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Algorithmic techniques in algebraic cryptanalysis and their applications

Makarim, R.H.

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Rusydi Hasan Makarim
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Promotor:
Prof.dr. R. Cramer (CWI Amsterdam & Universiteit Leiden)

Co-promotor:
Dr.ir. M. Stevens (CWI Amsterdam)

Promotiecommissie:
Prof.dr.ir. G.L.A. Derks
Prof.dr. P. Stevenhagen
Prof.dr. C. Cid (Simula UiB, Norway & OIST, Japan)
Prof.dr. A. May (Ruhr-University Bochum, Germany)
Dr. S. Samardjiska (Radboud Universiteit, Nijmegen)

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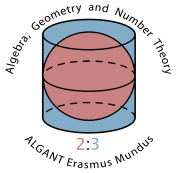

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Rusydi H. Makarim

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