Spectral localisers and aperiodic topological phases in noncommutative geometry

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Stellingen
Behorende bij het proefschrift
"Spectral localisers and aperiodic topological phases in noncommutative geometry"

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(1) (**Theorem 3.26**) There is a canonical comparison functor $\natural\colon \mathfrak{KK} \to \mathfrak{E}$ by universal property. Using the picture of unbounded Kasparov modules and asymptotic morphisms, such a canonical functor is described by the asymptotic morphism $(\Phi_t^D)_{t\in[1,\infty)}$, where

$$\Phi_t^D\colon C_0(\mathbb{R}) \otimes A \to \mathbb{K}_B(\mathcal{E}), \quad f \otimes a \mapsto f(t^{-1}D)a.$$

(2) (**Theorem 4.14, Theorem 4.20**) Asymptotic morphisms provide a convenient way to compute index pairings. The resulting K-theory classes are represented by quasi-projections or quasi-idempotents, which can be "spectrally truncated" due to homotopy invariance and additivity of K-theory.

(3) (**Theorem 4.27**) The odd spectral localiser of Loring and Schulz-Baldes naturally arises as a truncated quasi-projection representative of the index pairing, computed using the asymptotic morphism $(\Phi_t^D)_{t\in[1,\infty)}$.

(4) (**Theorem 6.32**) The position spectral triples are a class of spectral triples constructed from the position operators. They detect the "strong" topological phases which survive in the K-theory of the Roe C*-algebra.

(5) There are two well-established C*-algebra models of aperiodic topological insulators: the groupoid C*-algebra $C^*(\mathcal{G}_\Lambda)$ and the Roe C*-algebra $C^*_{\mathrm{Roe}}(\Lambda)$. The regular representation of the groupoid C*-algebra yields a comparison map between them.

(6) In the one-dimensional case, every Delone set $\Lambda \subseteq \mathbb{R}$ containing zero carries a canonical $\mathbb{Z}$-labelling. This turns the groupoid of such a Delone set into an action groupoid by $\mathbb{Z}$. Its groupoid C*-algebra can be identified with a crossed product or a topological graph C*-algebra.

(7) There is no isomorphism between the groupoid of a d-dimensional Delone set and an action groupoid by $\mathbb{Z}^d$, as opposed to the one-dimensional case in Proposition (6). Extra conditions, like a $\mathbb{Z}^d$-labelling and the unique factorisation property, would be needed to establish such an isomorphism.

(8) The strong and weak topological invariants of topological insulators can be studied and distinguished using an inductive limit of equivariant Roe C*-algebras. The weak invariants take values in a divisible subgroup, whereas the strong invariants belong to a discrete subgroup.

(9) An ultimate mathematical theory of the universe is unlikely to exist, since increasing complexity gives rise to genuinely new phenomena, i.e. *more is different*.

(10) In the AI-driven era of mathematics, identifying the correct questions is more important than providing their answers.