of both the extracted processes and the *remaining* system to the full result.

Alternatively, applying eq. (5) to $\boldsymbol{s}$ and $\boldsymbol{s}^*$ and then applying eq. (7) allows us to assess the contribution matrices of both the *target* processes and *remaining* system, as well as their characterized versions by applying eq. (6). These last steps allow us to assess contributions to the partial target processes and *remaining* system.

Finally, a careful reader may have noticed we do not apply eq. (7) to the scaling vector $\boldsymbol{s}$ but to the inventory $\boldsymbol{g}$. We do this as we also change

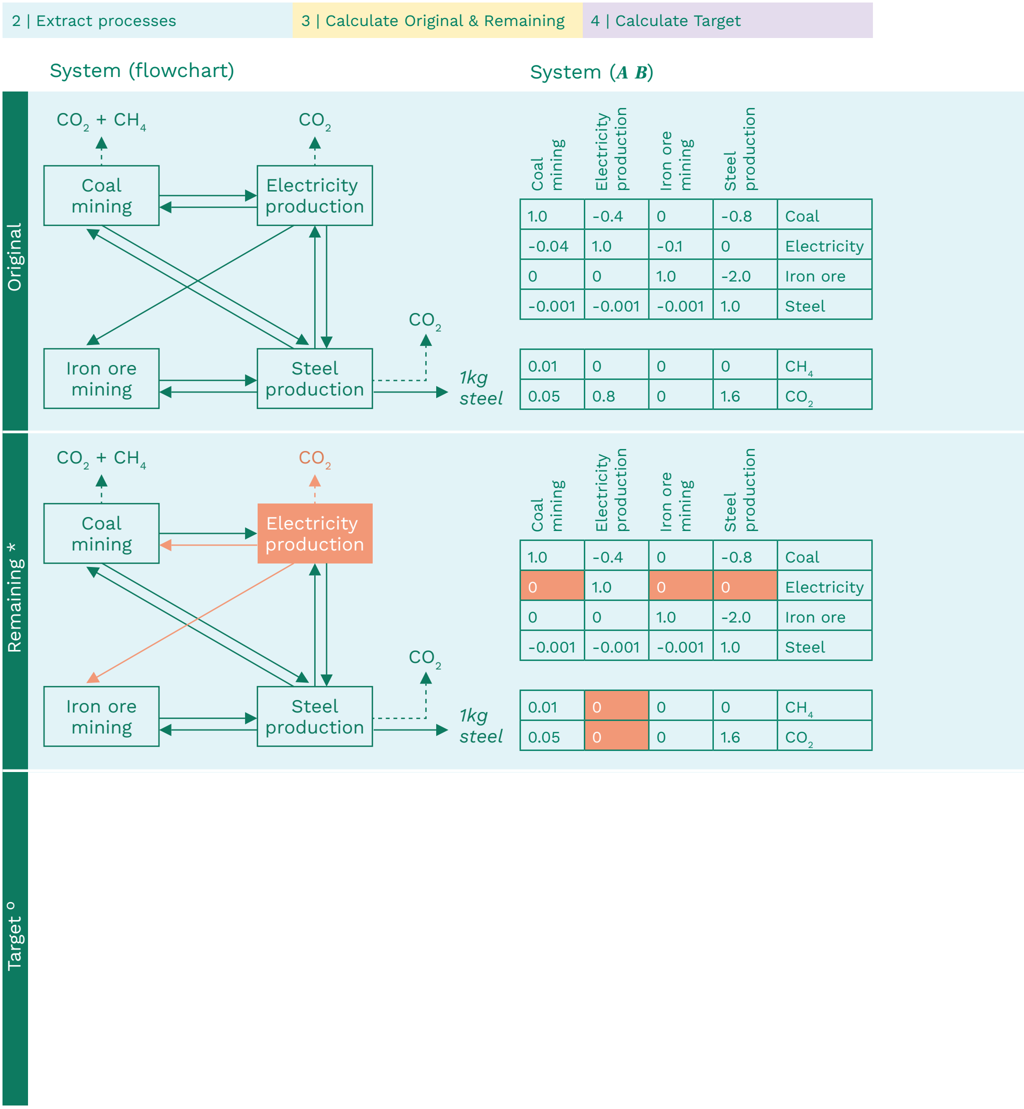

Figure 3.2 - Graphical overview of calculation steps for HEM. First, processes are extracted (blue), next, the original and remaining system are calculated (yellow) and finally, the target is calculated (pink).

the EF matrix $\boldsymbol{B}$, which would affect the inventory independent of the scaling vector. This detail is discussed further in SI 1.4. We provide a visual overview of the last three steps in Figure 3.2.

Scaling vector ($\boldsymbol{s}$)

$\boldsymbol{s} = \boldsymbol{A}^{-1} \boldsymbol{f}$

0.9	Coal mining
0.24	Electricity production
2.0063	Iron ore mining
1.0031	Steel production

$\boldsymbol{s}^* = \boldsymbol{A}^{*-1} \boldsymbol{f}$

0.8	Coal mining
0	Electricity production
2.0056	Iron ore mining
1.0028	Steel production

Inventory matrix ($\boldsymbol{G}$)

$\boldsymbol{G} = \boldsymbol{B}\, diag\,(\boldsymbol{s})$

Coal mining	Electricity production	Iron ore mining	Steel production	
0.009	0	0	0	CH_4
0.045	0.189	0	1.605	CO_2

$\boldsymbol{G}^* = \boldsymbol{B}^*\, diag\,(\boldsymbol{s}^*)$

Coal mining	Electricity production	Iron ore mining	Steel production	
0.008	0	0	0	CH_4
0.04	0	0	1.604	CO_2

$\boldsymbol{G}^{\circ} = \boldsymbol{G} - \boldsymbol{G}^*$

Coal mining	Electricity production	Iron ore mining	Steel production	
0.001	0	0	0	CH_4
0.005	0.189	0	0.001	CO_2

Characterization matrix ($\boldsymbol{H}$)

$\boldsymbol{H} = diag\,(\boldsymbol{q}^T)\ *\ \boldsymbol{G}$

Coal mining	Electricity production	Iron ore mining	Steel production	
0.267	0	0	0	CO_2eq
0.045	0.189	0	1.605	CO_2eq

$\Sigma \boldsymbol{H} = 2.106$ kg CO_2eq

$\boldsymbol{H}^* = diag\,(\boldsymbol{q}^T)\ *\ \boldsymbol{G}^*$

Coal mining	Electricity production	Iron ore mining	Steel production	
0.239	0	0	0	CO_2eq
0.04	0	0	1.604	CO_2eq

$\Sigma \boldsymbol{H}^* = 1.883$ kg CO_2eq (89.4%)

$\boldsymbol{H}^{\circ} = diag\,(\boldsymbol{q}^T)\ *\ \boldsymbol{G}^{\circ}$

Coal mining	Electricity production	Iron ore mining	Steel production	
0.028	0	0	0	CO_2eq
0.005	0.189	0	0.001	CO_2eq

$\Sigma \boldsymbol{H}^{\circ} = 0.223$ kg CO_2eq (10.6%)

3.3 Results

3.3.1 Interpreting HEM results

For our example case, we extract the *target* process 'electricity production' as shown in the method and calculate the results for a FU of '1kg steel'.

We calculate the following scaling vectors from equation (1). Note that values are rounded:

$$\boldsymbol{S}^T = \begin{bmatrix} 0.9 & 0.24 & 2.\,.. & 1.\,.. \end{bmatrix} \qquad \boldsymbol{S}^{*T} = \begin{bmatrix} 0.8 & 0 & 2.\,.. & 1.\,.. \end{bmatrix}$$

Based on these scaling vectors, we calculate the following LCI from eqs. (5) and (7) and characterized contribution matrices with eq. (6):

$$\boldsymbol{G} = \begin{bmatrix} 0.009 & 0 & 0 & 0 \\ 0.045 & 0.189 & 0 & 1.605 \end{bmatrix} \qquad \boldsymbol{H} = \begin{bmatrix} 0.267 & 0 & 0 & 0 \\ 0.045 & 0.189 & 0 & 1.605 \end{bmatrix}$$

$$\boldsymbol{G}^* = \begin{bmatrix} 0.008 & 0 & 0 & 0 \\ 0.04 & 0.189 & 0 & 1.604 \end{bmatrix} \qquad \boldsymbol{H}^* = \begin{bmatrix} 0.239 & 0 & 0 & 0 \\ 0.04 & 0 & 0 & 1.604 \end{bmatrix}$$

$$\boldsymbol{G}^o = \begin{bmatrix} 0.001 & 0 & 0 & 0 \\ 0.005 & 0.189 & 0 & 0.001 \end{bmatrix} \qquad \boldsymbol{H}^o = \begin{bmatrix} 0.028 & 0 & 0 & 0 \\ 0.005 & 0.189 & 0 & 0.001 \end{bmatrix}$$

Note again that as $\boldsymbol{G}^*$, $\boldsymbol{G}^o$ and $\boldsymbol{H}^*$, $\boldsymbol{H}^o$ are partial results, they sum to the total result $\boldsymbol{G}$ and $\boldsymbol{H}$ respectively. The results above are rounded, the full results are available in SI 2.1.

As $\boldsymbol{G}^o$ represents the partial inventory of the *target* processes, it does not just represent the direct, but also indirect elementary flows of processes in the system. This is because we 'extracted' not only the production of electricity, but also all production (coal, steel, iron ore for the steel etc.) needed for the production of that electricity, thus also including indirect impacts.

In HEM, the *target* processes never contribute to the *remaining* impact, but the *remaining* processes can contribute to the *target* impact. We show the results for the *target* and *remaining* system and the direct contributions to both in Figure 3.3 A. Here, we can see that the HEM *target* 'electricity' is contributing 10.58% of the impact to climate change to steel. In the first row of Table 3.3 we see this contribution is made up of both 8.98% direct contribution for 'electricity production', as well from indirect contributions from 'coal mining' and 'steel production' needed to produce that electricity. For the *remaining* system, the contributions are from 'coal mining' and 'steel production' without any contribution from 'electricity production'.

	Electricity production/ Electricity	Coal mining	Steel production	Connector	Total
HEM Target	8.98%	1.57%	0.03%		10.58%
Electricity life cycle stage	8.96%	1.56%	0.01%		10.52%
Path (electricity)	8.84%	1.71%	0.02%		10.58%
Causer/Connector	8.98%		1.74%	10.72%	

Tabel 3.3 - Direct and Indirect contribution results of 'electricity' or 'electricity production' for 'steel production'. Percentages have been rounded to 2 digits to show differences, complete results are available in SI 2.1.

Example case: Comparison of HEM to other Contribution Analysis approaches

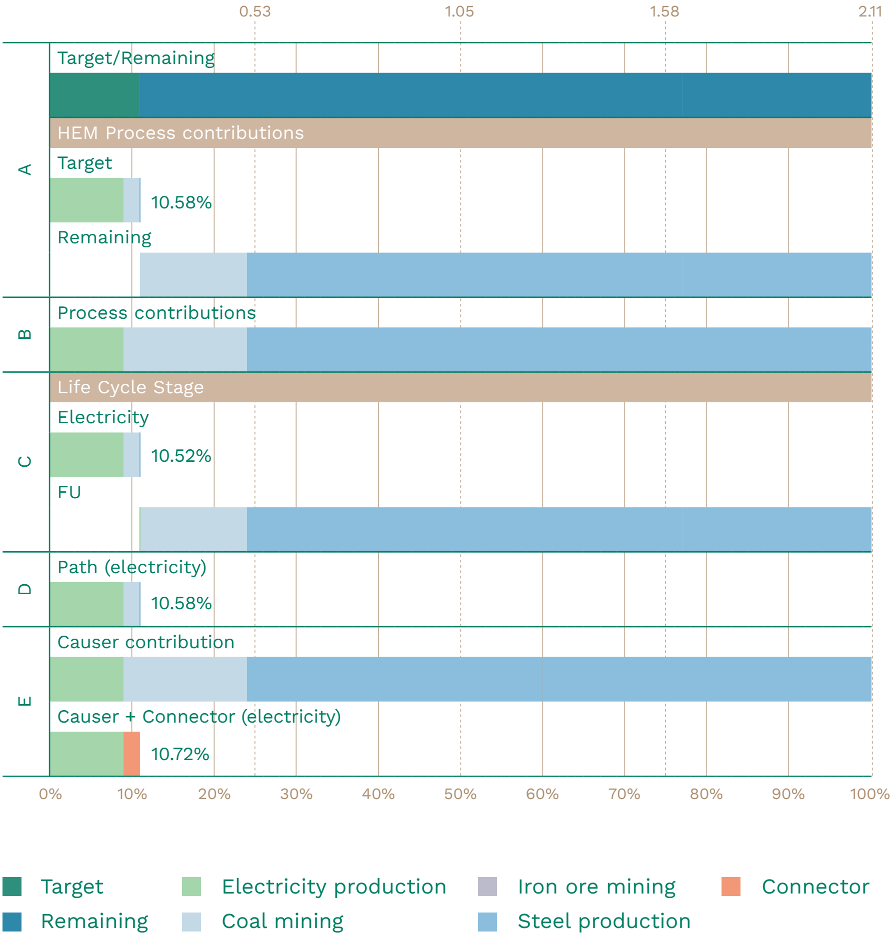

Figure 3.3 - Contribution Analysis results for different approaches for the production of 1kg steel and the extraction/contribution of 'electricity'. Top numbers are the climate change impact in kgCO_2eq., the bottom numbers are in percentage contribution. ***A*** *Contribution of both the target processes and remaining system. Also showing the process contributions to the target and remaining system.* ***B*** *Process contributions.* ***C*** *Process contributions to Life Cycle Stages.* ***D*** *Path contributions at 'electricity production'.* ***E*** *Causer and Connector contributions for 'electricity production'. Note that causer contributions are the same as process contributions.*

Extracting the Functional Unit
It could happen in an analysis that the practitioner wants to extract the processes that provide the FU. With case 3a we use for the extraction, this is possible. We demonstrate this with the example system by now taking '1kWh of Electricity' as a FU.

Intuitively one may assume that the contribution would just be 100%, as the extracted process is causing 100% of its direct and indirect contribution. However, the results show a different picture when using case 3a (though we do find the assumed 100% contribution when using the other extraction cases). Through the extraction of the direct impacts (in $\boldsymbol{B}^*$) and consumption of electricity by the other processes (in $\boldsymbol{A}^*$), we have created a situation where our *remaining* system contribution would show the impact of everything *needed* for the production of electricity, except for the electricity itself. The *target* would now show the direct contribution of 'Electricity production' and the indirect contribution of all electricity that would be consumed in the system to produce that electricity. Extracting the FU is the only instance where case 3a is different from cases 2a and 3b in terms of results. We discuss this further and show results for this in SI 1.3 and show results for all cases where the FU is extracted in SI 2.3.

3.3.2 Comparison with other approaches

For our comparison with other approaches, we extract 'electricity production' and use '1kg Steel' as FU. Full results are presented in SI 2.1.

Process contributions
When comparing HEM results against direct process contributions (Figure 3.3 B), we see that applying HEM to 'electricity production' not only accounts for the direct, but also indirect impacts of electricity production.

Life Cycle Stage Contribution Analysis
For the Life Cycle Stage contributions (van der Meide et al., 2025), we assume a life cycle stage consisting of the process 'electricity production' only. We show the process contributions to the life cycle stage and remainder of the system in Figure 3.3 C. We note the total impact is still the same (no double counting) but that the contribution of the life cycle stage is lower. The exact differences *within* the life cycle stage

are shown in Table 3.3. The difference with HEM is because the partial-FU of the life cycle stage only considers the direct use of electricity for the production of the FU. Other 'indirect' use, e.g. the electricity needed to produce coal to produce electricity is excluded. This is an important conceptual difference between HEM and Life Cycle Stage contributions, where HEM completely 'isolates' the processes through 'extraction' and Life Cycle Stage contributions instead consider direct use for the FU only.

Path Contribution Analysis

For Path contributions (Srocka & Montiel, 2021), the impact of 'Electricity' (Figure 3.3 D) is the same compared to HEM. In this case, we see that a lower contribution is attributed to 'Electricity production', while a higher contribution is attributed to 'Coal mining' in Table 3.3. Through the extraction of processes, HEM effectively breaks infinite loops between processes (e.g. coal is needed to produce electricity and electricity is needed to produce coal), where Path contributions does not, which leads to different attributions of contribution. As shown in Table 3.3, the contribution from 'Electricity production' is lower than the direct process contribution of the same process, we discuss this in more detail in SI 1.6.

As long as a singular process is extracted, the total result will be the same, however, when multiple processes are extracted, results may differ, as these could (indirectly) affect each other, this would lead to double counting in Path contributions. We discuss this in some more detail in SI 1.6.

Causer and Connector Contribution Analysis

Causer and Connector contributions (Reinhard et al., 2016) are more useful for systematic analysis of databases than 'regular' CA, but here we use this approach to assess the contributions of electricity production. We find two major differences compared to HEM: Firstly, we find higher contributions compared to HEM (Figure 3.3 E). Secondly we find only the aggregated contribution for 'connector' (indirect contributions), without any possible disaggregation of the processes that are in this group, which may not be ideal in CA.

While the direct contributions are the same (given that the FU amount is the same as the amount of product produced by that process), the indirect contributions are higher. Reinhard et al. (2016) point to double counting of their method for connector contributions, while they did not consider this a major issue for their specific use case, double counting should be avoided in CA.

3.3.3 Applying HEM using ecoinvent

Having demonstrated this approach to a small example, we now demonstrate HEM with a simple (but much larger) example using ecoinvent v3.11 (Wernet et al., 2016). The code for this example is available at (M. van der Meide, 2025a). We show the contribution of mining processes to the production of copper. As a Functional unit, we use 1 kg of 'copper cathode' from 'Market for copper, cathode, GLO'. We extract processes based on metadata, we use the CPC classification '14: Metal ores' (and processes with underlying classifications). All processes with this classification are extracted. To assess climate change impact, we use the GWP 100 method from the IPCC (2022).

Applying HEM to this example, we find the results in Figure 3.4 A. Here, we see that the *target* 'mining' contributes about 39% of the impact. To gain further insight, we then assess the process group contributions to both the *target* and *remaining* groups, grouping the processes by the reference product names. This shows us that both the *target* and *remaining* sectors are heavily affected by contributions from 'electricity', though the production of this electricity may happen in entirely different locations for the *target* and *remaining* systems.

We now compare HEM to grouped process contributions. The process group contributions only show the direct contributions. In Figure 3.4 B we have two groups; the processes in the *target* and *remaining* groups. Here, we find that the *target* 'mining' has almost no direct contribution. In Figure 3.4 C we also compare against the three main process groups and a remainder, grouped based on reference product. Here we find no mining processes. This shows us that while considering direct contributions may show *sources* of contributions, indirect contributions can show how these contributions connect to the FU (Reinhard et al., 2016).

From these results, a producer of copper may conclude that while they may be able to switch to more sustainable sources of electricity for their production processes, a large part of the electricity related impacts come from mining.

Ecoinvent: Comparison of HEM to other Contribution Analysis approaches

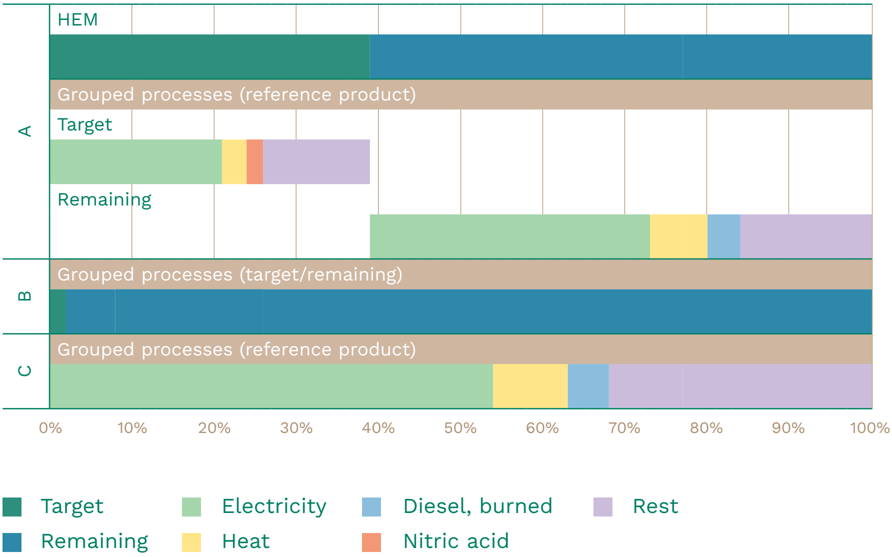

Figure 3.4 - Contribution Analysis results for different approaches for the production of 1kg 'copper, cathode' and the extraction/contribution of 'Metal, ores' in ecoinvent. The bottom numbers are in percentage contribution. ***A*** *Contribution of both the target processes and remaining system. Also showing the grouped process contributions to the target and remaining system. Grouped by reference product name, showing highest three contributing process groups and 'rest'.* ***B*** *Grouped process contributions, grouped by 'target' and 'remaining'.* ***C*** *Grouped process contributions, grouped by reference product name, showing highest three contributing process groups and 'rest'.*

Comparing approaches that show indirect contributions becomes more challenging in larger examples. Path Contributions can be calculated and are shown in SI 1.7. While calculating these results is easy, interpretation becomes complex as there are many production routes from 'ore' to the final product for the production of copper, some consisting of multiple processes in a chain. Combining these into a single 'minging' group is difficult due to interdependencies of these processes. For Life Cycle Stage contributions, we run into the same issue, where a life cycle stage would need to be defined for each production route, making this not only time consuming but also error-prone to ensure the life cycle stages don't overlap in this complex supply chain.

3.4 Discussion

Above, we have shown how HEM may be applied in LCA through: 1) defining a scenario, 2) extracting processes, 3) calculating results for the *original* and *remaining* system and 4) calculating results for the *target* processes. We have discussed interpretation of HEM results and have compared HEM against other CA approaches. That said, we want to emphasize that this should be considered as an additional approach for CA that may be useful in specific cases, not a replacement of existing ones.

The main advantage of HEM in LCA is that it allows practitioners to assess contributions of processes anywhere in the system, independent of their 'network' relationship between each other in the supply chain but also independent of their relationship to the FU. We have shown that this may be a useful approach when making use of large LCI databases and many processes may need to be assessed with our example using the ecoinvent LCI database (Wernet et al., 2016). Further uses could be to identify highly contributing sectors or countries, or this could be identifying suppliers or materials, that are important to a manufacturer.

3.4.1 Other applications for HEM

Another use-case of HEM may mirror that of Reinhard et al. (2016) where HEM can be used to systematically assess contributions from processes (or groups of processes) to other processes in an LCI database. These results could be used to prioritize database improvements while not double-counting impacts.

Furthermore, HEM could also be applied partially in two ways: 1) Reducing the 'extracted' amounts instead of completely removing them, making HEM this very similar to Sensitivity Analysis (Heijungs & Kleijn, 2001) (e.g. extracting half of the use of electricity). 2) Targeting the extraction of single uses of a product instead of all uses (e.g. the extraction of electricity use in coal mining), making the extraction more targeted. Both partial approaches may be useful, though the use-cases for this may be limited and it would be very important to carefully document what is extracted.

3.4.2 Comparison against other CA approaches

Life Cycle Stage CA bears a lot of similarity to HEM and both approaches avoid double counting, though the main difference is that HEM considers the use of all of the reference product from the *target* processes, where Life Cycle Stage CA only considers the direct use from the FU. Similarly, Path contributions are also close in results, though Path CA under-counts the contribution compared to process CA, implying that the contributions would be lower than they actually are.

As we have shown, both Life Cycle Stage CA and Path CA quickly become challenging to apply and interpret once many processes are involved. This use-case is where HEM becomes particularly useful; large systems and many *target* processes, especially if the direct 'supply chain' link between the FU and the *target* processes may be unclear.

Another comparison we made was with 'causer' and 'connector' contributions (Reinhard et al., 2016), here we find that we can calculate the same kind of results, but without double counting. Furthermore, HEM can apply this method to groups of processes, making it more useful for larger systems, where the contribution of a single process is likely to be low. Another advantage to HEM is that we are able to apply process contributions to the *remaining* system and *target* processes, allowing additional insight into results that was previously not possible.

One approach to HEM in LCA that we have not yet discussed but is technically similar is how Volkart et al. (2018) extract processes from their system to avoid double counting in a coupling of LCA with Partial Equilibrium models. While their stated goal is entirely different (avoiding double counting), they effectively use a similar approach (extracting processes similar to case 3a). Another difference is that we also explore the use of process contributions within the *remaining* system and *target* processes to gain more insight into results.

3.4.3 Limitations of HEM

CA is generally applied to answer one of two kinds of questions: 'What entities contribute to the total?' and 'What is the contribution of this entity to the total?'. The former is more an explorative question, while the latter is more an investigative question. While some approaches may be applied to answer both questions (Direct CA, First Tier CA, and Path CA), HEM and Life Cycle Stage CA are most useful for the last type of question, where the practitioner wants to gain deeper insight into a specific part of their model. This will limit the amount of use-cases HEM is relevant in LCA.

Furthermore, a main limitation is that there are currently no standard software tools to apply HEM in LCA for practitioners. That said, we do share the code we used to apply HEM in our ecoinvent example in van der Meide (2025a). This could be modified into a more generic software package in the future.

Next, one limitation that applies both to HEM and all other CA approaches; the usefulness of the approach will entirely depend on the quality of the data used and whether that is sufficient for the question at hand.

One more limitation comes when HEM in LCA is compared to HEM in IOA, in the latter, it is often used to analyze trade relationships, through extracting regions (Miller & Lahr, 2001). This use-case would be harder in process-LCA as such data is not well represented in LCI databases such as ecoinvent (Wernet et al., 2016). That said, especially since version 3.10, ecoinvent has included more trade information in every version, so it remains to be seen how much of a problem this will remain into the future.

Finally, changes to the technology matrix are made, this brings two possible questions. The first is on stability of results, when looking closely at each of the cases to implement HEM in LCA, it turns out each already has commonly used analogs in LCA and they pose no risk to calculations, as discussed further in SI 1.5. The second is on the computational load of re-calculating the scaling vector, one additional solution to the scaling vector needs to be found for each HEM calculation, which common LCA software such as Brightway2 (Mutel, 2017a) can do in well under a second, meaning doing that calculation twice should not be an issue. Lastly, more exotic approaches to LCA such as dynamic LCA have not been considered in this study, the interplay of such approaches to LCA and contribution analysis should be studied in more detail.

3.5 Conclusion

In this study we introduced HEM from IOA as a useful CA approach in LCA and have shown how HEM may provide different insights into LCA results compared to existing CA approaches. Through our demonstration of HEM in a simplified example and a larger example using ecoinvent (Wernet et al., 2016), we have shown how HEM can be used and what insights it can provide in LCA. By comparing against several established CA approaches we have shown how and when results are different and how this can help practitioners to gain insight into results.

As we have shown, this approach can help practitioners gain deeper understanding of impacts within their systems. The methodological connections between IOA and LCA should keep being investigated by both sides to keep improving both methods.

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