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Probabilistic graph inspections through forests

Koperberg, V.T.

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Probabilistic Graph Inspections Through Forests

Twan Koperberg

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Probabilistic Graph Inspections Through Forests

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Twan Koperberg
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Promotor:

Prof.dr. W.Th.F. den Hollander

Co-promotores:

Dr. L. Avena

(Università degli Studi di Firenze)

Dr. A. Gaudillière

(Université d'Aix-Marseille)

Promotiecommissie:

Prof.dr.ir. G.L.A. Derks

Prof.dr. P. Stevenhagen

Prof.dr. N. Enriquez

(Université de Paris)

Dr. W. Ruszel

(Universiteit Utrecht)

Prof.dr. C. Sabot

(Université de Lyon)

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