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Quantum methods for machine learning and classical dynamics

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Citation

Barthe, A. M. (2026, March 20). *Quantum methods for machine learning and classical dynamics*. Retrieved from <https://hdl.handle.net/1887/4297482>

Version: Publisher's Version

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Quantum Methods for Machine Learning and Classical Dynamics

Proefschrift

ter verkrijging van
de graad van doctor aan de Universiteit Leiden,
op gezag van rector magnificus prof. dr. S. de Rijcke,
volgens besluit van het college voor promoties
te verdedigen op vrijdag 20 maart 2026
klokke 13:00 uur

door

Alice Barthe

geboren te Suresnes, Frankrijk
in 1993

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This thesis was funded thanks to the CERN Quantum Technology Initiative and to ESA ϕ -lab Quantum Computing for Earth Observation (QC4EO) initiative.

To those who make me feel at home

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