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Stochastic amplitude modulation of nonlinear dispersive waves

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Stochastic Amplitude Modulation of Nonlinear Dispersive Waves

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