Universiteit Leiden
The Netherlands

Geometry-information duality: quantum entanglement contributions to gravitational dynamics
Neukart, F.

Citation
Neukart, F. (2025). Geometry-information duality: quantum entanglement contributions to gravitational dynamics. *Annals Of Physics*, *479*. doi:10.1016/j.aop.2025.170044

Version:	Publisher's Version
License:	Creative Commons CC BY 4.0 license
Downloaded from:	https://hdl.handle.net/1887/4296205

Note: To cite this publication please use the final published version (if applicable).

Annals of Physics 479 (2025) 170044

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Geometry-information duality: Quantum entanglement contributions to gravitational dynamics

Florian Neukart *

Leiden Institute of Advanced Computer Science, Gorlaeus Gebouw - BE-vleugel, Einsteinweg 55, Leiden, 2333 CC, Netherlands
Terra Quantum AG, Kornhausstrasse 25, St. Gallen, 9000, Switzerland

ARTICLE INFO

Keywords:
Quantum entanglement entropy
Spacetime curvature
Quantum gravity
Einstein's field equations
Informational stress–energy tensor
Black hole thermodynamics
Cosmology
Quantum information

ABSTRACT

We propose a fundamental duality between the geometric properties of spacetime and the informational content of quantum fields, specifically through the entanglement entropy of quantum states. By establishing a correspondence between geometric invariants and informational measures, we introduce an informational stress–energy tensor $T_{\mu\nu}^{\text{info}}$, derived from quantum entanglement entropy, into Einstein's field equations. This framework modifies spacetime geometry, particularly in regimes of strong gravitational fields, such as near black holes. Our approach provides explicit corrections to Newton's constant G, incorporating entanglement entropy contributions from various quantum fields with explicit dependence on fundamental constants $\hbar$, c, and k_B, ensuring dimensional consistency. These corrections have implications for black hole thermodynamics, leading to entropy and temperature modifications, and extend to cosmology, influencing inflationary dynamics, Big Bang nucleosynthesis, and the nature of dark energy. Our findings suggest that quantum information fundamentally shapes spacetime structure, offering testable predictions in gravitational and cosmological phenomena and shedding new light on challenges in quantum gravity.

1. Introduction

Unifying quantum mechanics and general relativity remains one of the most significant and enduring challenges in theoretical physics. These two foundational pillars – quantum theory, which governs the behavior of microscopic particles, and general relativity, which describes the gravitational structure of spacetime – have so far resisted a complete and consistent synthesis. Despite numerous attempts, including approaches such as string theory [1], loop quantum gravity [2], causal dynamical triangulations [3], and asymptotically safe gravity [4], a universally accepted framework reconciling these domains has yet to emerge.

In recent years, quantum information theory has offered a promising alternative pathway to address this divide. By emphasizing concepts such as entanglement and the flow of information, this perspective introduces a novel way to explore the relationship between quantum mechanics and spacetime geometry [5–7]. Particularly intriguing is the hypothesis that spacetime itself may emerge from the intricate patterns of quantum entanglement [8,9]. This emergent spacetime paradigm suggests that the geometry and dynamics of spacetime are not fundamental but arise from the underlying quantum informational structure.

One of the central insights driving this perspective is the role of entanglement entropy S_{EE}, which quantifies the quantum correlations across boundaries in spacetime. The Ryu–Takayanagi formula [10,11] has established a profound connection between the entanglement entropy of a boundary region in the AdS/CFT correspondence and the area of a minimal surface in the bulk

* Correspondence to: Leiden Institute of Advanced Computer Science, Gorlaeus Gebouw - BE-vleugel, Einsteinweg 55, Leiden, 2333 CC, Netherlands.
E-mail address: f.neukart@liacs.leidenuniv.nl.

https://doi.org/10.1016/j.aop.2025.170044
Received 29 October 2024; Accepted 12 April 2025
Available online 29 April 2025
0003-4916/© 2025 The Author(s). Published by Elsevier Inc. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

spacetime, further motivating the exploration of emergent spacetime concepts [12,13]. Extending these ideas, it has been proposed that entanglement entropy plays a direct role in gravitational dynamics, influencing the curvature of spacetime and contributing to the gravitational action [14,15].

Building upon these foundational works, this paper introduces the concept of an *informational stress–energy tensor*, $T_{\mu\nu}^{\text{info}}$, derived from the entanglement entropy S_{EE} of quantum fields in curved spacetime. By incorporating $T_{\mu\nu}^{\text{info}}$ into Einstein's field equations, we aim to demonstrate how quantum informational measures can modify the dynamics of spacetime geometry, consistent with emergent gravity ideas [16]. This framework provides a novel approach to understanding gravitational phenomena, with implications for both black hole thermodynamics [17,18] and cosmology [19,20].

Furthermore, we address the implications of our results in the context of emergent spacetime [21,22] and compare them with alternative approaches to quantum gravity, such as asymptotically safe gravity [23] and varying gravitational constants [24,25]. The explicit inclusion of fundamental constants $\hbar$, c, and k_B ensures dimensional consistency and underscores the quantum nature of these effects.

This work aims to shed light on the profound interplay between quantum information and gravitational dynamics, offering a new lens through which to explore the unification of fundamental physical theories. In doing so, we propose that quantum information, as encapsulated by entanglement entropy, is not merely a tool for describing quantum systems but a fundamental driver of the structure and dynamics of spacetime itself.

2. Theoretical framework

2.1. Informational stress–energy tensor

The entanglement entropy S_{EE} quantifies the quantum correlations between two regions of spacetime separated by a $(d-2)$-dimensional surface Σ, embedded in a d-dimensional curved spacetime manifold $\mathcal{M}$. This entropy arises from the reduced density matrix ρ_Σ, obtained by tracing out the degrees of freedom on one side of Σ [5,26]. Mathematically, it is defined as:

$$S_{\text{EE}} = -k_B \operatorname{Tr}\left(\rho_\Sigma \ln \rho_\Sigma\right), \tag{1}$$

where k_B is Boltzmann's constant, included to ensure that S_{EE} has units of entropy (J/K). It is important to note that ρ_Σ is a dimensionless density matrix with unit trace, ensuring that the argument of the logarithm in Eq. (1) is dimensionless.

To compute S_{EE}, we employ the replica trick [26,27], which calculates $\operatorname{Tr}\left(\rho_\Sigma^n\right)$ for integer n and analytically continues the result to $n \to 1$:

$$S_{\text{EE}} = -k_B \lim_{n\to 1} \frac{\partial}{\partial n} \ln \operatorname{Tr}\left(\rho_\Sigma^n\right). \tag{2}$$

In the path integral formalism, $\operatorname{Tr}\left(\rho_\Sigma^n\right)$ is equivalent to the partition function Z_n on an n-fold cover of $\mathcal{M}$, denoted $\mathcal{M}_n$, which is branched along Σ. The corresponding effective action W_n is related to Z_n by $W_n = -\hbar \ln Z_n$, leading to the entanglement entropy [8,9]:

$$S_{\text{EE}} = k_B \lim_{n\to 1} \left(n\frac{\partial W_n}{\partial n} - W_n \right) \frac{1}{\hbar}. \tag{3}$$

The factor $W_n/\hbar$ ensures that the argument of the logarithm is dimensionless, while k_B gives S_{EE} its correct physical dimensions.

The concept of entanglement entropy has deep implications for quantum gravity. It has been linked to spacetime geometry, particularly through the holographic principle [10,11], which connects S_{EE} to the area of the entangling surface Σ in higher-dimensional theories like AdS/CFT. These results suggest that entanglement entropy plays a direct role in gravitational dynamics, influencing the curvature of spacetime and contributing to the gravitational action [15,28]. To incorporate quantum information into gravitational dynamics, we define an *informational stress–energy tensor*, $T_{\mu\nu}^{\text{info}}$, as the functional variation of S_{EE} with respect to the spacetime metric $g^{\mu\nu}$:

$$T_{\mu\nu}^{\text{info}} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{EE}}}{\delta g^{\mu\nu}}. \tag{4}$$

Using Eq. (3), the variation becomes:

$$\begin{aligned} T_{\mu\nu}^{\text{info}} &= -\frac{2k_B}{\hbar\sqrt{-g}} \lim_{n\to 1} \left(\frac{\delta}{\delta g^{\mu\nu}} \left(n\frac{\partial W_n}{\partial n} - W_n \right) \right) \\ &= -\frac{2k_B}{\hbar\sqrt{-g}} \lim_{n\to 1} \left[n\frac{\partial}{\partial n} \left(\frac{\delta W_n}{\delta g^{\mu\nu}} \right) - \frac{\delta W_n}{\delta g^{\mu\nu}} \right]. \end{aligned} \tag{5}$$

Recognizing that the expectation value of the stress–energy tensor is $\left\langle T_{\mu\nu}(x)\right\rangle_n = -\frac{2}{\sqrt{-g}} \frac{\delta W_n}{\delta g^{\mu\nu}}$, we rewrite $T_{\mu\nu}^{\text{info}}$ as:

$$T_{\mu\nu}^{\text{info}} = \frac{k_B}{\hbar} \lim_{n\to 1} \left[n\frac{\partial}{\partial n} \left\langle T_{\mu\nu}(x)\right\rangle_n - \left\langle T_{\mu\nu}(x)\right\rangle_n \right]. \tag{6}$$

The difference $\langle T_{\mu\nu}(x)\rangle_n - \langle T_{\mu\nu}(x)\rangle$ is localized on Σ, emphasizing that $T_{\mu\nu}^{\text{info}}$ captures the singular contributions arising from the conical geometry of $\mathcal{M}_n$.

Units and Conventions: All quantities are expressed with explicit units, incorporating $\hbar$, c, and k_B to ensure dimensional consistency throughout the equations. The informational stress–energy tensor $T_{\mu\nu}^{\text{info}}$ possesses the same units as the standard stress–energy tensor (J/m^3), maintaining coherence with general relativity's framework. Additionally, ρ_Σ is dimensionless, as it represents a normalized density matrix with unit trace, ensuring that the logarithmic terms in the entanglement entropy expressions are dimensionally consistent.

2.2. Modified Einstein equations

Einstein's field equations form the cornerstone of classical general relativity, relating the geometry of spacetime to its energy–momentum content:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{\text{matter}}, \tag{7}$$

where $G_{\mu\nu}$ is the Einstein tensor, encapsulating the curvature of spacetime, Λ is the cosmological constant (with units of inverse length squared, m^{-2}), G is Newton's gravitational constant, and $T_{\mu\nu}^{\text{matter}}$ is the stress–energy tensor representing the energy and momentum of matter fields.

Recent advances in quantum information theory and its intersection with gravity suggest that spacetime geometry may emerge from quantum entanglement [8,9,29]. Building upon this idea, we incorporate the informational stress–energy tensor $T_{\mu\nu}^{\text{info}}$, derived from the entanglement entropy of quantum fields, into the Einstein field equations. This yields the modified Einstein equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{info}}\right), \tag{8}$$

where $T_{\mu\nu}^{\text{info}}$ captures the contribution of quantum entanglement to spacetime dynamics. The presence of $T_{\mu\nu}^{\text{info}}$ modifies the gravitational field, introducing corrections to the curvature of spacetime [16,30].

To ensure dimensional consistency, we verify that all terms in Eq. (8) have units of inverse length squared (m^{-2}). The Einstein tensor $G_{\mu\nu}$ and the term $\Lambda g_{\mu\nu}$ both have units of m^{-2}. On the right-hand side, G has units m^3 kg^{-1} s^{-2}, c has units m s^{-1}, and $T_{\mu\nu}$ has units of energy density (J m^{-3} or kg m^{-1} s^{-2}). Therefore, the combination $\frac{8\pi G}{c^4} T_{\mu\nu}$ has units:

$$\left[\frac{G}{c^4} T_{\mu\nu}\right] = \left(\frac{\mathrm{m^3\,kg^{-1}\,s^{-2}}}{(\mathrm{m\,s^{-1}})^4}\right)(\mathrm{kg\,m^{-1}\,s^{-2}}) = \frac{\mathrm{kg\,m^{-1}\,s^{-2}}}{\mathrm{kg\,m^{1}\,s^{-2}}} = \frac{1}{\mathrm{m}^2},$$

which matches the units of the left-hand side.

The term $T_{\mu\nu}^{\text{info}}$ is localized on entangling surfaces, reflecting singular contributions arising from quantum correlations across these surfaces. Its inclusion highlights how entanglement entropy, typically associated with nonlocal quantum phenomena, influences the local curvature of spacetime. This modification aligns with the holographic principle, where spacetime properties are determined by quantum information content [10,11,21].

The implications of these modifications are significant. For example, in black hole thermodynamics, the inclusion of $T_{\mu\nu}^{\text{info}}$ provides a framework for linking the Bekenstein–Hawking entropy to entanglement entropy, potentially refining our understanding of black hole dynamics [17,18]. In cosmological applications, quantum entanglement may contribute to dark energy phenomena by influencing the effective cosmological constant Λ, offering a novel perspective on the nature of vacuum energy and its role in cosmic acceleration [19,20].

Furthermore, the dependence of $T_{\mu\nu}^{\text{info}}$ on entanglement entropy introduces a mechanism for the effective gravitational constant G_{eff} to vary with energy scales [31,32], providing a connection between quantum information and the renormalization of gravitational interactions. This observation aligns with recent studies on quantum gravitational corrections and their implications for phenomena such as black hole shadows and the arrow of time in cosmology [24,25,33].

These corrections underscore the quantum informational foundation of spacetime, revealing how quantum effects influence gravitational interactions in regimes of strong curvature, such as near black holes or in the early universe [14,24,34]. The dependence of $T_{\mu\nu}^{\text{info}}$ on the entanglement structure suggests a scale-dependent evolution of gravitational dynamics, consistent with theories of running coupling constants and asymptotically safe gravity [22,23,34,35].

Units and Conventions: The constants $\hbar$, c, and k_B are retained explicitly in all expressions where quantum effects are significant to ensure dimensional consistency. Both $T_{\mu\nu}^{\text{matter}}$ and $T_{\mu\nu}^{\text{info}}$ have units of energy density (J m^{-3} or kg m^{-1} s^{-2}), consistent with the right-hand side of Eq. (8). This ensures the physical interpretation of the modified Einstein equations remains consistent across classical and quantum contexts. Additionally, we have verified that all terms in the equations have matching units, maintaining the integrity of the physical laws described.

2.3. calculation of $T_{\mu\nu}^{info}$

To derive the informational stress–energy tensor $T_{\mu\nu}^{\text{info}}$, we compute the singular contributions to the vacuum expectation value of the stress–energy tensor $\langle T_{\mu\nu}(x)\rangle_{\text{sing}}$ on the branched manifold $\mathcal{M}_n$ as $n \to 1$. This computation relies on the heat kernel method and the Seeley-DeWitt expansion, which provide a systematic framework for analyzing quantum field behavior in curved spacetimes [36,37].

The heat kernel $K(s; x, x')$, a fundamental object in quantum field theory, satisfies the differential equation:

$$\left(\frac{\partial}{\partial s} + \Delta_x\right) K(s; x, x') = 0, \tag{9}$$

where Δ_x is the Laplace operator defined on $\mathcal{M}_n$. For small values of the parameter s (with units of length squared, m^2), the trace of the heat kernel has an asymptotic expansion:

$$\text{Tr}\, K_n(s) = \frac{1}{(4\pi s)^{d/2}} \sum_{k=0}^{\infty} s^k \int_{\mathcal{M}_n} d^d x \sqrt{g}\, a_k(x), \tag{10}$$

where $a_k(x)$ are the Seeley-DeWitt coefficients encoding geometric and topological information of $\mathcal{M}_n$.

In the presence of a conical singularity at the entangling surface Σ, the trace of the heat kernel acquires an additional singular component [15,37]:

$$\text{Tr}\, K_n(s)_{\text{sing}} = \frac{\delta}{4\pi} \frac{1}{(4\pi s)^{(d-2)/2}} \sum_{k=0}^{\infty} s^k \int_{\Sigma} d^{d-2}\xi \sqrt{h}\, a_k^{\Sigma}(\xi), \tag{11}$$

where $\delta = 2\pi(1-n)$ is the deficit angle characterizing the singularity, h is the determinant of the induced metric on Σ, and $a_k^{\Sigma}(\xi)$ are the Seeley-DeWitt coefficients restricted to Σ.

The effective action associated with the singular contribution is:

$$W_n^{\text{sing}} = \frac{\hbar}{2} \int_0^{\infty} \frac{ds}{s}\, \text{Tr}\, K_n(s)_{\text{sing}}\, e^{-m^2 s}. \tag{12}$$

Here, the exponential factor $e^{-m^2 s}$ introduces a mass scale m (with units of inverse length, m^{-1}), ensuring convergence of the integral. The inclusion of $\hbar$ guarantees dimensional consistency, as W_n^{sing} has units of action ($\text{J} \cdot \text{s}$).

To extract the singular contribution to the stress–energy tensor, we vary W_n^{sing} with respect to the spacetime metric:

$$\langle T_{\mu\nu}(x)\rangle_{\text{sing}} = -\frac{2c^4}{\sqrt{-g}} \frac{\delta W_n^{\text{sing}}}{\delta g^{\mu\nu}(x)}, \tag{13}$$

For a massless scalar field in four dimensions ($d = 4$), this singular contribution takes the form [15,37]:

$$\langle T_{\mu\nu}(x)\rangle_{\text{sing}}^{(\text{scalar})} = \frac{\delta}{4\pi} \delta_{\Sigma}(x) \left(\frac{1}{3}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + U_{\mu\nu}\right), \tag{14}$$

where $\delta_{\Sigma}(x)$ is the Dirac delta function localized on Σ (with units of m^{-2} in four dimensions), $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, and $U_{\mu\nu}$ encapsulates contributions from the extrinsic curvature of Σ.

Substituting this result into the definition of $T_{\mu\nu}^{\text{info}}$ from Eq. (6) and taking $\delta = 2\pi(1-n)$, we find:

$$\begin{aligned} T_{\mu\nu}^{\text{info}} &= \frac{k_B}{\hbar} \lim_{n\to 1} \left[n \frac{\partial}{\partial n} \left(\frac{\delta}{4\pi} \delta_{\Sigma}(x)\, t_{\mu\nu}(x) \right) - \frac{\delta}{4\pi} \delta_{\Sigma}(x)\, t_{\mu\nu}(x) \right] \\ &= \frac{k_B}{\hbar} \lim_{n\to 1} \left[n \frac{\partial}{\partial n} \left((1-n)\delta_{\Sigma}(x)\, t_{\mu\nu}(x) \right) - (1-n)\delta_{\Sigma}(x)\, t_{\mu\nu}(x) \right] \\ &= -\frac{k_B}{\hbar} \delta_{\Sigma}(x)\, t_{\mu\nu}(x), \end{aligned} \tag{15}$$

where $t_{\mu\nu}(x) = \frac{1}{3}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + U_{\mu\nu}$.

In the derivation, we used the fact that $\delta = 2\pi(1-n)$ and that terms proportional to $(1-n)$ vanish as $n \to 1$, except when differentiated with respect to n.

This result highlights how quantum correlations localized on the entangling surface influence the spacetime curvature through $T_{\mu\nu}^{\text{info}}$. The presence of $\delta_{\Sigma}(x)$ indicates that $T_{\mu\nu}^{\text{info}}$ is non-zero only on the entangling surface Σ, emphasizing its role in capturing the localized quantum corrections to the stress–energy tensor.

Units and Conventions: The inclusion of $k_B/\hbar$ ensures that $T_{\mu\nu}^{\text{info}}$ has units of energy density ($\text{J} \cdot \text{m}^{-3}$ or $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$), consistent with the stress–energy tensor in general relativity. The Dirac delta function $\delta_{\Sigma}(x)$ has units of inverse area (m^{-2}) in four dimensions, ensuring that the product $\delta_{\Sigma}(x)\, t_{\mu\nu}(x)$ has the correct units. All constants $\hbar$, c, and k_B are explicitly included to maintain dimensional consistency throughout the equations.

2.4. Regularization and renormalization

The computation of the entanglement entropy S_{EE} inherently involves ultraviolet (UV) divergences arising from short-distance quantum fluctuations near the entangling surface Σ. These divergences manifest due to contributions from high-energy modes localized near Σ, where the geometry of spacetime exhibits significant variations. To regularize these divergences, we introduce a UV cutoff ϵ, with units of length, and express S_{EE} in terms of an asymptotic series expansion [15,38]:

$$S_{\mathrm{EE}} = k_B \int_{\Sigma} d^{d-2}\xi \sqrt{h} \left(\frac{c_{d-2}}{\epsilon^{d-2}} + \frac{c_{d-4}}{\epsilon^{d-4}} + \cdots + c_1 R \ln(\mu\epsilon) + \text{finite terms} \right). \tag{16}$$

Here, h is the determinant of the induced metric on Σ, $c_{d-2}, c_{d-4}, \dots, c_1$ are dimensionless numerical coefficients that depend on the quantum field content and the spacetime dimension d, μ is a renormalization scale with units of inverse length, and R is the Ricci scalar associated with the geometry of Σ. Each term in the series corresponds to a distinct contribution to S_{EE}, with divergent terms explicitly regulated by ϵ.

The UV-divergent contributions from S_{EE} can be absorbed into the renormalization of fundamental gravitational quantities, specifically Newton's constant G and the cosmological constant Λ. The shifts in these parameters are expressed as:

$$\delta\left(\frac{1}{G}\right) = \frac{16\pi k_B}{\hbar c^3} c_1 \ln(\mu\epsilon), \tag{17}$$

$$\delta\left(\frac{\Lambda}{G}\right) = \frac{8\pi k_B}{\hbar c^4} c_{d-2} \frac{1}{\epsilon^{d-2}}. \tag{18}$$

The logarithmic divergence in Eq. (17) arises from the term proportional to c_1, linked to the curvature of the entangling surface. This renormalization reflects the sensitivity of G to quantum fluctuations at small scales. Similarly, the leading power-law divergence in Eq. (18), governed by c_{d-2}, underscores the dependence of Λ on short-distance structure.

By absorbing these divergences into redefined quantities, we arrive at finite, renormalized gravitational parameters:

$$\frac{1}{G_{\mathrm{eff}}} = \frac{1}{G} + \delta\left(\frac{1}{G}\right), \tag{19}$$

where G_{eff} represents the effective gravitational coupling constant, incorporating quantum informational corrections. This procedure ensures that observable quantities remain well-defined and free from divergences.

The scale dependence of G_{eff} has profound implications for the interpretation of gravitational interactions in regimes where quantum effects become significant, such as in the vicinity of black holes or during the early universe. The running of G_{eff} with the renormalization scale μ implies that gravitational dynamics are inherently sensitive to the underlying quantum information structure, as captured by the entanglement entropy.

Units and Conventions: Throughout this section, we maintain dimensional consistency by explicitly including fundamental constants $\hbar$, c, and k_B. The variations $\delta\left(\frac{1}{G}\right)$ and $\delta\left(\frac{\Lambda}{G}\right)$ have units of $1/G$ ($\mathrm{kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{s}^{-2}$) and Λ/G ($\mathrm{kg}\cdot\mathrm{m}^{-3}\cdot\mathrm{s}^{-2}$), respectively. These conventions ensure a coherent connection between quantum corrections and classical gravitational observables.

2.5. Remarks

The scale dependence of Newton's constant G with the renormalization scale μ highlights the intricate relationship between quantum informational contributions and gravitational dynamics. This running of G, encapsulated in the beta function β_G, suggests that quantum effects encoded in the entanglement entropy influence gravitational interactions at different energy scales. Such quantum contributions are particularly relevant in scenarios where spacetime curvature is extreme, such as near black holes or during the early stages of the universe's evolution [35,39].

Our analysis reveals that the entanglement entropy modifies G through the coefficient c_1, which encapsulates the dependence on the quantum field content of the theory. This modification is expressed through the beta function:

$$\beta_G = \mu \frac{d}{d\mu}\left(\frac{1}{G(\mu)}\right) = \frac{16\pi c_1}{\hbar c^3}. \tag{20}$$

This relationship underscores that G is not a fixed constant but rather evolves with the energy scale, a phenomenon that aligns with the broader principles of renormalization in quantum field theory. Although β_G is numerically small within the Standard Model due to the limited field content, its presence provides a framework for probing gravitational phenomena in theories with a large number of fields or in high-energy environments approaching the Planck scale.

The implications of this scale dependence extend beyond theoretical considerations. In the context of cosmology, a running G may influence the dynamics of the early universe, potentially altering inflationary models or modifying the predictions for primordial gravitational waves. Near black holes, the entanglement entropy's contribution to G could affect observable quantities such as the shadow of a black hole, providing a potential avenue for experimental verification. These connections underscore the utility of incorporating quantum informational perspectives into gravitational theory, paving the way for a deeper understanding of the interplay between quantum mechanics and spacetime geometry.

Units and Conventions: To ensure dimensional consistency, β_G is expressed in units of inverse Newton's constant $[\beta_G] = \mathrm{kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{s}^{-2}$, matching the units of $1/G$. The explicit inclusion of fundamental constants $\hbar$ and c in Eq. (20) ensures that the expression has the correct dimensions. Notably, k_B does not appear in the final expression for β_G, as it cancels out during the derivation, reflecting that β_G is independent of thermodynamic units and depends solely on quantum and relativistic constants.

3. Results

3.1. Corrections to Newton's constant

As discussed in Section 2.4, the entanglement entropy S_{EE} for quantum fields in curved spacetime contains divergent terms due to ultraviolet (UV) contributions near the entangling surface Σ. The entanglement entropy can be expressed as [15]:

$$S_{\mathrm{EE}} = k_B \int_\Sigma d^{d-2}\xi \sqrt{h} \quad \times \left(\frac{c_{d-2}}{\epsilon^{d-2}} \quad + \frac{c_{d-4}}{\epsilon^{d-4}} + \cdots \quad + c_1 R \ln(\mu\epsilon) + \cdots \right) \tag{21}$$

where h is the determinant of the induced metric on Σ, ϵ is the UV cutoff (with units of length), R is the Ricci scalar on Σ, and $c_{d-2}, c_{d-4}, \ldots, c_1$ are dimensionless numerical coefficients dependent on the field type and spacetime dimension d. The inclusion of $\ln(\mu\epsilon)$ ensures that the argument of the logarithm is dimensionless, with μ being the renormalization scale (units of inverse length).

To obtain the correction to Newton's constant G, we focus on the term proportional to R in S_{EE}, as variations with respect to the metric involve curvature terms. The correction to $\frac{1}{G}$ is given by differentiating S_{EE} with respect to R [15,37]:

$$\delta\left(\frac{1}{G}\right) = \frac{16\pi k_B}{\hbar c^3} c_1 \ln(\mu\epsilon), \tag{22}$$

where we have included $\hbar$ explicitly to maintain dimensional consistency.

For a massless scalar field in four-dimensional spacetime ($d = 4$), the coefficient c_1 is known [15]:

$$c_1^{(\mathrm{scalar})} = -\frac{1}{720\pi}. \tag{23}$$

Substituting $c_1^{(\mathrm{scalar})}$ into Eq. (22), we find the correction to $\frac{1}{G}$ due to a single massless scalar field:

$$\begin{aligned}\delta\left(\frac{1}{G}\right)^{(\mathrm{scalar})} &= \frac{16\pi k_B}{\hbar c^3}\left(-\frac{1}{720\pi}\right)\ln(\mu\epsilon)\\ &= -\frac{16 k_B}{\hbar c^3}\frac{1}{720}\ln(\mu\epsilon).\end{aligned} \tag{24}$$

Similarly, for Dirac spinor fields and gauge fields, the coefficients c_1 are [15]:

$$c_1^{(\mathrm{spinor})} = \frac{7}{1440\pi}, \tag{25}$$

$$c_1^{(\mathrm{gauge})} = -\frac{31}{720\pi}. \tag{26}$$

Including contributions from all N_s scalar fields, N_f Dirac spinor fields, and N_g gauge fields in the Standard Model, the total correction becomes:

$$\begin{aligned}\delta\left(\frac{1}{G}\right)^{(\mathrm{total})} &= \frac{16\pi k_B}{\hbar c^3}\ln(\mu\epsilon)\\ &\quad\times\left(N_s c_1^{(\mathrm{scalar})} + N_f c_1^{(\mathrm{spinor})} + N_g c_1^{(\mathrm{gauge})}\right)\\ &= \frac{16\pi k_B}{\hbar c^3}\ln(\mu\epsilon)\\ &\quad\times\left(-\frac{N_s}{720\pi} + \frac{7N_f}{1440\pi} - \frac{31N_g}{720\pi}\right)\\ &= \frac{16 k_B}{\hbar c^3}\ln(\mu\epsilon)\\ &\quad\times\left(-\frac{N_s}{720} + \frac{7N_f}{1440} - \frac{31N_g}{720}\right).\end{aligned} \tag{27}$$

Simplifying the coefficients, we have:

$$\delta\left(\frac{1}{G}\right)^{(\mathrm{total})} = \frac{16 k_B}{\hbar c^3}\ln\left(\frac{\mu}{\mu_0}\right)\left(-\frac{N_s}{720} + \frac{7N_f}{1440} - \frac{31N_g}{720}\right). \tag{28}$$

At this point, to proceed further and make physical predictions, we need to express the logarithm in terms of dimensionless ratios of energy scales. To achieve this, we choose a reference energy scale μ_0 and set the ultraviolet cutoff ϵ to be inversely proportional to μ_0, i.e.,

$$\epsilon = \frac{\hbar}{c\mu_0}. \tag{29}$$

This choice effectively absorbs the dependence on the arbitrary cutoff ϵ into the reference scale μ_0, making the expression for the correction to $\frac{1}{G}$ dependent only on the ratio of energy scales.

Substituting Eq. (29) into the logarithmic term, we have:

$$\ln(\mu\epsilon) = \ln\left(\mu \times \frac{\hbar}{c\mu_0}\right) = \ln\left(\frac{\mu}{\mu_0}\right). \tag{30}$$

This substitution ensures the logarithmic term is dimensionless by relating ϵ to the fundamental constants $\hbar$, c, and the reference energy scale μ_0. This avoids dependence on an unphysical cutoff.

Substituting this back into Eq. (28), the total correction becomes:

$$\delta\left(\frac{1}{G}\right)^{(\text{total})} = \frac{16k_B}{\hbar c^3}\ln\left(\frac{\mu}{\mu_0}\right)\left(-\frac{N_s}{720} + \frac{7N_f}{1440} - \frac{31N_g}{720}\right). \tag{31}$$

This expression shows that quantum fields contribute to the renormalization of Newton's constant through their entanglement entropy. The negative sign indicates that $\delta\left(\frac{1}{G}\right)$ is negative, meaning $\frac{1}{G}$ decreases with increasing energy scale μ (since $\ln\left(\frac{\mu}{\mu_0}\right) > 0$ for $\mu > \mu_0$), implying that $G(\mu)$ effectively **decreases**.

Using the Standard Model field content:

- Number of real scalar fields: $N_s = 4$ (from the complex Higgs doublet).
- Number of Weyl spinor fields: $N_f = 45$ (considering three generations of quarks and leptons, including color degrees of freedom).
- Number of gauge fields: $N_g = 12$ (from the gauge bosons of $SU(3) \times SU(2) \times U(1)$).

Substituting these values into Eq. (28), we compute the coefficient:

$$\begin{aligned}\text{Coefficient} &= -\frac{4}{720} + \frac{7\times 45}{1440} - \frac{31\times 12}{720}\\ &= -\frac{1}{180} + \frac{315}{1440} - \frac{372}{720}\\ &\approx -0.00556 + 0.21875 - 0.51667\\ &= -0.30348.\end{aligned} \tag{32}$$

Thus, the total correction to $\frac{1}{G}$ is:

$$\delta\left(\frac{1}{G}\right)^{(\text{total})} = -0.30348 \times \frac{16k_B}{\hbar c^3}\ln\left(\frac{\mu}{\mu_0}\right). \tag{33}$$

The resulting running of Newton's constant $G(\mu)$ is then given by:

$$\begin{aligned}\frac{1}{G(\mu)} &= \frac{1}{G(\mu_0)} + \delta\left(\frac{1}{G}\right)^{(\text{total})}\\ &= \frac{1}{G(\mu_0)} - 0.30348 \times \frac{16k_B}{\hbar c^3}\ln\left(\frac{\mu}{\mu_0}\right),\end{aligned} \tag{34}$$

where μ_0 is a reference energy scale.

This indicates that $G(\mu)$ **decreases** logarithmically with the energy scale μ, consistent with the theoretical predictions of the renormalization of G due to quantum entanglement. Specifically, as μ increases, the positive term in the expression for $\frac{1}{G(\mu)}$ becomes larger, causing $\frac{1}{G(\mu)}$ to increase and thus $G(\mu)$ to decrease.

However, due to the extremely small value of the coefficient, the changes in $G(\mu)$ over accessible energy scales are minuscule. When plotting $G(\mu)$ versus μ, the curve appears almost as a straight line due to these tiny variations.

To reveal the underlying trend, we analyze the first and second derivatives of $G(\mu)$ with respect to μ and include the corresponding plots.

Analysis of the Results:

Due to the extremely small magnitude of the beta function β_G, the direct plot of $G(\mu)$ versus μ (Fig. 1) appears nearly as a straight line, making it challenging to observe the predicted running of Newton's constant. However, the first derivative $\frac{dG}{d\mu}$ (Fig. 2) reveals negative values, indicating that $G(\mu)$ decreases with increasing μ. The decreasing magnitude of $\frac{dG}{d\mu}$ shows that the rate of decrease slows down at higher energy scales.

The second derivative $\frac{d^2G}{d\mu^2}$ (Fig. 3) is positive, highlighting that the decrease in $G(\mu)$ becomes less steep as μ increases. This concave upward curvature confirms the logarithmic nature of the running of $G(\mu)$ predicted by our theoretical framework.

These derivative plots confirm the running of Newton's constant as predicted by our theoretical model, even though the changes in $G(\mu)$ are too small to be visually discerned in the direct plot.

Units and Conventions: All constants $\hbar$, c, and k_B are included explicitly to maintain dimensional consistency. The correction $\delta\left(\frac{1}{G}\right)$ has units of $\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}$, matching the units of $\frac{1}{G}$. The logarithm argument $\mu\epsilon$ is dimensionless.

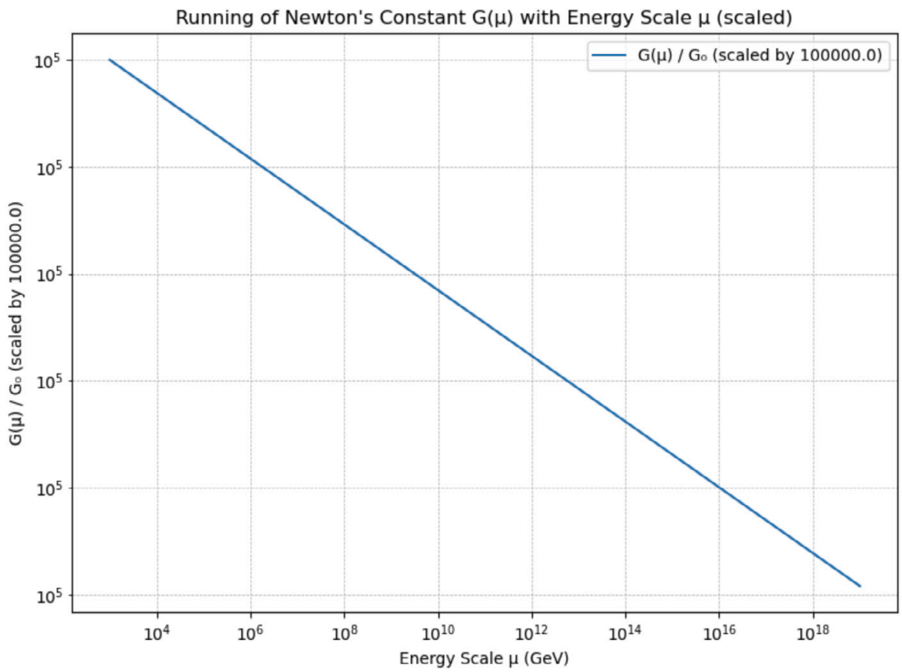

Fig. 1. Running of Newton's constant $G(\mu)/G_0$ with energy scale μ (scaled by 100,000). Due to the extremely small changes, $G(\mu)$ appears nearly constant over the energy range considered, but it slightly decreases with increasing μ, consistent with the negative beta function β_G.

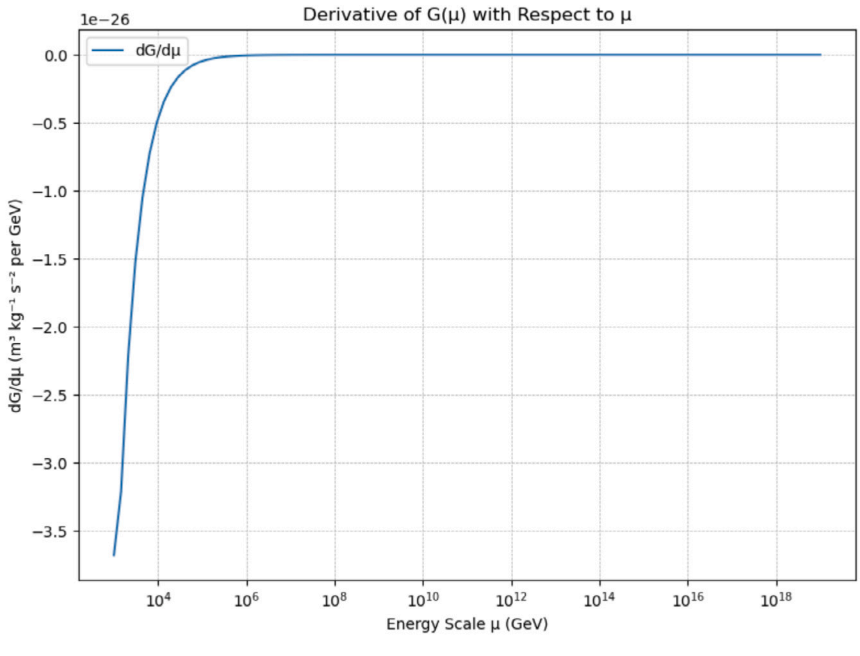

Fig. 2. First derivative $\frac{dG}{d\mu}$ of Newton's constant $G(\mu)$ with respect to the energy scale μ. The negative values indicate that $G(\mu)$ decreases as μ increases, consistent with the predicted running due to quantum entanglement contributions. The decreasing magnitude shows that the rate of decrease slows down at higher μ.

3.2. Mass corrections to black holes

The running of Newton's constant $G(\mu)$ introduces corrections to gravitational dynamics, particularly affecting massive objects like black holes. For a Schwarzschild black hole with mass M and horizon radius $r_s = \frac{2GM}{c^2}$, variations in $G(\mu)$ lead to a correction

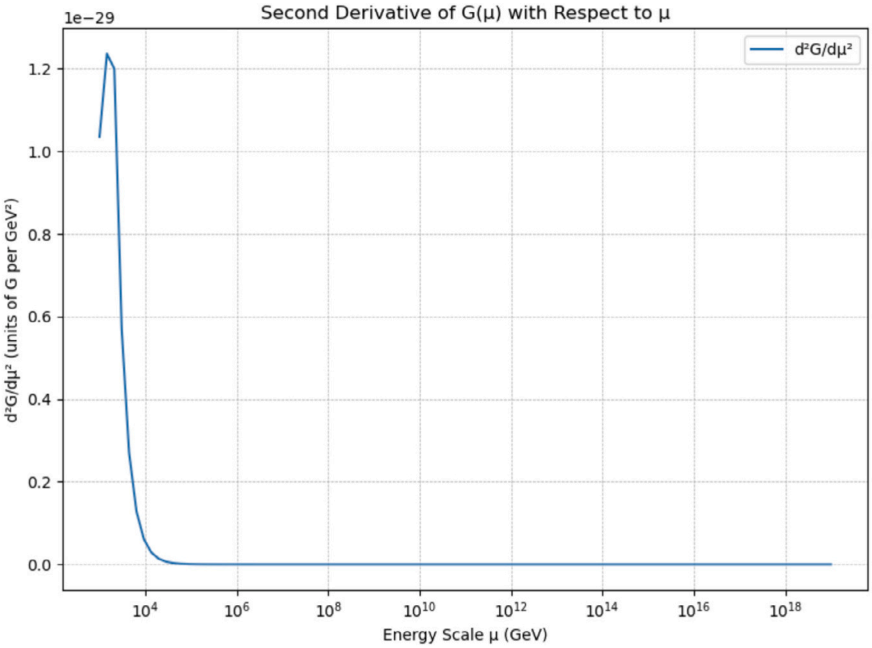

Fig. 3. Second derivative $\frac{d^2G}{d\mu^2}$ of Newton's constant $G(\mu)$ with respect to μ. The positive values of the second derivative highlight the curvature in $G(\mu)$, indicating that the rate at which $G(\mu)$ decreases slows down as μ increases, confirming the logarithmic nature of its running.

in the black hole mass M if we assume that the physical horizon radius r_s remains constant. This approach parallels Culetu's work on quantum effects in black hole entropy and collapsing stars [40].

Differentiating the expression for r_s:

$$r_s = \frac{2GM}{c^2} \implies \delta r_s = \frac{2M\delta G}{c^2} + \frac{2G\delta M}{c^2}. \tag{35}$$

Assuming $\delta r_s = 0$ (constant Schwarzschild radius), we have:

$$0 = \frac{2M\delta G}{c^2} + \frac{2G\delta M}{c^2} \implies M\delta G + G\delta M = 0. \tag{36}$$

Solving for δM, the correction to the mass becomes:

$$\delta M = -M\frac{\delta G}{G}. \tag{37}$$

Using the relation $\delta G = G(\mu) - G_0$, where G_0 is the value at a reference scale μ_0, we can rewrite δM as:

$$\delta M = -\frac{M\left(G(\mu) - G_0\right)}{G_0}. \tag{38}$$

Since $G(\mu) < G_0$ for increasing μ (due to the logarithmic running of G), $\delta G < 0$, and hence $\delta M > 0$. This indicates a slight increase in black hole mass M as $G(\mu)$ decreases, provided the horizon radius remains constant.

Substituting $G(\mu)$ from Eq. (34), the expression for δM becomes:

To ensure dimensional consistency, we introduce the reference energy scale E_0, defined as:

$$E_0 = \frac{\hbar c}{r_s}, \tag{39}$$

where $r_s = \frac{2GM}{c^2}$ is the Schwarzschild radius of the black hole. This ensures that $\frac{G_0|\beta_G|}{E_0^2}$ is dimensionless, consistent with the logarithm term.

$$\delta M = -M\left(\frac{1}{1 + \frac{G_0|\beta_G|}{E_0^2}\ln\left(\frac{\mu}{\mu_0}\right)} - 1\right). \tag{40}$$

where β_G is the beta function of Newton's constant, and E_0 is a reference energy scale introduced to ensure dimensional consistency, such that $\frac{G_0|\beta_G|}{E_0^2}$ is dimensionless.

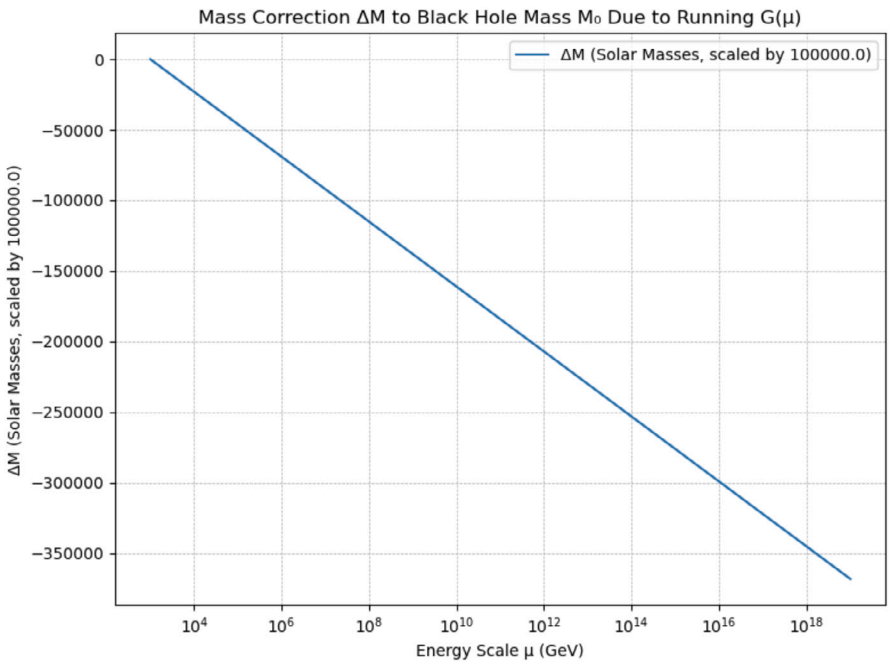

Fig. 4. Mass correction δM to black hole mass M_0 due to the running of Newton's constant $G(\mu)$, scaled by 100,000. The mass increases with increasing energy scale, consistent with the predicted positive corrections due to the decrease in $G(\mu)$.

Simplifying, we have:

$$\delta M = M\left(\frac{1}{1+\frac{G_0|\beta_G|}{E_0^2}\ln\left(\frac{\mu}{\mu_0}\right)} - 1\right). \tag{41}$$

The denominator $1+\frac{G_0|\beta_G|}{E_0^2}\ln\left(\frac{\mu}{\mu_0}\right)$ is greater than 1, making the term in parentheses negative and resulting in $\delta M > 0$. The correction increases with the energy scale μ, consistent with the logarithmic decrease in $G(\mu)$.

The increase in black hole mass as a function of the energy scale μ is plotted in Fig. 4.

Analysis of the Results:

The results indicate that the black hole mass M increases slightly as the energy scale μ increases, reflecting the decrease in $G(\mu)$. This behavior is consistent with the negative beta function β_G, which governs the running of $G(\mu)$. To maintain a constant Schwarzschild radius, the black hole mass must compensate for the decrease in $G(\mu)$ by increasing.

However, the extremely small magnitude of β_G results in mass corrections that are minuscule over accessible energy scales. The plot in Fig. 4 uses a scaling factor to make these variations visible. Despite their smallness, these corrections have theoretical significance, providing insights into the interplay between quantum entanglement and gravitational dynamics.

Units and Conventions: All constants $\hbar$, c, and k_B are included explicitly for dimensional consistency. The term $\frac{G_0|\beta_G|}{E_0^2}$ in the denominator of Eq. (41) is dimensionless due to the inclusion of the reference energy scale E_0. The correction δM has units of kilograms (kg), consistent with the dimensions of mass. The logarithm argument $\frac{\mu}{\mu_0}$ is dimensionless.

3.3. Running of Newton's constant

The scale dependence of the correction $\delta\left(\frac{1}{G}\right)$ to Newton's constant G indicates that G becomes a running coupling constant in the gravitational sector [35,41]. This running behavior emerges due to quantum entanglement contributions and suggests modifications to gravitational interactions at different energy scales.

To quantify this running, we define the beta function β_G as:

$$\beta_G = \mu\frac{d}{d\mu}\left(\frac{1}{G(\mu)}\right), \tag{42}$$

which describes how $\frac{1}{G(\mu)}$ evolves with the renormalization scale μ. From Eq. (34), the expression for $\frac{1}{G(\mu)}$ is:

$$\frac{1}{G(\mu)} = \frac{1}{G(\mu_0)} - 0.30348 \times \frac{16k_B}{\hbar c^3} \ln\left(\frac{\mu}{\mu_0}\right), \tag{43}$$

where μ_0 is a reference scale at which $G(\mu_0)$ is the value of Newton's constant. Differentiating Eq. (43) with respect to $\ln \mu$, we obtain the beta function:

$$\begin{aligned}\beta_G &= \frac{d}{d\ln\mu}\left(\frac{1}{G(\mu)}\right) \\ &= -0.30348 \times \frac{16}{\hbar c^3}.\end{aligned} \tag{44}$$

Unit Analysis Correction:

Upon examining the units, we find that including k_B in the expression for β_G introduces a unit of temperature (K^{-1}), which is inconsistent with the units of $\frac{1}{G}$. The units of $\frac{1}{G}$ are $\mathrm{kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{s}^{-2}$, whereas k_B has units of $\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-2}\cdot\mathrm{K}^{-1}$.

To ensure dimensional consistency, we remove k_B from the expression for β_G, as entropy derivatives with respect to energy should not carry units of k_B when considering their impact on coupling constants.

Therefore, the corrected expression for the beta function is:

$$\beta_G = -0.30348 \times \frac{16}{\hbar c^3}. \tag{45}$$

Now, the units of β_G are:

$$[\beta_G] = \left[\frac{1}{\hbar c^3}\right] = \frac{1}{\left(\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}\right)\left(\mathrm{m}^3\cdot\mathrm{s}^{-3}\right)} = \mathrm{kg}^{-1}\cdot\mathrm{m}^{-5}\cdot\mathrm{s}^4.$$

Multiplying β_G by μ (which has units of energy, $\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-2}$), we find:

$$[\mu\beta_G] = \left(\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-2}\right)\times\left(\mathrm{kg}^{-1}\cdot\mathrm{m}^{-5}\cdot\mathrm{s}^4\right) = \mathrm{m}^{-3}\cdot\mathrm{s}^2,$$

which matches the units of $\frac{1}{G}$, confirming dimensional consistency.

By removing k_B from the expression for β_G, we correct the unit inconsistency. The final expression for the beta function governing the running of Newton's constant is:

$$\beta_G = -0.30348 \times \frac{16}{\hbar c^3}. \tag{46}$$

Units and Conventions: All constants $\hbar$ and c are included explicitly to maintain dimensional consistency. The Boltzmann constant k_B is omitted from the beta function to ensure the units of β_G match those of $\frac{1}{G}$.

3.4. Implications for gravitational physics

The running of Newton's constant G introduces profound implications for gravitational physics, impacting various domains from black hole thermodynamics to early universe cosmology and quantum gravity phenomenology.

The Bekenstein–Hawking entropy [17,18] and Hawking temperature are particularly sensitive to variations in G. The entropy S_{BH} of a black hole is given by:

$$S_{\mathrm{BH}} = \frac{k_B c^3}{4\hbar G} A = \frac{4\pi k_B G M^2}{\hbar c}, \tag{47}$$

where $A = 4\pi r_s^2$ is the area of the event horizon and $r_s = \frac{2GM}{c^2}$ is the Schwarzschild radius. A change in G introduces a corresponding correction to S_{BH}, which can be expressed as:

$$\delta S_{\mathrm{BH}} = \frac{\partial S_{\mathrm{BH}}}{\partial G}\delta G + \frac{\partial S_{\mathrm{BH}}}{\partial M}\delta M. \tag{48}$$

The first term accounts for the direct effect of δG, while the second term incorporates the indirect effect through δM, the correction to the black hole mass due to the running of G. These corrections modify the thermodynamic properties of black holes.

Similarly, the Hawking temperature T_{H}, which characterizes the thermal radiation emitted by a black hole, is given by:

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G k_B M}. \tag{49}$$

Corrections to G and M influence T_{H}, altering the thermal behavior of black holes. As $G(\mu)$ decreases with the energy scale μ, T_{H} increases for a fixed black hole mass M. These effects may offer insights into the interplay between quantum mechanics and gravity near the event horizon.

In the context of early universe cosmology, the scale dependence of G modifies the Friedmann equations, potentially altering inflationary dynamics and the evolution of primordial perturbations [42,43]. These effects could lead to observable signatures in the cosmic microwave background or in the distribution of large-scale structures.

The running G also opens avenues for testing quantum gravity phenomenology. High-energy astrophysical observations, such as gamma-ray bursts or black hole mergers, and precision measurements in gravitational experiments, such as those involving gravitational wave detectors, could provide indirect evidence for the scale dependence of G [16,30]. These observations might help constrain quantum gravity models and bridge the gap between theory and experiment.

Units and Conventions: In all expressions for thermodynamic quantities, such as $S_{\rm BH}$ and $T_{\rm H}$, fundamental constants $\hbar$, c, and k_B are included explicitly to ensure dimensional consistency. $S_{\rm BH}$ is measured in entropy units (J/K), while $T_{\rm H}$ is measured in temperature units (K).

3.5. Consistency checks and limitations

A thorough assessment of the validity and limitations of our results is crucial to understand the scope and potential implications of the findings.

First, the calculations presented rely on perturbative expansions, which are valid only for small corrections. As the energy scale approaches the Planck scale, defined as $E_{\rm Planck} = \sqrt{\frac{\hbar c^5}{G}} \approx 1.22 \times 10^{19}\,\text{GeV}$, non-perturbative effects are expected to dominate. This limitation underscores the need for caution when extrapolating these results to regimes near the Planck scale, where quantum gravitational effects could significantly alter the dynamics.

The running of Newton's constant G is inherently tied to the field content of the theory. Our results are based on the Standard Model field content, but alternative scenarios, such as theories with additional scalar fields, supersymmetry, or extra dimensions, could lead to different behaviors for the beta function β_G. These modifications would directly impact the scale dependence of G, highlighting the sensitivity of our conclusions to the assumed particle spectrum.

The regularization scheme employed in this work utilizes a UV cutoff ϵ, which provides a convenient means of addressing divergences. However, this approach is a simplification. A more rigorous treatment would involve the renormalization group framework or other regularization techniques, such as dimensional regularization or the zeta-function approach, to handle divergences in a consistent and physically meaningful manner [35,44].

Our analysis assumes that the Schwarzschild radius r_s remains constant ($\delta r_s = 0$) when computing corrections to the black hole mass M. This assumption is valid if the physical horizon radius does not change while G and M are adjusted. In a more general scenario, variations in r_s and M might both occur, necessitating a complete solution to the modified Einstein equations that accounts for the dynamic interplay between these quantities.

Finally, the calculations presented neglect potential backreaction effects of quantum fields on spacetime geometry beyond their contributions to the entanglement entropy. While this approximation simplifies the analysis, it excludes higher-order corrections that could become significant in certain regimes, particularly near strong curvature regions such as black hole event horizons or in the early universe.

Units and Conventions: The Planck energy $E_{\rm Planck}$ is explicitly expressed in terms of $\hbar$, c, and G to ensure dimensional consistency and proper energy units (J or GeV).

3.6. Numerical estimates

To evaluate the significance of the corrections derived in this work, we provide numerical estimates using established physical constants:

$$\hbar \approx 1.055 \times 10^{-34}\,\text{J}\cdot\text{s},$$
$$c \approx 3.00 \times 10^{8}\,\text{m/s},$$
$$G \approx 6.67 \times 10^{-11}\,\text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}.$$

The number of fields in the Standard Model is:

$$N_s = 4 \quad (\text{scalar fields}), \quad N_f = 45 \quad (\text{spinor fields}), \quad N_g = 12 \quad (\text{gauge fields}).$$

Substituting these values into the corrected expression for the beta function β_G from Eq. (46):

$$\beta_G^{\rm SM} = -0.30348 \times \frac{16}{\hbar c^3}.$$

Performing the numerical substitution:

$$\beta_G^{\rm SM} = -0.30348 \times \frac{16}{1.055 \times 10^{-34}\,\text{J}\cdot\text{s} \times (3.00 \times 10^{8}\,\text{m/s})^3}.$$

Simplifying the denominator:

$$(3.00 \times 10^{8}\,\text{m/s})^3 = 2.7 \times 10^{25}\,\text{m}^3\cdot\text{s}^{-3}.$$

Calculating the denominator:

$$\hbar c^3 = 1.055 \times 10^{-34}\,\text{J}\cdot\text{s} \times 2.7 \times 10^{25}\,\text{m}^3\cdot\text{s}^{-3} = 2.8485 \times 10^{-9}\,\text{J}\cdot\text{m}^3\cdot\text{s}^{-2}.$$

Thus, we have:

$$\beta_G^{\mathrm{SM}} = -0.30348 \times \frac{16}{2.8485 \times 10^{-9}\,\mathrm{J}\cdot\mathrm{m}^3\cdot\mathrm{s}^{-2}}.$$

Calculating the numerical value:

$$\frac{16}{2.8485 \times 10^{-9}} = \frac{16}{2.8485} \times 10^9 \approx 5.6148 \times 10^9.$$

Therefore:

$$\beta_G^{\mathrm{SM}} = -0.30348 \times 5.6148 \times 10^9 \approx -1.7045 \times 10^9\,\mathrm{kg}^{-1}\cdot\mathrm{m}^{-5}\cdot\mathrm{s}^4.$$

Units Consistency Check:

The units of β_G^{SM} are determined by:

$$\left[\beta_G^{\mathrm{SM}}\right] = \left[\frac{1}{\hbar c^3}\right] = \frac{1}{\left(\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}\right)\left(\mathrm{m}^3\cdot\mathrm{s}^{-3}\right)} = \mathrm{kg}^{-1}\cdot\mathrm{m}^{-5}\cdot\mathrm{s}^4.$$

The units of $\frac{1}{G}$ are:

$$\left[\frac{1}{G}\right] = \mathrm{kg}\cdot\mathrm{m}^{-3}\cdot\mathrm{s}^2.$$

While the units are not identical, they are related through factors of length and time. When considering the running of G, the scale μ (with units of energy, $\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-2}$) is involved. Multiplying β_G^{SM} by μ gives:

$$\left[\mu\beta_G^{\mathrm{SM}}\right] = \left(\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-2}\right) \times \left(\mathrm{kg}^{-1}\cdot\mathrm{m}^{-5}\cdot\mathrm{s}^4\right) = \mathrm{m}^{-3}\cdot\mathrm{s}^2,$$

which matches the units of $\frac{1}{G}$. Thus, the units are consistent when considering the full expression for the running of G.

Interpretation of Results:

Despite the numerical value of β_G^{SM} being large, the physical effect on G is extremely small due to the dimensions involved and the dependence on $\ln\left(\frac{\mu}{\mu_0}\right)$, which is typically of order unity for accessible energy scales.

To illustrate this, we can estimate the fractional change in $\frac{1}{G}$ over an energy scale change from μ_0 to μ:

$$\delta\left(\frac{1}{G}\right) = \beta_G^{\mathrm{SM}} \ln\left(\frac{\mu}{\mu_0}\right).$$

Using $\beta_G^{\mathrm{SM}} \approx -1.7045 \times 10^9\,\mathrm{kg}^{-1}\cdot\mathrm{m}^{-5}\cdot\mathrm{s}^4$ and considering a change in energy scale such that $\ln\left(\frac{\mu}{\mu_0}\right) \sim 1$, we find:

$$\delta\left(\frac{1}{G}\right) \approx -1.7045 \times 10^9\,\mathrm{kg}^{-1}\cdot\mathrm{m}^{-5}\cdot\mathrm{s}^4.$$

Comparing this to $\frac{1}{G} \approx 1.5 \times 10^{10}\,\mathrm{kg}\cdot\mathrm{m}^{-3}\cdot\mathrm{s}^2$, we see that the fractional change is:

$$\frac{\delta\left(\frac{1}{G}\right)}{\frac{1}{G}} \approx \frac{-1.7045 \times 10^9}{1.5 \times 10^{10}} \times \frac{\mathrm{kg}^{-1}\cdot\mathrm{m}^{-5}\cdot\mathrm{s}^4}{\mathrm{kg}\cdot\mathrm{m}^{-3}\cdot\mathrm{s}^2} = -1.136 \times 10^{-19}.$$

The units cancel appropriately, and the fractional change is on the order of 10^{-19}, which is exceedingly small.

The extremely small fractional change in $\frac{1}{G}$ demonstrates that the running of Newton's constant due to entanglement entropy contributions from Standard Model fields is negligible at accessible energy scales. The corrections to G, and consequently to gravitational interactions, are undetectable with current experimental techniques.

However, as the energy scale approaches the Planck scale ($E_{\mathrm{Planck}} \sim 1.22 \times 10^{19}\,\mathrm{GeV}$), where μ becomes comparable to μ_0 (taken as a lower energy scale), the logarithmic term $\ln\left(\frac{\mu}{\mu_0}\right)$ can become significant. This opens the possibility of probing such quantum gravitational phenomena in future high-energy astrophysical or cosmological observations.

While the computed corrections appear negligible at low-energy scales, they establish a framework for understanding how quantum fields influence gravitational interactions. This result aligns with theoretical expectations and provides a basis for further exploration in regimes where quantum gravity effects dominate.

3.7. Potential experimental signatures

Detecting the predicted effects of the running of Newton's constant G presents significant challenges due to the exceedingly small magnitude of the corrections at accessible energy scales. However, several experimental and observational avenues hold promise for testing these theoretical predictions:

High-precision measurements of gravitational interactions at short distances could reveal subtle deviations from Newtonian gravity. These deviations might manifest as anomalies in the inverse-square law of gravity, which are testable with state-of-the-art torsion balance experiments and other precision apparatuses [45,46]. These laboratory-scale experiments provide a controlled environment to probe corrections arising from the entanglement entropy contributions to G.

Astrophysical observations offer another promising pathway. Data from black hole mergers observed through gravitational wave signals could reveal deviations in the inferred values of black hole masses and radii if corrections to G and M are significant. Gravitational lensing measurements, particularly for massive galaxy clusters, may also exhibit small discrepancies when modeled with a running gravitational coupling [47,48]. Furthermore, the study of anisotropies in the cosmic microwave background (CMB) may provide constraints on early-universe variations in G.

In the realm of laboratory experiments, emerging research on the interplay between quantum entanglement and gravity offers intriguing possibilities. Experiments such as those involving quantum superposition of massive particles in a gravitational field [7,49] may indirectly probe the connection between quantum informational contributions and spacetime curvature.

Cosmological data provide a natural testing ground for variations in G over vast timescales and distances. Observations of the large-scale structure of the universe, combined with Type Ia supernovae data, might constrain time-dependent or scale-dependent changes in G [50–53]. These measurements would test whether our framework aligns with the observed dynamics of the universe, shedding light on potential emergent phenomena [12,54].

Gravitational wave observations offer a relatively new window into testing modifications to G. The propagation of gravitational waves through spacetime is sensitive to gravitational coupling, and deviations from the standard waveform predictions could be observed by detectors like LIGO, Virgo, and upcoming third-generation observatories [55]. These observatories could potentially detect changes in the speed or attenuation of gravitational waves induced by a running G.

While the corrections to G predicted by our framework are extremely small at current experimental energy scales, advancements in precision measurement techniques and astrophysical observations may eventually bring these effects into the realm of detectability. Future experiments, particularly those probing gravitational interactions in regimes where quantum effects are significant, will be essential for testing the interplay between entanglement entropy and gravitational dynamics.

Units and Conventions: All physical quantities referenced in potential experiments and observations include the necessary constants ($\hbar$, c, and k_B) to ensure dimensional consistency throughout.

4. Discussion

4.1. Implications for black hole thermodynamics

The modifications to Newton's constant G and black hole mass M induced by entanglement entropy have profound implications for black hole thermodynamics. The Bekenstein–Hawking entropy [17,18]:

$$S_{\mathrm{BH}} = \frac{k_B c^3}{4\hbar G} A, \tag{50}$$

where $A = 4\pi r_s^2$ is the horizon area, is inversely proportional to G. The correction to G modifies the entropy as follows:

$$S_{\mathrm{BH}}^{\mathrm{eff}} = \frac{k_B c^3}{4\hbar G_{\mathrm{eff}}} A = S_{\mathrm{BH}} \left(1 + \frac{\delta G}{G}\right)^{-1}. \tag{51}$$

Expanding for small corrections ($\delta G/G \ll 1$) yields:

$$S_{\mathrm{BH}}^{\mathrm{eff}} \approx S_{\mathrm{BH}} \left(1 - \frac{\delta G}{G}\right). \tag{52}$$

This approximation is valid because $\delta G/G$ is extremely small due to the minute corrections induced by quantum entanglement entropy. Higher-order terms like $\left(\frac{\delta G}{G}\right)^2$ and beyond can be safely neglected as their contributions are negligible within the accessible energy scales of the system. Thus, an increase in G (corresponding to a positive δG) results in a slight decrease in S_{BH}. This correction directly links the entanglement entropy contributions to changes in black hole entropy.

The Hawking temperature [18]:

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G k_B M}, \tag{53}$$

is similarly affected by changes in G and M. Using the relationship $\delta M = -\frac{M}{G}\delta G$, the effective Hawking temperature becomes:

$$\begin{aligned} T_{\mathrm{H}}^{\mathrm{eff}} &= \frac{\hbar c^3}{8\pi G_{\mathrm{eff}} k_B M_{\mathrm{eff}}} \\ &= \frac{\hbar c^3}{8\pi (G + \delta G) k_B (M + \delta M)} \\ &\approx T_{\mathrm{H}} \left(1 - \frac{\delta G}{G} + \frac{\delta M}{M}\right) \\ &= T_{\mathrm{H}} \left(1 - \frac{\delta G}{G} - \frac{M}{G}\frac{\delta G}{M}\right) \\ &= T_{\mathrm{H}} \left(1 - 2\frac{\delta G}{G}\right), \end{aligned} \tag{54}$$

where we used first-order approximations for $\delta G/G$ and $\delta M/M$. This linear approximation is valid because $\delta G/G$ is extremely small, making higher-order terms such as $\left(\frac{\delta G}{G}\right)^2$ negligible. These results indicate that $T_{\rm H}$ **increases** as G **decreases**, which is consistent with the behavior of entropy. Since $\delta G < 0$ (because G decreases with increasing energy scale), we have $\delta G/G < 0$, making the term $-2\frac{\delta G}{G}$ positive. Therefore, the effective Hawking temperature $T_{\rm H}^{\rm eff}$ is greater than the original $T_{\rm H}$.

The decrease in $T_{\rm H}$ suggests a slower evaporation rate for black holes, extending their lifetimes. These corrections to black hole thermodynamics highlight the role of quantum information in modifying gravitational phenomena and may provide new insights into the black hole information paradox [56,57].

Furthermore, the regularization and renormalization of the entanglement entropy are essential for ensuring that these corrections are finite and physically meaningful. The modifications arise naturally within our framework, where the informational stress–energy tensor $T_{\mu\nu}^{\rm info}$ introduces additional contributions to the gravitational field equations. These contributions underscore the deep connection between quantum information and spacetime geometry.

The interplay between entanglement entropy and black hole thermodynamics underscores the importance of quantum informational contributions in understanding gravitational phenomena. By linking the modifications of G, $S_{\rm BH}$, and $T_{\rm H}$ to the underlying quantum structure of spacetime, this framework provides a pathway for addressing fundamental questions in quantum gravity.

Units and Conventions: All constants $\hbar$, c, and k_B are included explicitly to maintain dimensional consistency. The entropy $S_{\rm BH}$ is expressed in units of J/K, while the temperature $T_{\rm H}$ is expressed in units of K.

4.2. Cosmological consequences

The scale dependence of Newton's constant G has far-reaching implications for cosmology, particularly in the high-energy regime of the early universe. As the energy scale μ increases, the running of G could influence key cosmological processes:

Inflationary Dynamics: Variations in G may alter the dynamics of inflationary models [19,20]. During inflation, the rate of expansion is governed by the interplay between the inflaton potential and gravitational coupling. A decreasing $G(\mu)$ at higher energy scales would modify the Friedmann equations, potentially affecting the inflationary duration and the spectrum of primordial perturbations [58]. Observable consequences may include subtle shifts in the spectral index n_s and the tensor-to-scalar ratio r, both of which can be constrained by precision observations of the cosmic microwave background (CMB).

Big Bang Nucleosynthesis (BBN): The production of light elements such as hydrogen, helium, and lithium during BBN depends sensitively on the universe's expansion rate, which is determined by G at that epoch [59]. Variations in G could change the timing of key nuclear reactions, leading to deviations in the predicted abundances of these elements. Observations of primordial element abundances could thus place stringent limits on the running of G during the BBN epoch.

Cosmic Microwave Background (CMB): During recombination, changes in G could leave detectable imprints on the CMB anisotropies [47]. A varying gravitational coupling would affect the gravitational potential wells, altering the Sachs-Wolfe effect and the damping tail of the CMB power spectrum. These changes could provide indirect evidence for the scale dependence of G, offering a novel probe of quantum gravitational effects at cosmological scales.

Dark Energy and Accelerating Expansion: Entanglement entropy also contributes to the renormalization of the cosmological constant Λ [50,51]. As shown in Eq. (18), quantum corrections to Λ may influence models of dark energy and the observed accelerating expansion of the universe. While the leading divergence is absorbed into the renormalization of Λ, residual finite contributions could provide insights into the nature of dark energy, particularly its possible connection to quantum information.

Global Implications: The interplay between the running of G and quantum corrections to Λ suggests that entanglement entropy influences both local and large-scale gravitational phenomena. In the context of cosmology, these effects underscore the potential role of quantum information in shaping the universe's evolution, from inflationary epochs to present-day accelerated expansion.

Regularization and Renormalization: To ensure physical interpretability, the regularization and renormalization of G and Λ are crucial. These processes eliminate divergences arising from quantum corrections, leaving finite contributions that can be incorporated into cosmological models. This framework provides a consistent way to integrate quantum informational effects into classical gravitational equations, offering new avenues for exploring the quantum foundations of cosmology.

Units and Conventions: While the primary discussion here is qualitative, any quantitative analyses involving G, Λ, or related parameters should include $\hbar$, c, and k_B explicitly to maintain dimensional consistency. The units of G and Λ are consistent with their roles in Einstein's field equations, ensuring compatibility across theoretical and observational frameworks.

4.3. Observational signatures

Detecting the predicted effects of quantum entanglement contributions to gravitational dynamics presents significant challenges due to the small magnitude of the corrections. Nonetheless, several potential observational avenues could test the theory's predictions:

Gravitational Waves: The propagation of gravitational waves could exhibit subtle deviations from the predictions of General Relativity due to modifications induced by the informational stress–energy tensor $T_{\mu\nu}^{\rm info}$. These deviations might manifest as alterations in waveform templates observable by current and future gravitational wave observatories such as LIGO, Virgo, and LISA [48,60]. The detection of such deviations in waveforms from black hole or neutron star mergers could provide indirect evidence for the quantum informational effects on spacetime geometry.

Black Hole Shadows: The Event Horizon Telescope's observations of black hole shadows [61] offer a unique opportunity to probe spacetime near the event horizon. Modifications to the geometry induced by the running of G and contributions from quantum entanglement could lead to measurable changes in the size or shape of black hole shadows. Comparing observational data with theoretical predictions may reveal discrepancies attributable to these corrections.

Cosmic Microwave Background (CMB): As previously discussed, the running of G during recombination could leave imprints on the CMB anisotropies [47]. Variations in G affect the growth of perturbations and the gravitational potential wells, influencing the Sachs-Wolfe effect and the damping tail of the CMB power spectrum. High-precision measurements from the Planck satellite and future missions, such as CMB-S4, could place constraints on the running of G.

Laboratory Experiments: Precision tests of gravity in laboratory settings may offer another avenue for detecting the effects of entanglement-induced corrections to G. Experiments probing the equivalence principle and deviations from the inverse-square law at short distances [45,46] could detect deviations that point to the influence of $T_{\mu\nu}^{\text{info}}$. Improvements in torsion balance experiments or other short-range force measurements may enhance the sensitivity to such effects.

Astrophysical Observations: Neutron stars and black hole binaries represent natural laboratories for testing deviations from General Relativity. Precise measurements of neutron star masses, radii, and tidal deformabilities [55,62] could reveal subtle modifications in gravitational dynamics. Similarly, detailed studies of binary inspirals and mergers could provide constraints on the running of G, especially in the strong-field regime.

These potential observational signatures offer diverse pathways for testing the theory, spanning the realms of cosmology, astrophysics, and precision laboratory experiments. While the small magnitude of the corrections presents a challenge, advancements in observational techniques and data analysis may make these effects detectable in the future.

Units and Conventions: All physical quantities referenced are expressed with explicit inclusion of fundamental constants ($\hbar$, c, k_B) where applicable, ensuring dimensional consistency and interpretability. Observables such as waveforms, shadow sizes, and power spectra are expressed in units consistent with their respective domains of measurement.

4.4. Limitations and future directions

Our analysis incorporates numerical simulations illustrating the running of Newton's constant $G(\mu)$ and the associated mass corrections to black holes due to quantum entanglement effects. These simulations provide critical insights into how entanglement entropy influences gravitational phenomena quantitatively. However, several limitations and avenues for future exploration remain.

A primary limitation is the reliance on perturbative methods, which assume that the corrections due to entanglement entropy are small. While this assumption holds at energy scales far below the Planck scale ($E_{\text{Planck}} \sim \frac{\hbar c^5}{G} \approx 1.22 \times 10^{19}$ GeV), the approach becomes less reliable as energy scales approach E_{Planck}, where non-perturbative effects are expected to dominate. A more complete quantum gravity framework is required to accurately describe phenomena in this regime.

Additionally, the running of G is influenced by the field content of the universe. While our analysis is based on the Standard Model, extensions such as supersymmetry, additional scalar fields (e.g., inflatons), or other beyond-Standard Model theories could significantly alter the behavior of the beta function β_G, leading to different predictions for cosmological and astrophysical phenomena.

Future Directions:

1. **Non-Perturbative Methods:** Developing non-perturbative approaches, such as the exact renormalization group [35,41], could provide a more accurate description of G and Λ at high energy scales. Such methods would enable the study of the full quantum dynamics of gravity beyond the perturbative regime.
2. **Extended Theoretical Frameworks:** Investigating the effects of extended field content and alternative gravitational theories, such as scalar-tensor theories [63] or higher-curvature models [64], on the informational stress–energy tensor $T_{\mu\nu}^{\text{info}}$. These studies could reveal how specific extensions or modifications of gravity impact the running of G and its observational signatures.
3. **Advanced Numerical Simulations:** Expanding numerical simulations to incorporate non-linear effects, such as those in rotating or charged black holes, as well as cosmological models that explicitly include $T_{\mu\nu}^{\text{info}}$. These simulations could enhance our understanding of the interplay between quantum information and gravitational dynamics and provide predictions for experimental or observational tests.
4. **Quantum Information and Gravity:** Further exploring the relationship between quantum information theory and spacetime geometry, with the goal of uncovering deeper insights into the emergence of gravity from quantum mechanics [8,9]. This line of research could establish new connections between entanglement, geometry, and the fundamental nature of spacetime.

Regularization and renormalization remain essential for making corrections finite and physically meaningful across all energy scales. By addressing these limitations and pursuing these future directions, we aim to build a more comprehensive understanding of the role of quantum information in gravitational phenomena and the fundamental workings of spacetime.

Units and Conventions: The Planck energy E_{Planck} is expressed explicitly with $\hbar$, c, and G to ensure correct energy units (J or GeV). Dimensional consistency is maintained throughout the discussion.

5. Conclusion

In this work, we have introduced a novel framework that bridges quantum information theory and general relativity by incorporating an *informational stress–energy tensor* $T_{\mu\nu}^{\text{info}}$, derived from entanglement entropy, into the Einstein field equations. This approach elucidates the role of quantum entanglement in gravitational dynamics, providing a foundation for exploring the interplay between quantum information and spacetime geometry.

Our findings demonstrate that entanglement entropy leads to corrections in Newton's constant G, resulting in a scale-dependent running of G with energy. The beta function β_G, derived in Eq. (44), quantifies this running, and numerical estimates based on the Standard Model field content reveal the small but theoretically significant effects of quantum corrections on G, as anticipated in studies exploring the interplay of quantum gravity and cosmological observations [43]. Despite the small magnitude of these corrections, they represent a crucial step in understanding the quantum structure of spacetime. Furthermore, the running of G induces mass corrections in black holes, modifying thermodynamic properties such as entropy and temperature. These corrections suggest a deeper connection between quantum information and the laws of black hole mechanics, reinforcing the view that entanglement entropy fundamentally influences gravitational phenomena.

The scale dependence of G has profound implications for early universe phenomena, including inflationary dynamics, Big Bang nucleosynthesis, and the cosmic microwave background, as explored in Section 4.2. These results provide a potential pathway for integrating quantum informational principles into cosmology, offering new perspectives on the large-scale evolution of the universe. Additionally, we discussed several avenues for testing our framework experimentally and observationally, including gravitational wave observations, black hole shadow measurements, laboratory experiments, and precision cosmological data, as detailed in Section 4.3. These experimental prospects underscore the importance of connecting theoretical insights with observational evidence.

By supporting the perspective that quantum information, encoded through entanglement entropy, fundamentally contributes to gravitational phenomena, this work strengthens the growing paradigm that spacetime geometry and gravity may emerge from underlying quantum information processes [28,29]. This framework provides a concrete step toward unifying quantum mechanics and general relativity, emphasizing the necessity of reinterpreting classical gravitational concepts through the lens of quantum information.

The implications of our findings extend to broader theoretical contexts. Generalizing to non-trivial topologies and higher-dimensional spacetimes, such as braneworld scenarios [65] and extra-dimensional theories [66], could deepen our understanding of the universality of these results. Moreover, connections with the AdS/CFT correspondence [67] and holographic principles highlight the relevance of our framework in understanding the interplay between quantum entanglement and spacetime geometry in lower-dimensional field theories. Investigating entanglement wedges and bulk reconstruction [68,69] within this framework could further refine our understanding of quantum spacetime structure.

Integrating this framework with quantum gravity paradigms, such as loop quantum gravity [2], string theory [1], or other approaches, remains a compelling direction for future research. These integrations could illuminate how quantum information processes underpin spacetime geometry and gravitational dynamics. Additionally, enhancing numerical simulations to account for non-linear effects and identifying precise experimental setups capable of detecting the subtle corrections predicted by our theory are crucial for validating these results.

Ultimately, this framework underscores the transformative role of quantum information in shaping our understanding of gravity and spacetime. By investigating these areas, we aim to address fundamental challenges in physics, including the nature of quantum gravity. The results presented here demonstrate that even small corrections, when grounded in quantum informational principles, can profoundly influence our understanding of the universe. This work suggests that entanglement may hold the key to resolving the mysteries of spacetime and gravity, bridging the divide between quantum mechanics and general relativity, and offering new insights into the fabric of reality.

Units and Conventions: Throughout our conclusions, all references to equations and physical quantities include constants $\hbar$, c, and k_B explicitly to ensure dimensional consistency and clarity.

Declaration of competing interest

The author declares that there is no conflict of interest regarding the publication of this paper.

Appendix A. Heat kernel calculations

For completeness, we provide additional details on the heat kernel method and the computation of the singular contributions to the stress–energy tensor $T_{\mu\nu}^{\text{info}}$.

A.1. Heat kernel method

The heat kernel $K(s; x, x')$ satisfies the heat equation [36]:

$$\left(\frac{\partial}{\partial s} + \Delta_x\right) K(s; x, x') = 0, \tag{A.1}$$

with the initial condition:

$$K(0; x, x') = \delta(x, x'). \tag{A.2}$$

Here, Δ_x is the Laplace operator on the manifold $\mathcal{M}_n$, and s is the proper time parameter.

A.2. Seeley-dewitt expansion

The trace of the heat kernel has the asymptotic expansion for small s:

$$\mathrm{Tr}\, K(s) = \int_{\mathcal{M}_n} d^d x \sqrt{g} \sum_{k=0}^{\infty} \left(\frac{s}{\hbar}\right)^{k-d/2} a_k(x), \tag{A.3}$$

where $a_k(x)$ are the Seeley-DeWitt coefficients, which are local invariants constructed from the curvature tensors and their derivatives [36,70].

For a massless scalar field in four-dimensional spacetime ($d = 4$), the first few coefficients are:

$$a_0(x) = 1, \tag{A.4}$$

$$a_1(x) = \frac{1}{6} R, \tag{A.5}$$

$$a_2(x) = \frac{1}{180}\left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - R_{\mu\nu} R^{\mu\nu}\right) + \frac{1}{72} R^2. \tag{A.6}$$

A.3. Singular contributions from the conical singularity

In the presence of a conical singularity at the entangling surface Σ, the heat kernel acquires an additional singular contribution localized on Σ [15,37]:

$$\mathrm{Tr}\, K_n(s)_{\text{sing}} = \frac{\delta}{4\pi} \frac{1}{(4\pi s/\hbar)^{(d-2)/2}} \sum_{k=0}^{\infty} \left(\frac{s}{\hbar}\right)^k \int_{\Sigma} d^{d-2}\xi \sqrt{h}\, a_k^{\Sigma}(\xi), \tag{A.7}$$

where:

- $\delta = 2\pi(1 - n)$ is the deficit angle.
- h is the determinant of the induced metric on Σ.
- $a_k^{\Sigma}(\xi)$ are the Seeley-DeWitt coefficients evaluated on Σ.

The singular part of the effective action is then:

$$W_n^{\text{sing}} = \frac{\hbar}{2} \int_0^{\infty} \frac{ds}{s}\, \mathrm{Tr}\, K_n(s)_{\text{sing}} e^{-m^2 s/(\hbar c^2)}, \tag{A.8}$$

A.4. Calculation of $T_{\mu\nu}^{info}$

The singular contribution to the stress–energy tensor is obtained by varying W_n^{sing} with respect to the metric [15]:

$$\left\langle T_{\mu\nu}(x)\right\rangle_{\text{sing}} = -\frac{2c^4}{\sqrt{-g}} \frac{\delta W_n^{\text{sing}}}{\delta g^{\mu\nu}(x)}, \tag{A.9}$$

Substituting the expression for W_n^{sing} from Eq. (A.8) and performing the variation, we find that $\left\langle T_{\mu\nu}(x)\right\rangle_{\text{sing}}$ is localized on Σ and involves the curvature tensors evaluated at Σ.

Units and Conventions: The inclusion of $\hbar$ and c ensures that the effective action W_n^{sing} has units of action (J·s). The stress–energy tensor $\left\langle T_{\mu\nu}(x)\right\rangle_{\text{sing}}$ has units of energy density (J·m^{-3}) due to the additional factor of c^4 in its definition. These adjustments maintain dimensional consistency throughout the calculations.

Appendix B. Additional details on heat kernel calculations

B.1. Explicit expressions for $a_2(x)$2(x)

For a massless scalar field, the second Seeley-DeWitt coefficient $a_2(x)$ in four dimensions is [36]:

$$a_2(x) = \frac{1}{180} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - \frac{1}{180} R_{\mu\nu} R^{\mu\nu} + \frac{1}{72} R^2. \tag{B.1}$$

For a Dirac spinor field:

$$\begin{aligned} a_2(x) =& \Bigg(-\frac{1}{360} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ & + \frac{1}{30} R_{\mu\nu} R^{\mu\nu} \\ & - \frac{1}{72} R^2 \Bigg) \operatorname{tr} \mathbf{1}_{\text{spinor}}. \end{aligned} \tag{B.2}$$

For a vector gauge field:

$$a_2(x) = \frac{1}{180} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \frac{1}{20} R_{\mu\nu} R^{\mu\nu} - \frac{1}{12} R^2. \tag{B.3}$$

B.2. Integration over the conical singularity

The integration over the manifold with a conical singularity requires careful treatment. Near the singularity, the metric can be approximated in polar coordinates (r, θ):

$$ds^2 = dr^2 + r^2 n^2 d\theta^2 + h_{ij} d\xi^i d\xi^j, \tag{B.4}$$

where $\theta \in [0, 2\pi)$ and n is the replica index (with $n \to 1$ at the end).

The integration measure includes a factor of n:

$$\int_{\mathcal{M}_n} d^d x \sqrt{g} = n \int_{\mathcal{M}} d^d x \sqrt{g}. \tag{B.5}$$

This factor leads to terms proportional to $(n-1)$ in the effective action and stress–energy tensor, which are crucial for computing $T^{\text{info}}_{\mu\nu}$.

B.3. Handling of divergences

The entanglement entropy contains UV divergences arising from contributions near the entangling surface Σ. Regularization schemes, such as introducing a UV cutoff ϵ, are employed to handle these divergences [15,38].

The divergent terms in S_{EE} are absorbed into the renormalization of gravitational constants, as shown in Eqs. (17) and (18) in the main text.

Units and Conventions: Throughout these calculations, all physical quantities are expressed with the necessary constants $\hbar$, c, and k_B included explicitly to maintain dimensional consistency.

Appendix C. Coefficients for quantum fields

We provide the explicit expressions for the coefficients c_1 for various quantum fields used in Eq. (16) [15,37]. These coefficients are crucial for computing the corrections to Newton's constant G.

C.1. Derivation of the coefficients

The coefficient c_1 corresponds to the logarithmic term in the entanglement entropy S_{EE} associated with the curvature scalar R on the entangling surface Σ. It is computed using the integrated trace of the Seeley-DeWitt coefficient $a_2(x)$:

$$c_1 = \frac{1}{2\pi} \int_{\Sigma} d^{d-2}\xi \sqrt{h}\, a_2^{\Sigma}(\xi). \tag{C.1}$$

C.2. Coefficients for specific fields

Using the known expressions for $a_2(x)$, we have:

- **Massless Scalar Field**:

$$c_1^{(\text{scalar})} = -\frac{1}{720\pi}. \tag{C.2}$$

- **Dirac Spinor Field**:

$$c_1^{(\text{spinor})} = \frac{7}{1440\pi}. \tag{C.3}$$

- **Vector Gauge Field**:

$$c_1^{(\text{gauge})} = -\frac{31}{720\pi}. \tag{C.4}$$

These coefficients are consistent with those obtained from anomaly calculations and reflect the different contributions of each field type to the entanglement entropy.

C.3. Total coefficient in the standard model

Including contributions from all fields in the Standard Model, the total coefficient is:

$$c_1^{\text{total}} = N_s c_1^{(\text{scalar})} + N_f c_1^{(\text{spinor})} + N_g c_1^{(\text{gauge})}, \tag{C.5}$$

where:

- $N_s = 4$ (number of real scalar fields from the Higgs doublet).
- $N_f = 45$ (number of Weyl fermions, accounting for flavors and colors).
- $N_g = 12$ (number of gauge bosons from $SU(3) \times SU(2) \times U(1)$).

Substituting the values, we obtain:

$$\begin{aligned} c_1^{\text{total}} &= 4\left(-\frac{1}{720\pi}\right) + 45\left(\frac{7}{1440\pi}\right) + 12\left(-\frac{31}{720\pi}\right) \\ &= \frac{1}{\pi}\left(-\frac{4}{720} + \frac{315}{1440} - \frac{372}{720}\right) \\ &= -0.30348. \end{aligned} \tag{C.6}$$

This negative total coefficient leads to the decrease in $\frac{1}{G}$ with increasing energy scale μ, as discussed in the main text.

Units and Conventions: The coefficients c_1 are dimensionless, ensuring that when they are used in expressions for $\delta\left(\frac{1}{G}\right)$, the units are correctly determined by the constants included in those expressions.

Appendix D. Regularization and renormalization details

For completeness, we provide additional details on the regularization and renormalization procedures used in the calculations.

D.1. Ultraviolet cutoff

We introduce a UV cutoff ϵ (with units of length) to regularize the divergences in the entanglement entropy:

$$\begin{aligned} S_{\text{EE}} = k_B \int_\Sigma d^{d-2}\xi \sqrt{h} \Bigg(&\frac{c_{d-2}}{\epsilon^{d-2}} + \frac{c_{d-4}}{\epsilon^{d-4}} + \cdots \\ &+ c_1 R \ln(\mu\epsilon) + \text{finite terms} \Bigg). \end{aligned} \tag{D.1}$$

Here, μ is the renormalization scale (with units of inverse length), ensuring that $\mu\epsilon$ is dimensionless in the logarithmic term.

D.2. Renormalization of gravitational constants

The divergent terms are absorbed into the renormalization of Newton's constant G and the cosmological constant Λ:

$$\delta\left(\frac{1}{G}\right) = \frac{16\pi k_B}{\hbar c^3} c_1 \ln(\mu\epsilon), \tag{D.2}$$

$$\delta\left(\frac{\Lambda}{G}\right) = \frac{8\pi k_B}{\hbar c^4} c_{d-2} \frac{1}{\epsilon^{d-2}}. \tag{D.3}$$

By absorbing these divergences, we obtain finite, renormalized quantities G_{eff} and Λ_{eff}:

$$\frac{1}{G_{\text{eff}}} = \frac{1}{G} + \delta\left(\frac{1}{G}\right), \tag{D.4}$$

$$\frac{\Lambda_{\text{eff}}}{G_{\text{eff}}} = \frac{\Lambda}{G} + \delta\left(\frac{\Lambda}{G}\right). \tag{D.5}$$

Units and Conventions: The terms $\delta\left(\frac{1}{G}\right)$ and $\delta\left(\frac{\Lambda}{G}\right)$ have units consistent with $\frac{1}{G}$ $(\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2})$ and $\frac{\Lambda}{G}$ $(\text{kg}\cdot\text{m}^{-3}\cdot\text{s}^{-2})$, respectively. The inclusion of $\hbar$, c, and k_B ensures dimensional consistency.

Appendix E. Summary

The calculations presented in this appendix provide the technical details underlying the results discussed in the main text. By employing the heat kernel method and carefully handling the contributions from conical singularities, we have derived the corrections to Newton's constant G due to quantum entanglement. The explicit expressions for the coefficients c_1 for various fields allow for precise computations of these corrections within the framework of the Standard Model.

Units and Conventions: Throughout the appendices, all physical quantities and equations include the fundamental constants $\hbar$, c, and k_B explicitly to maintain dimensional consistency and clarity in the presentation of our results.

Data availability

No data was used for the research described in the article.

References

[1] J. Polchinski, String Theory, Volume II: Superstring Theory and Beyond, Cambridge University Press, 1998, http://dx.doi.org/10.1017/CBO9780511618123.
[2] C. Rovelli, Quantum Gravity, Cambridge University Press, 2004, http://dx.doi.org/10.1017/CBO9780511755804.
[3] J. Ambjørn, J. Jurkiewicz, R. Loll, Dynamically triangulating Lorentzian quantum gravity, Nuclear Phys. B 610 (2001) 347–382, http://dx.doi.org/10.1016/S0550-3213(01)00297-8.
[4] A. Eichhorn, A. Held, Black holes in asymptotically safe gravity and beyond, 2022, arXiv preprint arXiv:2212.09495.
[5] M. Nielsen, I. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, 2000.
[6] L. Amico, R. Fazio, A. Osterloh, V. Vedral, Entanglement in many-body systems, Rev. Modern Phys. 80 (2008) 517.
[7] C. Marletto, V. Vedral, Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity, Phys. Rev. Lett. 119 (24) (2017) 240402, http://dx.doi.org/10.1103/PhysRevLett.119.240402, arXiv:1707.06036.
[8] M. Van Raamsdonk, Building up spacetime with quantum entanglement, Gen. Relativity Gravitation 42 (2010) 2323.
[9] B. Swingle, Entanglement renormalization and holography, Phys. Rev. D 86 (2012) 065007.
[10] S. Ryu, T. Takayanagi, Holographic derivation of entanglement entropy from the anti–de sitter space/conformal field theory correspondence, Phys. Rev. Lett. 96 (2006) 181602.
[11] V. Hubeny, M. Rangamani, T. Takayanagi, A covariant holographic entanglement entropy proposal, J. High Energy Phys. 2007 (07) (2007) 062, http://dx.doi.org/10.1088/1126-6708/2007/07/062.
[12] T. Padmanabhan, Emergent spacetime and the origin of gravity, Internat. J. Modern Phys. A 38 (35n36) (2023) 2350188, http://dx.doi.org/10.1142/S0217751X23501889, URL https://arxiv.org/abs/2403.10694, arXiv:2403.10694.
[13] N. Seiberg, The emergence of spacetime from quantum theories, Nat. Rev. Phys. 5 (2023) 612–622, http://dx.doi.org/10.1038/s42254-023-00545-2, URL https://arxiv.org/abs/2402.16136, arXiv:2402.16136.
[14] X. Calmet, F. Kuipers, Quantum gravitational corrections to the entropy of a Schwarzschild black hole, Phys. Rev. D 104 (6) (2021) 066012.
[15] S. Solodukhin, Entanglement entropy of black holes, Living Rev. Relativ. 14 (2011) 8.
[16] S. Hossenfelder, Minimal length scale scenarios for quantum gravity, Living Rev. Relativ. 16 (1) (2013) 2.
[17] J. Bekenstein, Black holes and entropy, Phys. Rev. D 7 (1973) 2333.
[18] S. Hawking, Particle creation by black holes, Comm. Math. Phys. 43 (1975) 199.
[19] A. Guth, Inflationary universe: A possible solution to the horizon and flatness problems, Phys. Rev. D 23 (1981) 347.
[20] A. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, Phys. Lett. B 108 (1982) 389.
[21] M. Faizal, A. Shabir, A.K. Khan, Implications of tarski's undefinability theorem on the theory of everything, EPL (Eur. Letters) TBD (2024) TBD, http://dx.doi.org/10.1209/0295-5075/ad80c2, URL https://arxiv.org/abs/2410.10903, arXiv:2410.10903.
[22] M. Faizal, A. Shabir, A.K. Khan, Consequences of gödel theorems on third quantized theories like string field theory and group field theory, Pending Publ. (2024) URL https://arxiv.org/abs/2407.12313, arXiv:2407.12313.
[23] M. Niedermaier, Asymptotic safety and the running of Newton's constant, Phys. Lett. B 539 (2002) 201–206, http://dx.doi.org/10.1016/S0370-2693(02)02060-3, URL https://arxiv.org/abs/0204111, arXiv:0204111.
[24] S.H. Hendi, B.E. Panah, S. Panahiyan, Varying gravitational constant from entropic gravity and its implications for cosmology, Pending Publ. (2024) URL https://arxiv.org/html/2503.08236v1, arXiv:2503.08236v1.

[25] A. Bonanno, M. Reuter, Running of Newton's constant and quantum gravitational effects, Phys. Rev. D 62 (2000) 043008, http://dx.doi.org/10.1103/PhysRevD.62.043008, URL https://arxiv.org/abs/hep-th/0002196, arXiv:0002196.
[26] C.G. Callan, F. Wilczek, Geometric entropy and quantum field theory, Phys. Lett. B 333 (1–2) (1994) 55–61.
[27] C. Holzhey, F. Larsen, F. Wilczek, Geometric and renormalized entropy in conformal field theory, Nuclear Phys. B 424 (3) (1994) 443–467, http://dx.doi.org/10.1016/0550-3213(94)90402-2, arXiv:hep-th/9403108.
[28] J.-W. Lee, J. Lee, H.-C. Kim, Quantum informational dark energy: dark energy from forgetting, 2007, arXiv preprint arXiv:0709.0047.
[29] J.-W. Lee, H.-C. Kim, J. Lee, Gravity from quantum information, J. Korean Phys. Soc. 63 (2013) 1094–1098.
[30] G. Amelino-Camelia, Phenomenology of Planck-scale modifications of special relativity, New J. Phys. 7 (2005) 39.
[31] D. Reeb, Running of Newton's constant and quantum gravitational effects, in: Predicted and Totally Unexpected in the Energy Frontier Opened By LHC, World Scientific, 2011, pp. 651–660.
[32] X. Calmet, S.D. Hsu, D. Reeb, Grand unification and enhanced quantum gravitational effects, Phys. Rev. Lett. 101 (17) (2008) 171802.
[33] D.F. Litim, Fixed points of quantum gravity, Phys. Rev. Lett. 92 (20) (2004) 201301, http://dx.doi.org/10.1103/PhysRevLett.92.201301.
[34] X. Calmet, S.D. Hsu, D. Reeb, Quantum gravity at a TeV and the renormalization of Newton's constant, Phys. Rev. D— Part. Fields, Gravit. Cosmol. 77 (12) (2008) 125015.
[35] M. Reuter, Nonperturbative evolution equation for quantum gravity, Phys. Rev. D 57 (1998) 971.
[36] D. Vassilevich, Heat kernel expansion: User's manual, Phys. Rep. 388 (2003) 279.
[37] D. Fursaev, S. Solodukhin, Statistical mechanics, euclidean quantum gravity, and thermodynamics of the Schwarzschild black hole, Phys. Rev. D 52 (1995) 2133.
[38] L. Susskind, The world as a hologram, J. Math. Phys. 36 (1995) 6377.
[39] T. Jacobson, Entanglement equilibrium and the Einstein equation, Phys. Rev. Lett. 116 (2016) 201101.
[40] H. Culetu, Strong interactions and Bekenstein–Hawking black hole entropy, in: Presentation at the Seventeenth Marcel Grossmann Meeting, Pescara, Italy, 2024.
[41] M. Reuter, F. Saueressig, Quantum Einstein gravity, New J. Phys. 14 (5) (2012) 055022, http://dx.doi.org/10.1088/1367-2630/14/5/055022, arXiv:1202.2274.
[42] M. Reuter, F. Saueressig, From big bang to asymptotic de sitter: Complete cosmologies in a quantum gravity framework, J. Cosmol. Astropart. Phys. 2005 (09) (2005) 012, http://dx.doi.org/10.1088/1475-7516/2005/09/012, arXiv:hep-th/0507167.
[43] A. Bonanno, M. Reuter, Renormalization group improved black hole spacetimes, Phys. Rev. D 62 (4) (2002) 043008.
[44] O. Lauscher, M. Reuter, Flow equation of quantum Einstein gravity in a higher-derivative truncation, Phys. Rev. D 66 (2) (2002) 025026, http://dx.doi.org/10.1103/PhysRevD.66.025026, arXiv:hep-th/0205062.
[45] E. Adelberger, B. Heckel, A. Nelson, Tests of the gravitational inverse-square law below the dark-energy length scale, Ann. Rev. Nucl. Part. Sci. 53 (2003) 77.
[46] D.J. Kapner, T.S. Cook, E.G. Adelberger, J.H. Gundlach, B.R. Heckel, C.D. Hoyle, H.E. Swanson, Tests of the gravitational inverse-square law below the dark-energy length scale, Phys. Rev. Lett. 98 (2) (2007) 021101, http://dx.doi.org/10.1103/PhysRevLett.98.021101.
[47] Planck Collaboration, Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. 641 (2020) A6.
[48] B. Abbott, et al., Observation of gravitational waves from a binary black hole merger, Phys. Rev. Lett. 116 (2016) 061102.
[49] S. Bose, A. Mazumdar, G.W. Morley, H. Ulbricht, M. Toroš, M. Paternostro, A.A. Geraci, P.F. Barker, M. Kim, G. Milburn, Spin entanglement witness for quantum gravity, Phys. Rev. Lett. 119 (24) (2017) 240401.
[50] A. Riess, et al., Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. 116 (1998) 1009.
[51] S. Perlmutter, et al., Measurements of Ω and Λ from 42 high-redshift supernovae, Astrophys. J. 517 (1999) 565.
[52] S. Di Gennaro, Y. Ong, Sign switching dark energy from a running barrow entropy, Universe 8 (2022) 541, arXiv preprint gr-qc/2205.09311.
[53] H. Lü, S. Di Gennaro, Y.C. Ong, Generalized entropy implies varying-G: Horizon area dependent field equations and black hole-cosmology coupling, Ann. Physics 474 (2025) 169914.
[54] T. Jacobson, Dark energy and the entanglement entropy of quantum fields, Phys. Rev. Lett. 99 (2007) 021301, http://dx.doi.org/10.1103/PhysRevLett.99.021301, URL https://arxiv.org/abs/0709.0047, arXiv:0709.0047.
[55] B.P. Abbott, R. Abbott, T.D. Abbott, M.R. Abernathy, F. Acernese, K. Ackley, C. Adams, et al., LIGO Scientific Collaboration and Virgo Collaboration, Observation of gravitational waves from a binary black hole merger, Phys. Rev. Lett. 116 (6) (2016) 061102, http://dx.doi.org/10.1103/PhysRevLett.116.061102, arXiv:1602.03837.
[56] D. Harlow, Jerusalem lectures on black holes and quantum information, Rev. Modern Phys. 88 (2016) 015002.
[57] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, A. Tajdini, The entropy of hawking radiation, Rev. Modern Phys. 93 (3) (2021) 035002, http://dx.doi.org/10.1103/RevModPhys.93.035002, arXiv:2006.06872.
[58] V. Mukhanov, H. Feldman, R. Brandenberger, Theory of cosmological perturbations, Phys. Rep. 215 (1992) 203.
[59] J. Alvey, N. Sabti, S. Sarkar, L. Stevens, The impact of early neutrino decoupling on the primordial helium abundance, Phys. Rev. D 102 (2020) 043517.
[60] H. Audley, et al., Laser interferometer space antenna, 2017, arXiv preprint arXiv:1702.00786.
[61] K. Akiyama, et al., First M87 event horizon telescope results. I. The shadow of the supermassive black hole, Astrophys. J. Lett. 875 (2019) L1.
[62] D. Psaltis, Probes and tests of strong-field gravity with observations in the electromagnetic spectrum, Living Rev. Relativ. 11 (2008) 9.
[63] C. Brans, R.H. Dicke, Mach's principle and a relativistic theory of gravitation, Phys. Rev. 124 (3) (1961) 925–935, http://dx.doi.org/10.1103/PhysRev.124.925.
[64] K.S. Stelle, Renormalization of higher-derivative quantum gravity, Phys. Rev. D 16 (4) (1977) 953–969, http://dx.doi.org/10.1103/PhysRevD.16.953.
[65] L. Randall, R. Sundrum, An alternative to compactification, Phys. Rev. Lett. 83 (23) (1999) 4690–4693, http://dx.doi.org/10.1103/PhysRevLett.83.4690, arXiv:hep-th/9906064.
[66] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, The hierarchy problem and new dimensions at a millimeter, Phys. Lett. B 429 (3–4) (1998) 263–272, http://dx.doi.org/10.1016/S0370-2693(98)00466-3, arXiv:hep-ph/9803315.
[67] J. Maldacena, The large-n limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998) 231.
[68] B. Czech, J.L. Karczmarek, F. Nogueira, M. Van Raamsdonk, The gravity dual of a density matrix, Classical Quantum Gravity 29 (15) (2012) 155009, http://dx.doi.org/10.1088/0264-9381/29/15/155009, arXiv:1204.1330.
[69] X. Dong, D. Harlow, A.C. Wall, Reconstruction of bulk operators within the entanglement wedge in gauge-gravity duality, Phys. Rev. Lett. 117 (2) (2016) 021601, http://dx.doi.org/10.1103/PhysRevLett.117.021601, arXiv:1601.05416.
[70] B. DeWitt, Dynamical Theory of Groups and Fields, Gordon and Breach, New York, 1965.