Quantum computing, norms and polynomials

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Stellingen

Behorende bij het proefschrift *Quantum computing, norms and polynomials.*

1. The acceptance probability $\mathcal{A} : \{-1,1\}^n \to [0,1]$ of a quantum algorithm that makes t queries to a string $x \in \{-1,1\}^n$ equals the restriction to $\{-1,1\}^n$ of a polynomial $p \in \mathbb{R}[x_1, \ldots, x_n]$ of degree at most $2t$ (*Chapter 3 of this thesis*).

2. Littlewood's inequality ensures that the 2-norm of the small coefficients of bounded polynomials with degree 2 is negligible. More precisely, for a real polynomial $p(x) = \sum_{i,j\in[n]} c_{ij} x_i x_j$ that satisfies
$$\sup_{|x_1|,\ldots,|x_n|\leq 1} |p(x)| \leq 1,$$
we have that
$$\sum_{(i,j):\, |c_{ij}|<\varepsilon'} |c_{ij}|^2 \leq \varepsilon,$$
where $\varepsilon' = \Omega(\varepsilon^3)$ (*Chapter 6 of this thesis*).

3. Quantum Hamiltonians are a generalization of polynomials defined in the Boolean cube. Given a function $p : \{-1,1\}^n \to \mathbb{R}$, it can be written as a polynomial via the Fourier expansion,
$$p(x) = \sum_{s\in\{0,1\}^n} \widehat{p}(s) \prod_{i\in[n]} x_i^{s_i}.$$
The degree of p is the minimum d such that $\widehat{p}(s) = 0$ if $|s| > d$. Given such a p, we define the matrix $H_p = \mathrm{Diag}(p(x))_{x\in\{-1,1\}^n}$. H_p is a Hamiltonian whose Pauli expansion is
$$H_p = \sum_{s\in\{0,1\}^n} \widehat{p}(s) \otimes_{i\in[n]} Z^{s_i}.$$
In particular, H_p is d-local (*Chapter 7 of this thesis*).

4. The interaction between quantum mechanics and functional analysis is bidirectional (*Chapters 3 to 7 of this thesis*).

5. Quantum computers with 4096 logical qubits would endanger current cryptography, as a perfect implementation of Shor's algorithm would make RSA-based protocols no longer reliable.

6. No one in the field wants to consider it to be true, but it is by no means certain that fault-tolerant quantum computers will ever exist.

7. Fundamental research in computer science has solved central open questions in mathematics and physics, as witnessed by the MIP*=RE result, which implied the resolution of the Connes' embedding problem and the Tsirelson problem.

8. Research in quantum computing is valuable, even if no practical application ever occurs.

9. The current academic system, driven by (an apparent) productivity and sustained by temporary workers, is inefficient.

10. The academic publication system is a scam, where public employees do most of the work without any compensation, and private companies gather all the profit, which mainly comes from the payments of public institutions.

11. Science is political. Deciding which topics are worth doing research on is a political decision, and thus it concerns democracy.

By Francisco Escudero Gutiérrez – Leiden, 10th of February of 2026.