Spectral localisers and aperiodic topological phases in noncommutative geometry

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Acknowledgements

My first and foremost gratitude goes to my supervisor, mentor and teacher Bram Mesland. Bram is a role model of mine in vastly many aspects, not only in mathematics, but also in life and in how to be a kind and thoughtful person. Beyond his constant support and heartfelt encouragement throughout these years, I have learned from Bram the two essential virtues of a truly excellent mathematician: to be *honest* about what one knows and what one does not; and to be *brave* in the face of challenges, whether in mathematics or in life.

It has been a great pleasure to spend four years with my colleagues at Leiden University: Francesca Arici, Chun Ding, Jack Ekenstam, Onno van Gaans, Yufan Ge, Dimitris Gerontogiannis, Sander Hiller, Marcel de Jeu, Mirmukhsin Makhmudov, Mark Roelands, Samuel Tiersma and Torstein Ulsnæs. Among them, I owe special thanks to Francesca, Marcel and Yufan. It was Francesca's wonderful online talk that first brought me to Leiden in 2021. Marcel has consistently served as an example of responsibility and precision. Yufan's companionship and care sustained me through both the challenges and joys of this journey.

Göttingen has always held a special place in my heart: it is where my adventure in Europe and in mathematics began. Amongst my teachers and friends there, I dedicate my deepest gratitude and admiration to Ralf Meyer, for introducing me to the world of K-theory and topological phases; and for his sharp intuition across all fields of mathematics. I am also genuinely thankful to Christopher Wulff for his lecture course on E-theory, which has been one of the most essential tools in the first part of the thesis.

I have also greatly benefited from the stimulating conversations with Jean Bellissard, Boris Bilich, Chris Bourne, Eske Ewert, Nigel Higson, Xingni Jiang, Matthias Ludewig, Yunfeng Shi, Guo Chuan Thiang and Hang Wang. I am especially grateful to Guo Chuan for sharing his profound insight into mathematics and physics, and for initiating our joint project. I also owe my special gratitude to Chris for his illuminating comments and suggestions, which have lead to significant improvements in several results presented in the thesis.

I am sincerely thankful to my friends for their warmth and support, especially, Tianbo Chen, Jialong Deng, Dominik Gutwein, Yan Huang, Daniel Christoph

Jentsch, Collin Mark Joseph, Malte Leimbach, Zhiyuan Li, Jingmin Long, Xianyu Hu, Yi Shan, Stijn Velstra, Dan Wu, Qian Xiao and Jialing Yu. Your friendship has truly brightened my life in Europe.

Lastly, and most importantly, I wish to thank my parents for their eternal love: I am proud to be your son. I love you.

Curriculum Vitae

Yuezhao Li was born on 18 April, 1996 in Shenyang, China. From 2013 to 2018, he was an undergraduate student at Peking University, China. In 2018, he obtained a bachelor degree in physics, with a minor degree in mathematics. From 2018 to 2021, he studied at Georg-August-Universität Göttingen, Germany, with a specialisation in analysis, geometry and topology. He graduated from Göttingen in 2021 with a master degree in mathematics and a minor degree (*Nebenfach*) in physics. His master thesis, entitled *Invariants for topological insulators coming from decompositions of coarse spaces*, was supervised by Prof. Dr. Ralf Meyer.

Since October 2021, Yuezhao Li was a PhD student at Leiden University under the supervison of Dr. Bram Mesland, and funded by NWO project 613.009.142 *Noncommutative index theory of discrete dynamical systems*. His research focused on the application of noncommutative geometry (bivariant K-theory, groupoids and coarse geometry) in mathematical physics, and yielded the two preprints submitted for publication.

During his PhD, Yuezhao Li participated several conferences and workshops as a contributed speaker, including *Applications of noncommutative geometry to gauge theories, field theories, and quantum space-time* in Marseille in April 2025, C*-*algebras, coarse geometry and physics* in Greifswald in June 2025. He was invited to visit and give research talks at Sichuan University in August 2024, East China Normal University in September 2024, Universität Greifswald in January 2025, Delft University of Technology in March 2025, Georg-August-Universität Göttingen in July 2025, Leibniz Universität Hannover in July 2025, Max Planck Institute for Mathematics in November 2025, and Radboud University Nijmegen in December 2025.

Yuezhao Li took the course *Scientific Conduct* at Leiden University. He assisted with the undergraduate course *Differentiable Manifolds* at Leiden University, and the Dutch national Mastermath courses *Functional Analysis* and *Operator Algebras*. He was an organiser of the *Local Noncommutative Geometry Seminar* at Leiden University. He was also a co-organiser of the *5th Conference of Settat on Operator Algebras and Applications* in Marrakech in January 2023.

List of publications

(1) Y. Li and B. Mesland, *The odd spectral localiser via asymptotic morphisms and quasi-projections* (2025). arXiv: 2506.17143. To appear in Ann. K-Theory.

(2) Y. Li, *Robustness of topological phases on aperiodic lattices* (2025). arXiv: 2504.04817. Submitted.