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The emperor’s new arc: gigaparsec patterns abound in a Λ CDM universe

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ABSTRACT

Recent discoveries of apparent large-scale features in the structure of the Universe extending over many hundreds of megaparsecs, have been claimed to contradict the large-scale isotropy and homogeneity foundational to the standard (Lambda cold dark matter – Λ CDM) cosmological model. We explicitly test and refute this conjecture using FLAMINGO-10K, a new and very large cosmological simulation of the growth of structure in a Λ CDM context. Applying the same methods used in the observations, we show that patterns like the ‘Giant Arc’, supposedly in tension with the standard model, are, in fact, common and expected in a Λ CDM universe. We also show that their reported significant overdensities are an algorithmic artefact and unlikely to reflect any underlying structure.

Key words: methods: numerical – methods: statistical – large-scale structure of Universe – cosmology: theory.

1 INTRODUCTION

The standard paradigm of hierarchical structure formation predicts that the Universe, while highly inhomogeneous on the scales of galaxies and galaxy clusters, is statistically homogeneous on very large scales (Davis et al. 1985). Structures extending to several hundred comoving megaparsecs (cMpc),¹ have long been observed (Huchra & Geller 1982; Gott et al. 1989). Simulations in the cold dark matter (CDM) context have since shown that similar features are also predicted to exist in the standard model (White et al. 1987; Springel et al. 2005; Park et al. 2012; Schaller 2024) and are, in fact, required to explain the observed large-scale distribution of galaxies (e.g. Sawala et al. 2024). It is worth noting that not all previous claims of large-scale inhomogeneities have persisted: the ‘Giant Ring of Gamma Ray Bursts’ (Balázs et al. 2015) was subsequently shown to be statistically insignificant (Balázs, Rejtő & Tusnády 2018), while the reported gigaparsec-scale agglomerations of quasars (Clowes et al. 2013) were shown by several authors to be consistent with random fluctuations (Nadathur 2013; Park et al. 2015; Fujii 2024) and compatible with (Lambda cold dark matter – Λ CDM) expectations (Marinello et al. 2016).

Recently, the discoveries of two more gigaparsec-scale patterns, the ‘Giant Arc’ (Lopez, Clowes & Williger 2022) and the ‘Big Ring’ (Lopez, Clowes & Williger 2024b), have again been interpreted as invalidating the standard model (e.g. Kumar Aluri et al. 2023; Lopez, Clowes & Williger 2024a) or as signatures of non-standard physics

(e.g. Constantin et al. 2023; Lapi et al. 2023; Mazurenko, Banik & Kroupa 2025). This is partly motivated by the notion of a definite homogeneity scale of ≈ 370 Mpc (Yadav, Bagla & Khandai 2010) beyond which any structures would be in tension with the standard model.

We use a new, extremely large cosmological simulation of structure formation in Λ CDM, FLAMINGO-10K (Schaller et al., in preparation; see also Pizzati et al. 2024), to test directly the claim that the existence of features like the ‘Giant Arc’ contradicts Λ CDM and that its reported overdensity violates the assumption of large-scale homogeneity upon which the standard model is based.

2 THE ‘GIANT ARC’

The ‘Giant Arc’ was serendipitously discovered from a total sample of 63 876 Mg II absorbers, originally constructed from the catalogue of Zhu & Ménard (2013) paired with Sloan Digital Sky Survey (SDSS) DR7 and DR12 quasars, and later identified using a friends-of-friends (FoF) algorithm (Lopez et al. 2022). In their analysis, Lopez et al. (2022) considered a subvolume of $1541 \times 1615 \times 338$ cMpc³ in a redshift interval of $z = 0.742 - 0.862$ containing 504 absorbers. The central redshift, depth of the slice, and linking length of 95 cMpc were all specifically chosen to obtain the clearest identification of the ‘Giant Arc’ which, with these parameters, contains 44 members. Clearly, this strategy of setting parameter values post-hoc risks undermining a meaningful assessment of the statistical significance.

To ascertain the physical nature and significance of their discovery, Lopez et al. (2022) applied several statistical tests. They applied the FoF search to 1000 samples with randomized redshifts but did not find another pattern as significant as the ‘Giant Arc’, inferring a

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¹Here and throughout, distances are expressed in comoving coordinates to allow a fair comparison of structures at different redshifts.

probability of < 0.01 of such a pattern arising at random. Additionally, they computed the convex-hull overdensity of the ‘Giant Arc’, and compared it to overdensities of randomized point samples, attributing a 4.5σ significance to their structure. We will show that the probability of finding similar structures in both Λ CDM and in random patterns is much greater than claimed, while the significance of the associated point overdensity is much lower.

3 THE FLAMINGO-10K SIMULATION

To identify possible counterparts to the ‘Giant Arc’ in Λ CDM, we consider FLAMINGO-10K, a simulation that follows 10080^3 collisionless CDM and 5600^3 neutrino particles (1.2×10^{12} particles in total) in a periodic volume of 2.8^3 cGpc 3 , with cosmological parameters $h = 0.681$, $\Omega_m = 0.306$, $\Omega_\Lambda = 0.694$, $\Omega_b = 0.0486$, $n_s = 0.967$, and $\sigma_8 = 0.807$. FLAMINGO-10K was performed using the SWIFT simulation code (Schaller et al. 2024) and is part of the FLAMINGO suite (see Schaye et al. 2023 for a complete description), using the same phases as their flagship L2p8_m9 hydrodynamical simulation but with $8 \times$ more collisionless particles.

For comparison to the ‘Giant Arc’, we use the output at $z = 0.70$, the closest for which the full particle data are available, but we also examine different outputs between $z = 3$ and $z = 0$. From the outputs, self-bound subhaloes were identified in the simulation using the HBT + (Hierarchical Bound-Tracing) algorithm (Han et al. 2018; Forouhar Moreno et al. 2025). As analogues of the Mg II absorbers, we consider subhaloes in the mass range $M_{200,c} = 1\text{--}5 \times 10^{12} M_\odot$, of which FLAMINGO-10K contains $\approx 2.69 \times 10^7$ at $z = 0.7$. This mass range brackets results of studies based on clustering strength at similar redshifts (e.g. Lundgren et al. 2011; Gauthier et al. 2014). The precise halo masses of Mg II absorbers are not known, but as we will discuss, our results are not sensitive to the assumed mass.

The number density of subhaloes in this mass range in FLAMINGO-10K is $\approx 1.23 \times 10^{-3} \text{ cMpc}^{-3}$, about $2000 \times$ the density of Mg II absorbers in the slice of Lopez et al. (2022), which were selected on the basis of a (random) association with background quasars. To mimic this selection, we draw random subsamples of subhaloes with the same number density. To provide a baseline for the amount of structures found in the simulation, we follow Nadathur (2013) and construct random (Poisson) point patterns of equal density.

4 STRUCTURE FINDING

To identify extended structures in the simulation, we follow Lopez et al. (2022) and apply a FoF algorithm (Huchra & Geller 1982; Davis et al. 1985), i.e. a hierarchical single-link clustering algorithm with a Euclidean distance metric and the linking length as a free parameter. In the more familiar application of finding collapsed objects in cosmological simulations, a canonical linking length of $0.2 \times$ the mean interparticle distance is used. In general, the amount and type of structures found using a FoF algorithm are very sensitive to the relation between the linking length and the interparticle distance: a linking length that is too small results in the breakup of structures, while a linking length that is too large results in points being linked together independently of any underlying correlation.

The redshift slice examined in Lopez et al. (2022) has a volume of $V \approx 0.841 \text{ Gpc}^3$ and a mean point density of $N/V \approx 599 \text{ Gpc}^{-3}$, corresponding to a mean interparticle distance $\langle r \rangle = (V/N)^{1/3} \approx 119 \text{ cMpc}$. The fact that both the mean interparticle distance and the linking length (95 cMpc) are similar to the shortest dimension of the slice (338 cMpc), but much smaller than its other two dimensions

makes the geometry of the slice an important factor. Most points are separated by less than one linking length from either face of the slice, and any structures resembling the ‘Giant Arc’ will be much longer than its shortest dimension. This largely restricts such structures to two dimensions, which also makes them, by definition, highly anisotropic.

For a direct comparison, we must therefore not only adapt the same redshift, point density, and linking length, but also consider equally thin slices of the FLAMINGO-10K volume (or random point patterns). Specifically, we examine slices containing 1588 points within $2800 \times 2800 \times 338 \text{ cMpc}^3$ along each of the three orientations of the FLAMINGO-10K volume. We exclude structures separated by less than one linking length from any side of the box.

To characterize identified structures, we consider the membership number, N , and compute their *extent or length*, L , which we define as the maximum pairwise point distance. We also compute the *point overdensity*, δ_p , for which we calculate the density of the convex hull that encloses spheres of radius equal to the mean distance of the points belonging to the structure, centred on all member points (Clowes et al. 2012). In addition, we use the simulation particle data to compute the *matter overdensity*, δ_m , within the same convex hulls.

We characterize the shape or *anisotropy* of structures according to the ratio between the square roots of the second-largest and largest eigenvalues of their covariance matrix, b/a . Structures that extend along only one dimension, i.e. arcs or filaments, have one dominant eigenvalue, i.e. $c/a < b/a \ll 1$ (due to the particular geometry, for the extended structures we discuss here, $c \ll a$ by construction).

For the ‘Giant Arc’, only the membership number ($N = 44$) and overdensity ($\delta = 0.9$) are reported, the latter computed in the same way as our point overdensity, δ_p . The extent of the ‘Giant Arc’ is only described as ‘approximately 1 Gpc’. Based on Lopez et al. (2022), we are able to measure its projected extent as 770 cMpc. Because its depth is at most equal to that of the slice, 338 cMpc, its true extent, by our definition, is at least 770 cMpc and at most 841 cMpc.

5 STRUCTURES IN FLAMINGO-10K

We find that ‘Giant Arc’-like structures are very common features in FLAMINGO-10K. In Fig. 1, we show the ‘Giant Arc’ alongside eight examples of similarly extended structures found in the first random sampling (seed = 0) of the FLAMINGO-10K simulation. All eight structures are disjoint from one another, i.e. no point belonging to any structure is contained in any other. All eight have extents greater than 750 cMpc, contain more than 20 members, and all have point overdensities, δ_p , above 0.4. While no structure in FLAMINGO-10K is an exact copy of the ‘Giant Arc’ (just as no two structures are exact copies of one another), the ‘Giant Arc’ evidently has many counterparts in a FLAMINGO-10K-like Λ CDM volume.

5.1 Structure persistence

The greater number density of possible Mg II absorbers in FLAMINGO-10K compared to the Lopez et al. (2022) sample allows us to construct independent samples of the full population. This enables us to both extend our statistics and test whether any structure identified in one subsample persists in other subsamples. Structures identified from one sample that persist in other samples of the same volume indicate a real underlying structure, while structures that are only identified in one subsample indicate a spurious pattern.

In Fig. 2, we show the probability density function (PDF, top) and cumulative distribution function (CDF, bottom) of the length of the longest structures satisfying $b/a < 0.5$ found within random

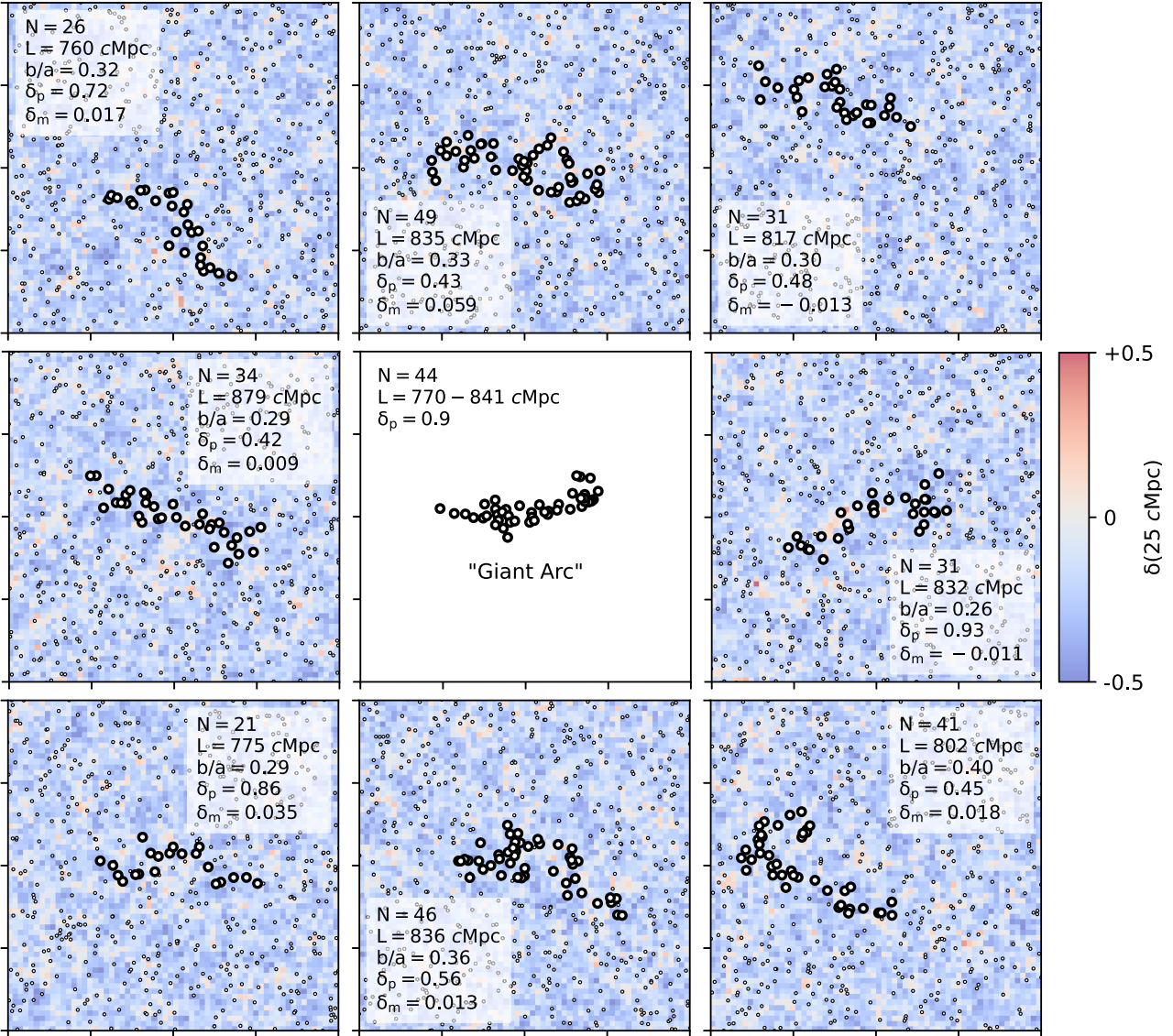


Figure 1. A selection of prominent structures identified in slices of the FLAMINGO-10K simulation in a single sampling at $z = 0.7$, using the same depth, points density, and linking length as in Lopez et al. (2022), alongside the ‘Giant Arc’ reproduced from Lopez et al. (2022). Each panel shows a region of $1600 \times 1600 \text{ cMpc}^2$ with a depth of 338 cMpc. The background of each simulation panel shows the projected matter surface overdensity within the slice on scales of 25 cMpc. Subhaloes are overlaid as white circles, with members belonging to the largest structures highlighted. For each structure, the number of points, N , maximum extent, L , anisotropy, b/a , and the convex-hull overdensity of points, δ_p , and of matter, δ_m , are quoted. Nearly every sampling of FLAMINGO-10K contains many structures that resemble or exceed the ‘Giant Arc’ in size and overdensity.

subsamples in 21 non-overlapping slices of FLAMINGO-10K. We find that the distributions for the length of the longest structures per slice are well approximated by a single log-normal distribution.

Almost every sample of every slice contains at least one structure longer than $\approx 400 \text{ cMpc}$, while the typical (median) length of the longest structure found in one random sample of any slice in FLAMINGO-10K is $\approx 600 \text{ cMpc}$. Approximately 19 percent and 11 percent of samples contain at least one structure longer than the lower and upper limits for the extent of the ‘Giant Arc’, respectively. Even accounting for the $\approx 3\times$ larger volume compared to that examined by Lopez et al. (2022), thin structures as extended as the ‘Giant Arc’ are very common in FLAMINGO-10K.

We also find that, in any given slice, different random subsamples of the underlying subhalo population lead to completely different structures, but the PDFs of maximum lengths across different slices

are very similar. ‘Giant-Arc’ analogues are not only common, but largely spurious, rather than real underlying structures.

5.2 Sensitivity to the linking length

The number of structures identified with a FoF algorithm is expected to depend on the linking length. In Fig. 3, we show the average number of ‘Giant Arc’ analogues per slice, either in samples of subhaloes of different masses in FLAMINGO-10K at $z = 0.7$ (coloured lines), or in random point samples (grey), as a function of linking length. We find that the number of structures is very sensitive to the linking length, and that for all but the highest subhalo masses, the number found in FLAMINGO-10K is very similar to that in random point samples. Notably, the linking length originally chosen to detect the ‘Giant Arc’ by Lopez et al. (2022), $l = 95 \text{ cMpc}$, is close

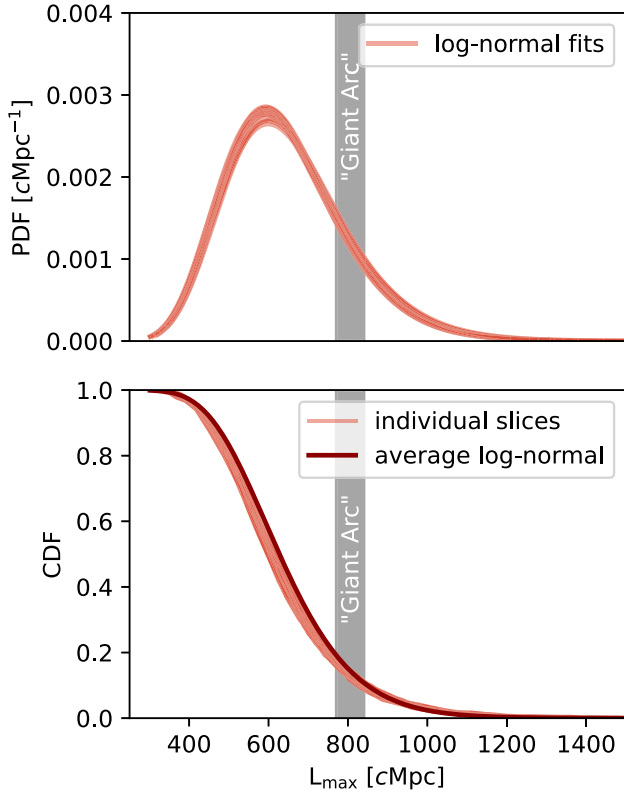


Figure 2. Probability density (PDF, top) and cumulative distribution (CDF, bottom) of the lengths of the longest structures with $b/a < 0.5$ in random subsamples within 21 non-overlapping slices. The top panel shows log-normal fits to the individual PDFs for the 21 individual slices. The bottom panel shows the raw CDFs for each slice, and a log-normal fit to the average PDF. The grey band indicates the lower and upper limit of the extent of the ‘Giant Arc’ ($770 \text{ cMpc} < L_{\text{max}} < 841 \text{ cMpc}$).

to the one that maximizes the number of structures identified in both the simulation and in random point patterns.

5.3 Overdensities

As shown in Fig. 1, structures resembling the ‘Giant Arc’ in FLAMINGO-10K often have point overdensities, δ_p , of order unity, and comparable to the stated overdensity of the ‘Giant Arc’ ($\delta_p = 0.9$). Physical overdensities of order unity on these scales would certainly be inconsistent with the Λ CDM model. However, for the same structures, we find matter overdensities, δ_m , of at most a few per cent, and in some cases, even slight underdensities.

In the top panel of Fig. 4, we show histograms of the point overdensities, δ_p , of extended structures in FLAMINGO-10K and in random samples. Structures in FLAMINGO-10 are very slightly denser than those in random point patterns. Both sets are clearly overdense, and the point overdensity of the ‘Giant Arc’ is not exceptional, ranking in the 87th and 89th percentiles among structures in FLAMINGO-10K and random patterns, respectively. This contradicts the statement by Lopez et al. (2022), who found that its convex hull overdensity represents a ‘ 4.5σ outlier’. The difference most likely arises from the fact that, while Lopez et al. (2022) considered the density of convex hulls enclosing N points randomly distributed within the slice, we consider the volumes that enclose N points already identified as belonging to FoF structures of extent similar to the ‘Giant Arc’.

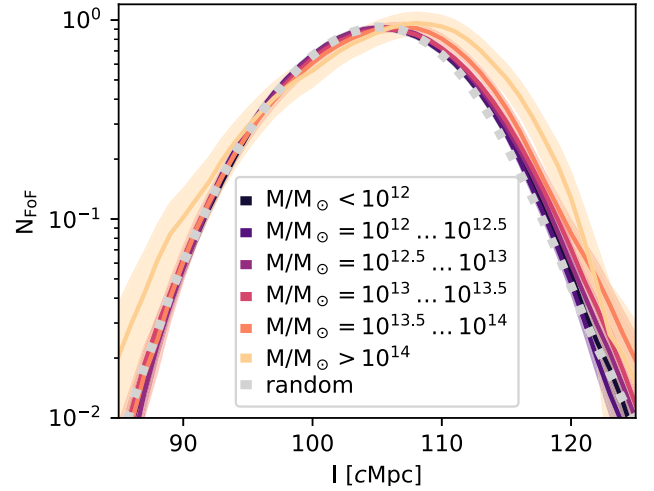


Figure 3. Average number of FoF groups per slice more extended than $L = 700 \text{ cMpc}$ and more anisotropic than $b/a = 0.5$, as a function of linking length, l . Different colours or shades indicate samples of different subhalo mass ranges in the FLAMINGO-10K simulation at $z = 0.7$, as indicated in the legend; the dotted grey line shows the result for random point samples. The number of groups found is a strong function of the linking length and the value of 95 cMpc that maximizes the appearance of the ‘Giant Arc’ is close to the value that maximizes the number of groups found both in the simulation and in random point patterns. The number of groups in FLAMINGO-10K is very similar to that in random point patterns for all mass ranges, except for a slight excess when considering only the most massive haloes.

Groups selected like the ‘Giant Arc’, with a linking length below the mean interparticle distance, are overdense almost by definition.

In the bottom panel of Fig. 4, we show the distribution of matter overdensities, δ_m , for the same structures in FLAMINGO-10K. Despite point overdensities of order unity, the average matter overdensity in these structures is only slightly above zero. Our results strongly suggest that this also applies to the region of the Universe that contains the ‘Giant Arc’.

5.4 Scale and time dependent number of structures

In Fig. 5, we show the number of extended structures of different scales and at different redshifts in slices of the FLAMINGO-10K simulation, normalized by the corresponding numbers in random point patterns. For consistency, for the largest slices, we choose the default parameters for the volume, $2800 \times 2800 \times 338 \text{ cMpc}^3$, number of points, $N = 1588$, and linking length, $l = 95 \text{ cMpc}$, but note that these provide a rather arbitrary anchor point.

To identify structures at different scales, we set a minimum extent of $8 \times l$, corresponding to 760 cMpc for the largest slice (≈ 20 per cent of slices contain such structures). For all smaller scales, we change all three dimensions of the slice and the linking length proportionally, maintaining a constant geometry, constant number of points, and constant ratio of the linking length to the mean interparticle separation. In this set-up, the average number of structures found in random point patterns is constant and serves as a normalization.

In the paradigm of hierarchical structure formation, the number of structures is both time- and scale-dependent. As expected, we find relatively more structures at later times (lower z) and on smaller scales in FLAMINGO-10K. However, even where the number of structures found in the simulation is the same or below that of random

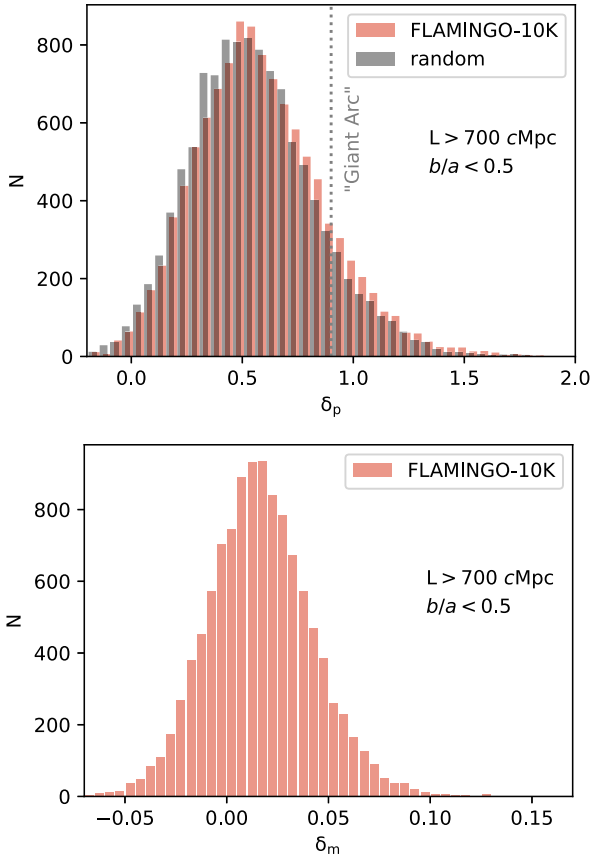


Figure 4. Distribution of overdensities associated with giant structures ($L > 700\text{cMpc}$). The top panel shows the point overdensities of groups in samples of FLAMINGO-10K at $z = 0.7$ and in random point patterns. Structures in FLAMINGO-10K are slightly more overdense than those in random patterns, but the reported point overdensity of the ‘Giant Arc’, $\delta_p = 0.9$ is entirely consistent with both FLAMINGO-10K (87th percentile) and random patterns (89th percentile). The bottom panel shows the matter overdensities, δ_m , of the same groups in FLAMINGO-10K. The matter overdensities are much lower, with a median of $\delta_m = 0.02$.

samples, it never decreases to zero: the ΛCDM model predicts structures at all times and on all scales.

6 SUMMARY

Observed patterns like the ‘Giant Arc’ do not contradict the ΛCDM paradigm. Applying the same detection algorithm with the same parameters assumed by Lopez et al. (2022) for their sample of Mg II absorbers to corresponding samples of subhaloes at the same redshift, the FLAMINGO-10K simulation contains many structures that are as extended, thin, and appear as overdense as the ‘Giant Arc’. Its reported overdensity holds no additional significance: FoF-groups resembling the ‘Giant Arc’ naturally possess a large points overdensity, but this does not translate into an underlying matter overdensity. Considering that the sample of Mg II absorbers only represents one of many possible, extremely sparse samples of the underlying galaxy population, and that similar structures are found in random point samples, there is no reason to believe that the ‘Giant Arc’ traces any underlying structure in the Universe.

We hope that our results will dispel the misconception that no inhomogeneity can be found in a standard model universe beyond some finite size. Instead, any given realization of an isotropic universe

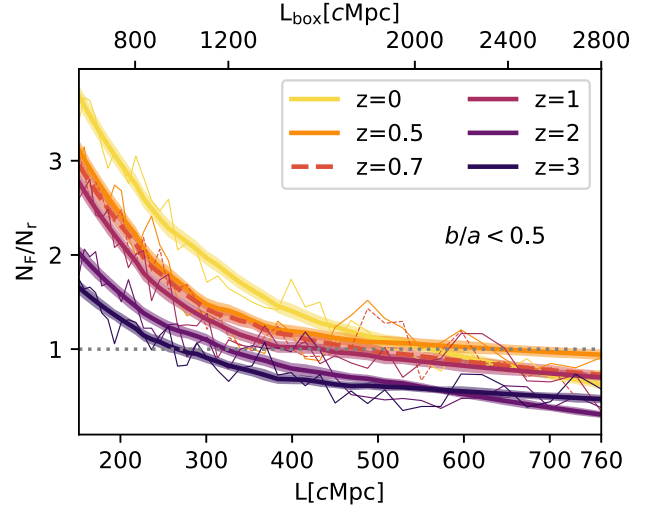


Figure 5. Number of groups more extended than $8\times$ the linking length and more anisotropic than $b/a = 0.5$ in slices of the FLAMINGO-10K simulation at different redshifts, normalized by the corresponding number in random point patterns. Thin lines show raw measurements, thick lines are interpolated. As the length of the slice (L_{box} , top x -axis) is varied, the linking length, l , and minimum structure size, (L , bottom x -axis), are scaled proportionally. As expected in the ΛCDM paradigm, structures grow hierarchically over time. At $z = 3$, FLAMINGO-10K shows an excess of structures up to $L \approx 300\text{cMpc}$, while at $z = 0$, the excess extends up to $L \approx 500\text{cMpc}$.

comprises a time- and scale-dependent population of structures from which patterns can be identified on any scale.

Structures in FLAMINGO-10K that resemble the ‘Giant Arc’ are not collapsed, gravitationally bound, or even particularly overdense. Our results strongly suggest that the same applies to the ‘Giant Arc’ and other similarly large agglomerations. Without these physical characteristics commonly associated with cosmic structures, it may be more appropriate to term them ‘patterns’ rather than structures.

Quantifying the frequency of direct counterparts to the ‘Giant Arc’ has been complicated by the fact that both the precise volume and the parameters of the identification algorithm were defined post-hoc, with the explicit goal of obtaining the clearest detection of a previously identified structure. For consistency, we adopt the same parameters, but this strategy clearly entails a significant look-elsewhere effect. However, the fact that we find so many counterparts in FLAMINGO-10K even with these particular parameters clearly demonstrates that its discovery is fully consistent with the standard model.

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Time allocation in 2023/24, under the auspices of the UKRI-funded DiRAC Federation Project. We used open source software, including MATPLOTLIB (Hunter 2007), SCIPY (Virtanen et al. 2020), SCIKIT-LEARN (Pedregosa et al. 2011), and NUMPY (Harris et al. 2020).

DATA AVAILABILITY STATEMENT

The script used to produce all figures and numbers presented in this work is available at <https://github.com/TillSawala/GiantArc>. Access to the underlying simulation data will be provided on reasonable request to the authors.

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