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Quantum methods for machine learning and classical dynamics

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Quantum Methods for Machine Learning and Classical Dynamics

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To those who make me feel at home

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