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Probabilistic graph inspections through forests

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Probabilistic Graph Inspections Through Forests

Twan Koperberg

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Probabilistic Graph Inspections Through Forests

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