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# Spectral localisers and aperiodic topological phases in noncommutative geometry

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# Spectral localisers and aperiodic topological phases in noncommutative geometry

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# Contents


**1. Introduction** **7**
- 1.1. Spectral truncation of index pairings . . . 8
- 1.2. Topological phases of aperiodic matter . . . 13
- 1.3. Structure of the thesis . . . 19

**I. Spectral truncation of index pairings** **21**

**2. Bivariant K-theories** **23**
- 2.1. Introduction . . . 23
- 2.2. Kasparov theory . . . 28
- 2.3. Connes–Higson E-theory . . . 40
- 2.4. Index pairings . . . 48

**3. Mapping KK-theory to E-theory** **55**
- 3.1. Introduction . . . 55
- 3.2. Unbounded Kasparov modules . . . 57
- 3.3. The Connes–Higson construction . . . 63
- 3.4. Asymptotic morphisms from unbounded Kasparov modules . . . 68
- 3.5. Some commutator estimates . . . 73

**4. Spectral localiser via asymptotic morphisms** **79**
- 4.1. Introduction . . . 79
- 4.2. Spectral localiser . . . 81
- 4.3. Quasi-projections and quasi-idempotents . . . 83
- 4.4. Pairing unitaries with asymptotic morphisms . . . 86
- 4.5. Truncating quasi-projections to spectral submodules . . . 90
- 4.6. Emergence of the spectral localiser . . . 96
- 4.7. Numerical index pairings and their truncations . . . 103

**II. Topological phases on aperiodic lattices** **115**

**5. Modelling topological phases on aperiodic lattices** **117**
5.1. Introduction . . . 117
5.2. The groupoid model . . . 121
5.3. Coarse-geometric model . . . 125

**6. Robustness of topological phases** **131**
6.1. Introduction . . . 131
6.2. Topological phases and K-theory . . . 132
6.3. Position spectral triples and index pairings . . . 141
6.4. Robustness of topological phases . . . 155
6.5. A coarse groupoid perspective . . . 163

**7. Dynamical aspects of one-dimensional Delone sets** **165**
7.1. Introduction . . . 165
7.2. Canonical $\mathbb{Z}$-labelling and unique factorisation . . . 166
7.3. Topological graph description . . . 167
7.4. Semigroup action groupoid description . . . 172

**8. Symmetry-breaking topological phases** **179**
8.1. Introduction . . . 179
8.2. Group actions and equivariant Roe C*-algebras . . . 181
8.3. Symmetry breaking . . . 191
8.4. Symmetry-breaking Roe C*-algebras on $\mathbb{R}^d$ . . . 197

**9. Conclusion and outlook** **209**

**Appendix** **217**
Bibliography . . . 217
Notation and symbols . . . 225
Index . . . 229
Summary . . . 231
Samenvatting . . . 235
Acknowledgements . . . 241
Curriculum Vitae . . . 243
List of publications . . . 245

*Wir müssen wissen, wir werden wissen.*

David Hilbert