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Appendix

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Notation and symbols

Categories and universal functors

$\mathfrak{Ab}$	category of abelian groups, with group homomorphisms as arrows
$\mathfrak{C}^*\mathfrak{Sep}$	category of separable C*-algebras, with *-homomorphisms as arrows
$\mathfrak{NC}^*$	intersection of the bootstrap category with $\mathfrak{C}^*\mathfrak{Sep}$
$\mathfrak{Asym}$	homotopy category of asymptotic morphisms
$\mathfrak{KK}$	Kasparov category
$\mathfrak{E}$	E-theory category
$\natural\colon \mathfrak{KK} \to \mathfrak{E}$	the universal comparison functor $\mathfrak{KK} \to \mathfrak{E}$

C*-algebras and Hilbert C*-modules

B^+	unitisation of B
B_+	positive cone of B
$J \triangleleft B$	J is an ideal of B
$\mathrm{C}_0(X, B)$	C*-algebra of continuous functions $X \to B$ vanishing at infinity
$\mathrm{C}_\mathrm{b}(X, B)$	C*-algebra of bounded functions $X \to B$
$\mathrm{S}B$	$:= \mathrm{C}_0(\mathbb{R}) \otimes B$
$\mathrm{S}^i B$	$:= \mathrm{C}_0(\mathbb{R}^i) \otimes B$
$\mathrm{C}B$	$:= \mathrm{C}_0((-\infty, +\infty]) \otimes B$
$\mathcal{M}(B)$	multiplier algebra of B
$\mathcal{Q}(B)$	$:= \mathcal{M}(B)/B$, Calkin algebra of B
$\mathcal{H}_B$	$:= \ell^2(\mathbb{N}) \otimes B$, the standard countably generated Hilbert B-module
$\mathbb{K}$	$:= \mathbb{K}(\mathcal{H})$, C*-algebra of compact operators on a separable Hilbert space
$\mathbb{K}_B(\mathcal{E})$	C*-algebra of B-linear compact operators on $\mathcal{E}$
$\mathrm{Fin}_B(\mathcal{E})$	algebra of B-linear finite-rank operators on $\mathcal{E}$
$\mathbb{B}_B(\mathcal{E})$	C*-algebra of B-linear bounded adjointable operators on $\mathcal{E}$
$\mathbb{B}_B(\mathcal{E}, \mathcal{F})$	space of B-linear bounded adjointable operators $\mathcal{E} \to \mathcal{F}$
$\mathbb{C}\ell_n$	complex Clifford algebra of n generators
$\mathbb{C}\ell_{p,q}$	"real" Clifford algebra of p self-adjoint and q anti-self-adjoint generators
$\mathrm{C}\ell_{p,q}$	real subalgebra of $\mathbb{C}\ell_{p,q}$

Operators on Hilbert C*-modules

$\operatorname{dom} T$	domain of T
$\operatorname{ran} T$	range of T
$\mathfrak{G}(T)$	$:= \{(x, Tx) \in \mathcal{E} \oplus \mathcal{F} \mid x \in \operatorname{dom} T\}$, graph of T
$\mathbb{1}_{\mathcal{E}}$	the identity operator on a Hilbert module $\mathcal{E}$
$\mathbb{1}_{n\times n}$	the identity $n \times n$-matrix
$\lvert x\rangle\langle y\rvert$	rank-one operator $\lvert x\rangle\langle y\rvert z := x\langle y, z\rangle$
$\mathbf{X}_j$	position operator $\mathbf{X}_j f(x) := x_j f(x)$ for $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$

Kasparov theory and E-theory

$\mathbb{KK}(A, B)$	set of Kasparov A-B–modules
$\mathbb{KK}_1(A, B)$	set of odd (=ungraded) Kasparov A-B–modules
$\Psi(A, B)$	set of unbounded Kasparov A-B–modules
$\Psi_1(A, B)$	set of odd (=ungraded) unbounded Kasparov A-B–modules
$\alpha \times \beta$	Kasparov product of KK-classes α and β
$\alpha \boxtimes \beta$	exterior Kasparov product of KK-classes α and β
$\mathbf{1}_A$	unit of KK-ring $\mathrm{KK}(A, A)$
$\mathfrak{A}B$	$:= \mathrm{C_b}([1,\infty), B)/\mathrm{C_0}([1,\infty), B)$, asymptotic algebra of B
$\Phi\colon A \dashrightarrow B$	asymptotic morphism $(\Phi_t\colon A \to B)_{t\in[1,\infty)}$
$[\![A, B]\!]$	set of homotopy equivalence classes of asymptotic morphisms $A \dashrightarrow B$

Spectral localiser and (semi-finite) index pairing

$[v]$	$\mathrm{K}_1(A)$-class of a unitary $v \in A$
$[D]$	$\mathrm{KK}_1(A, B)$-class of an odd unbounded Kasparov A-B–module $(\mathscr{A}, \mathcal{E}, D)$
$\langle [v], [D]\rangle$	$\mathrm{K}_0(\mathbb{K}_B(\mathcal{E}))$-valued index pairing of $[v] \in \mathrm{K}_1(A)$ and $[D] \in \mathrm{KK}_1(A, B)$
$\widehat{\mathcal{E}}$	$:= \mathcal{E} \oplus \mathcal{E}$
$\mathcal{E}^{\uparrow}_{\lambda}$	upper spectral submodule with threshold λ
$\mathcal{E}^{\downarrow}_{\lambda}$	lower spectral submodule with threshold λ
$L_{\kappa,\lambda}$	spectral localiser with tuning parameter κ and spectral threshold λ
$\widehat{\tau}$	transferred trace $\widehat{\tau}\colon \mathbb{B}_B(\mathcal{E}) \to [0, +\infty]$ of a finite trace $\tau\colon B \to \mathbb{C}$
$L^2(B, \tau)$	GNS-Hilbert space induced by a finite trace $\tau\colon B \to \mathbb{C}$
$\mathcal{H}^{\tau}$	$:= \mathcal{E} \otimes_B L^2(B, \tau)$, localised Hilbert space induced by τ
ϱ^{τ}	$:= \varrho \otimes \mathbb{1}_{L^2(B,\tau)}$, representation on $\mathcal{H}^{\tau}$ induced by τ
D^{τ}	$:= D \otimes \mathbb{1}_{L^2(B,\tau)}$, unbounded self-adjoint operator on $\mathcal{H}^{\tau}$ induced by τ

Groupoid of a Delone set and its C*-algebra

$\mathcal{G} \rightrightarrows \mathcal{G}^0$	groupoid $\mathcal{G}$ with unit space $\mathcal{G}^0$
$\mathcal{G}^2$	$:= \{(\gamma_1, \gamma_2) \in \mathcal{G} \times \mathcal{G} \mid s(\gamma_1) = r(\gamma_2)\}$
$\mathrm{Bis}(\mathcal{G})$	inverse semigroup of local bisections of a groupoid $\mathcal{G}$
$\mathrm{C}^*(\mathcal{G})$	reduced groupoid C*-algebra of a groupoid $\mathcal{G}$
$\mathrm{Ball}(x, R)$	open ball centered at x with radius R
$\mathrm{Del}_{(r,R)}(\mathbb{R}^d)$	space of (r, R)-Delone sets
Ω_Λ	closure of the orbit of $\Lambda \in \mathrm{Del}_{(r,R)}(\mathbb{R}^d)$ under the translation action by $\mathbb{R}^d$
Ω_0	$:= \{\omega \in \Omega_\Lambda \mid 0 \in \omega\}$, canonical transversal of $\Omega_\Lambda \rtimes \mathbb{R}^d$
$\mathcal{G}_\Lambda$	$:= \Omega_\Lambda \rtimes \mathbb{R}^d\big\vert_{\Omega_0}^{\Omega_0}$, groupoid of a Delone set Λ
π_ω	localised regular representation $\mathrm{C}^*(\mathcal{G}_\Lambda) \to \mathbb{B}(\ell^2(\omega))$
$\mathcal{E}_\Lambda$	topological graph associated to a one-dimensional Delone set Λ

Coarse geometry, group actions and symmetry-breaking

$\mathrm{C}^*(G)$	reduced group C*-algebra of group G
$\mathrm{C}^*_\lambda(G)$	reduced group C*-algebra generated by left regular representation
$\mathrm{C}^*_\rho(G)$	reduced group C*-algebra generated by right regular representation
$G \curvearrowright X$	G acts on X
$\mathcal{H}_X$	ample X-module
$\mathrm{C}^*_{\mathrm{Roe}}(X)$	Roe C*-algebra of a space X
$\mathrm{C}^*_{\mathrm{Roe}}(X)^\Gamma$	Γ-equivariant Roe C*-algebra of a Γ-space X
$\mathfrak{S}$	symmetry-breaking chain
$\mathrm{C}^*_{\mathrm{Roe}}(X)^{\mathfrak{S}}$	symmetry-breaking Roe C*-algebra of X for symmetry-breaking chain $\mathfrak{S}$

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