Probabilistic graph inspections through forests

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Stellingen

behorende bij het proefschrift

Probabilistic Graph Inspections Through Forests

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1. The probabilistic analysis of Wilson's algorithm is a powerful tool in the study of Kirchhoff forests and should be considered the default option. Nonetheless, other methods, e.g. combinatorial argumentation, can be employed effectively and should not be overlooked.

2. On cycle-free graphs, control on the two-point connectivity function of a Kirchhoff forest can be obtained by controlling the end-points of two killed random walks, which in many cases can be achieved.

3. The occupation field process of Wilson's algorithm admits a complete description in terms of Poisson point processes associated to closed walks on the graph. Under some mild conditions, an additional description of this process can be given, that highlights the Laplacian spectrum of the graph.

4. By coupling together Kirchhoff forests of different intensities, the cycles removed during the coupled version of Wilson's algorithm can be used to construct a random walk loop-soup.

5. There is a large class of problems, including e.g. bipartite matching, coupling of probability measures, and network flows, for which local feasibility is a sufficient condition for the existence of a global solution. For problems in this class, instances can often be translated into each other.

6. Coupling two probability measures can be seen as constructing a matching between 'pieces' of probability mass of both measures.

7. A time-inhomogeneous Markov chain with independent increments can be succinctly described as an integral over an inhomogeneous Poisson point process. Such a description is unavailable whenever the Markov chain does not have independent increments.

8. Obtaining a simple answer from a complicated mathematical computation hints at the existence of a simple argument that gives at least a heuristic explanation of the answer.

9. Gaining a better understanding of the concept of probability should not be a problem that is regarded only in the philosophy of science, but should be important for applied mathematicians working in stochastics as well.

10. Just as being a professional athlete requires full dedication and constant training of the body, so does being a professional research mathematician require full dedication and constant training of the mind.

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