Universiteit
Leiden
The Netherlands

Quantum computing, norms and polynomials

Escudero Gutiérrez, F.

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Part III

Bonus

Chapter 8

Cute remarks

In this chapter, we gather new proofs of known results that we find elegant and concise. All of them relate to functions defined on the Boolean cube.[1]

8.1 Generalizing a work of Kalai and Schulman

Kalai and Schulman studied the influences of multilinear polynomials with $\{-1, 0, 1\}$-valued coefficients [KS19] (we refer to their work for motivation). They showed an upper bound of $\sum_{i\in[n]} \sqrt{\mathrm{Inf}_i[p]}$ in terms of $\|p\|_\infty$. They proved that

$$\sum_{i\in[n]} \sqrt{\mathrm{Inf}_i[p]} \leq 3^d d^{5/2} \|p\|_\infty$$

for unimodular polynomials. We can improve this bound, generalize it to arbitrary polynomials (not necessarily unimodular), and show that the exponential dependence on d is necessary. Our proof is simple, short and based on hypercontractivity [Bon70] and a bound on the sum of L_1 influences [BB14, FHKL16].

Before diving into the proof of the main result of this section, Proposition 8.3, we need to the define the L_q influences and state two results that we use as lemmas. The L_q influence is defined by

$$\mathrm{Inf}_i^q[p] = \mathbb{E}_x \left[\left| \frac{p(x^{i\to 1}) - p(x^{i\to -1})}{2} \right|^q \right],$$

[1] The results of Section 8.1 were derived in a conversation with Miquel Saucedo during a research stay in Hausdorff Institute for Mathematics, in Bonn, Germany.

where the expectation is taken with respect to the uniform distribution on $\{-1,1\}^n$. Note that $\mathrm{Inf}^2[p]$ equals the $\mathrm{Inf}[p]$ defined in Section 2.5. In the Boolean case, $\mathrm{Inf}_i^2[p] = \mathrm{Inf}_i^q[p]$ for every $q \in [1,\infty)$.

Theorem 8.1 (Hypercontractivity)**.** *Let $p : \{-1,1\}^n \to \mathbb{R}$ be a polynomial of degree at most d. Then,*

$$\sqrt{\mathbb{E}_x\,|p(x)|^2} \le e^d \mathbb{E}_x |p(x)|.$$

Theorem 8.2 (Bound on sum of L_1 influences)**.** *Let $p : \{-1,1\}^n \to \mathbb{R}$ be a polynomial of degree at most d. Then,*

$$\sum_{i\in[n]} \mathrm{Inf}_i^1[p] \le d^2 \|p\|_\infty.$$

Proposition 8.3. *Let $p : \{-1,1\}^n \to \mathbb{R}$ be a polynomial of degree d. Then,*

$$\sum_{i\in[n]} \sqrt{\mathrm{Inf}_i^2[p]} \le e^d d^2 \|p\|_\infty.$$

In addition, there is a unimodular degree d polynomial p such that

$$\sum_{i\in[n]} \sqrt{\mathrm{Inf}_i^2[p]} \ge \sqrt{2}^{d-2} \|p\|_\infty.$$

Proof. By Theorem 8.1 it follows that for every $i \in [n]$

$$\sqrt{\mathrm{Inf}_i^2[p]} \le e^d \mathrm{Inf}_i^1[p].$$

Now, taking the sum over $i \in [n]$ and applying Theorem 8.2 we arrive at the claimed result.

Let $n = 2^{d-1}$. The (unnormalized) address function of $p : (\{-1,1\})^n)^d \to \mathbb{R}$ of degree d is defined as

$$p(x) = \sum_{a\in\{-1,1\}^{d-1}} \underbrace{(x_1(1) - a_1 x_1(2)) \dots (x_{d-1}(1) - a_{d-1} x_{d-1}(2))}_{g_a(x_1,\dots,x_{d-1})} x_d(a), \tag{8.1}$$

where we identify $\{-1,1\}^{d-1}$ with $[2^{d-1}]$. It is satisfied that $\|p\|_\infty = 2^{d-1}$, because given $(x_1,\dots,x_{d-1}) \in (\{-1,1\}^n)^{d-1}$ there is only one $a \in \{-1,1\}^{d-1}$ such that $g_a(x_1,\dots,x_{d-1})$ is not 0, in which case it takes the value $\pm 2^{d-1}$. For every of the 2^{d-1} variables $x_d(a)$ we have that $\mathrm{Inf}_{d,a}^2[p] = 2^{d-2}$, so $\sum \sqrt{\mathrm{Inf}_i^2[p]} \ge (2^{d-2})^{3/2}$. □

8.2 The adversary method via Grothendieck's inequality

In the first part of this thesis, we have focused on the polynomial method. Here, we will revisit the other main method to prove quantum query lower bounds: the adversary method [Amb00, HLv07] (see [LS21] for a survey). To define the adversary bound we must introduce some notation. Let $f : \{-1,1\}^n \to \{-1,1\}$. A matrix $\Gamma \in M_{2^n}(\mathbb{C})$ is an *adversary matrix for* f if it is Hermitian and for every $x, y \in \{-1,1\}^n$ such that $f(x) = f(y)$ we have that

$$\langle x|\Gamma|y\rangle = 0,$$

where $\{|x\rangle\}$ is an orthonormal basis of $\mathbb{C}^{2^n}$. Given $i \in [n]$, $D_i \in M_{2^n}$ is the matrix defined by

$$\langle x|D_i|y\rangle = \begin{cases} 0 & \text{if } x_i = y_i, \\ 1 & \text{if } x_i \neq y_i. \end{cases}$$

The *adversary bound of* f is defined by

$$\mathrm{Adv}(f) := \sup_{\Gamma} \frac{\|\Gamma\|_{\mathrm{op}}}{\max_{i\in[n]} \|\Gamma \circ D_i\|_{\mathrm{op}}}, \tag{8.2}$$

where the supremum runs over all adversary matrices Γ and $\circ$ denotes the entry-wise matrix product, namely $(A \circ B)_{ij} = A_{ij}B_{ij}$. In this section, we will give a, to the best of our knowledge, novel proof of the following result via Grothendieck's inequality (see Section 2.7.1).

Proposition 8.4. *Let* $f : \{-1,1\}^n \to \{-1,1\}$. *Then,* $Q(f) = \Omega(\mathrm{Adv}(f))$.

Before diving into the proof, we give an intuition of why such a result holds. Consider an algorithm whose bias approximates f with high probability. Before making any query, the algorithm prepares a state that does not depend on the input x. In terms of adversary matrices Γ, this will mean that some closeness measure, to be defined below, will have value $\|\Gamma\|_{\mathrm{op}}$ before making any query. Also, at the end of the algorithm, the state prepared on a pair of inputs x and y such that $f(x) \neq f(y)$ must be *far away*, so the algorithm can distinguish them with a measurement. In terms of Γ, this will be formalized via Grothendieck's inequality and will mean that the closeness measure will have value $\leq K_G/5 \cdot \|\Gamma\|_{\mathrm{op}}$ at the end of the algorithm. Finally, it will follow from a simple argument that the algorithm can only separate the states prepared when querying x and y a bounded amount per query. In terms of

Γ, this will mean that the closeness measure can only decrease $2\max_i \|\Gamma \circ D_i\|_{\mathrm{op}}$ per query. Putting everything together we have that the algorithm must make at least

$$\frac{\|\Gamma\|_{\mathrm{op}} - \frac{K_G}{5}\|\Gamma\|_{\mathrm{op}}}{2\max_{i\in[n]} \|\Gamma \circ D_i\|_{\mathrm{op}}}$$

queries.

Proof. We will show that $Q_{2/100}(f) = \Omega(\mathrm{Adv}(f))$. Let $\mathcal{A}$ be an algorithm that makes t queries and whose bias $2/100$ approximates $f(x)$. This means, that $\mathcal{A}$ fails with probability $\|\Pi_{-f(x)}|\psi_x^t\rangle\|_2^2 \leq 1/100$. Let Γ be an adversary matrix for f. Let $|\delta\rangle$ be such that $|\langle\delta|\Gamma|\delta\rangle| = \|\Gamma\|_{\mathrm{op}}$. We define the closeness measure at step $s \in \{0,\ldots,t\}$ as

$$\mathcal{C}^s := |\sum_{x,y} \Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^s|\psi_y^s\rangle|,$$

where $|\psi_x^s\rangle$ is the state prepared by the algorithm on input x just after the sth query. We divide the rest of the proof in three steps. First, we note that

$$\mathcal{C}^0 = |\sum_{x,y} \Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^0|\psi_y^0\rangle| = |\sum_{x,y} \Gamma_{xy}\delta_x^*\delta_y| = |\langle\delta|\Gamma|\delta\rangle| = \|\Gamma\|_{\mathrm{op}},$$

where we have used that $|\psi_x^0\rangle$ does not depend on x because no queries have been made.

Second, we claim that

$$\mathcal{C}^t \leq \frac{K_G}{5}\|\Gamma\|_{\mathrm{op}}$$

where K_G is the (complex) Grothendieck's constant, which is strictly smaller than 5. Indeed, let Π_{-1}, Π_1 be the measurement performed by the algorithm, then

$$\begin{aligned}
\mathcal{C}^t &= |\sum_{x,y} \Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^t|\psi_y^t\rangle| = |\sum_{x,y:f(x)\neq f(y)} \Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^t|(\Pi_{-1}+\Pi_1)|\psi_y^t\rangle| \\
&\leq |\sum_{x,y:f(x)\neq f(y)} \Gamma_{xy}\delta_x^*\delta_y(\langle\psi_x^t|\Pi_{-f(x)})|\psi_y^t\rangle| + |\sum_{x,y:f(x)\neq f(y)} \Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^t|(\Pi_{-f(y)}|\psi_y^t\rangle)| \\
&\leq \frac{2}{10}\sup_{\|u_x\|_2,\|v_y\|_2\leq 1} |\sum_{x,y} \Gamma_{xy}\delta_x^*\delta_y\langle u_x, v_y\rangle| \\
&\leq \frac{2K_G}{10}\sup_{\alpha_x,\beta_y\in\{-1,1\}} |\sum_{x,y} \Gamma_{xy}\delta_x^*\delta_y\alpha_x\beta_y| \\
&= \frac{K_G}{5}\|\Gamma\|_{\mathrm{op}},
\end{aligned}$$

where in the first line we have used that Γ is an adversary matrix; in the third line that $\|\Pi_{-f(x)}|\psi_x^t\rangle\|_2^2$ is the failure probability, so it is smaller than $1/100$; and in the fourth line we have used Grothendieck's inequality, Theorem 2.19.

Finally, we claim that

$$\mathcal{C}^s - \mathcal{C}^{s-1} \leq 2 \max \|\Gamma \circ D_i\|_{\text{op}}$$

for $s \in [t]$. Let U^s be the unitary in between the $(s-1)$th and the sth queries. Recall that O_x acts as the controlled version of $|i\rangle \to x_i|i\rangle$, so $O_x|0\rangle|i\rangle = |0\rangle|i\rangle$ and $O_x|1\rangle|i\rangle = x_i|1\rangle|i\rangle$. Let d be the extra dimensions of the algorithm as in Eq. (2.3). Then,

$$\begin{aligned}
\mathcal{C}^s - \mathcal{C}^{s-1} &\leq |\sum_{x,y} \Gamma_{xy}\delta_x^*\delta_y(\langle\psi_x^s|\psi_y^s\rangle - \langle\psi_x^{s-1}|\psi_y^{s-1}\rangle)| \\
&= |\sum_{x,y} \Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^{s-1}|(O_x \otimes \mathrm{Id}_d)\underbrace{U_s^\dagger U_s}_{\mathrm{Id}_{2nd}}(O_y \otimes \mathrm{Id}_d) - (\mathrm{Id}_{2nd})|\psi_y^{s-1}\rangle| \\
&= |\sum_{ij}\sum_{x,y} \Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^{s-1}|(|1i\rangle\langle 1i| \otimes \mathrm{Id}_d)\underbrace{(x_i y_j - 1)}_{-2(D_i)_{xy}}(|1j\rangle\langle 1j| \otimes \mathrm{Id}_d)|\psi_y^{s-1}\rangle| \\
&= 2|\sum_i \sum_{x,y} (\Gamma \circ D_i)_{xy}\delta_x^*\langle\psi_x^{s-1}|(|1i\rangle \otimes \mathrm{Id}_d)(\langle 1i| \otimes \mathrm{Id}_d)|\psi_y^{s-1}\rangle\delta_y|.
\end{aligned}$$

Now, if we define $\widetilde{\Gamma}$ as the block diagonal matrix with $\Gamma \circ D_i$ as diagonal blocks for $i \in [n]$, and G as the block diagonal matrix whose blocks are the Gram matrices of $\{\delta_x(\langle 1i| \otimes \mathrm{Id}_d)|\psi_x^{s-1}\rangle\}_x$, we have that

$$\begin{aligned}
\mathcal{C}^s - \mathcal{C}^{s-1} &= 2|\langle\widetilde{\Gamma}, G\rangle| \leq 2\|\widetilde{\Gamma}\|_{\text{op}}\|G\|_{\text{tr}} \\
&= 2\max_{i\in[n]}\|\Gamma \circ D_i\|_{\text{op}} \sum_i \mathrm{tr}[\mathrm{Gram}(\{\delta_x(\langle 1i| \otimes \mathrm{Id}_d)|\psi_x^{s-1}\rangle\}_x)] \\
&= 2\max_{i\in[n]}\|\Gamma \circ D_i\|_{\text{op}} \underbrace{\sum_x |\delta_x|^2}_{=\langle\delta,\delta\rangle=1} \underbrace{\sum_i \langle\psi_x^{s-1}|(|1i\rangle\langle 1i| \otimes \mathrm{Id}_d)|\psi_x^{s-1}\rangle}_{\leq\langle\psi_x^{s-1}|\psi_x^{s-1}\rangle=1} \\
&= 2\max_{i\in[n]}\|\Gamma \circ D_i\|_{\text{op}}.
\end{aligned}$$

Putting everything together, it follows that

$$t \geq \frac{\mathcal{C}^t - \mathcal{C}^0}{2\max_{i\in[n]}\|\Gamma \circ D_i\|_{\text{op}}} = \Omega\left(\frac{\|\Gamma\|_{\text{op}}}{\max_{i\in[n]}\|\Gamma \circ D_i\|_{\text{op}}}\right). \qquad \square$$

8.3 Average sensitivity lower bounds all reasonable complexity measures

We will show that the average sensitivity $\overline{s}(f)$ of a Boolean function lower bounds all the *reasonable* complexity measures of a Boolean function, which is the list of well-studied complexity measures considered in [ABDK+21]. For total Boolean functions, all of these measures are polynomially related to classical and quantum query complexity. In particular, we will show that the average sensitivity lower bounds the spectral sensitivity of a Boolean function $\lambda(f)$. This is enough, as $\lambda(f)$ lower bounds, up to constant factors, all the reasonable complexity measures. From there, we can easily show that all reasonable complexity measures are $\Omega(n)$ for almost all Boolean functions, concisely reproving previous results such as $Q(f) = \Omega(n)$ for almost all total Boolean functions [Amb99, ABSdW13]. More formally, given $f : \{-1,1\}^n \to \{-1,1\}$ its average sensitivity is defined by

$$\overline{s}(f) := \mathbb{E}_x \sum_{i\in[n]} \left[\left(\frac{f(x) - f(x^{\oplus i})}{2} \right)^2 \right],$$

which also equals the sum of the influences, $\sum_{i\in[n]} \mathrm{Inf}_i^2[f]$. Its spectral sensitivity is given by

$$\lambda(f) := \sup_{\Gamma} \frac{\|\Gamma\|}{\max_{i\in[n]} \|\Gamma \circ D_i\|},$$

where the supremum runs over all adversary matrices that satisfy $\Gamma[x,y] = 0$ if the Hamming distance between x and y is not 1 (see Section 8.2 for the definitions of adversary matrix and D_i).

Proposition 8.5. *Let $f : \{-1,1\}^n \to \{-1,1\}$. Then, $\overline{s}(f) \leq \lambda(f)$. Furthermore, the inequality is tight for $f = \chi_{[n]}$.*

Proof. Let Γ be the adversary matrix such that $\Gamma_{x,y} = 1$ if the Hamming distance between x and y is exactly one and $f(x) \neq f(y)$ and 0 in the other case. Note that Γ can be written as

$$\Gamma_{x,y} = \delta_{x^{\oplus i},y}\delta_{f(x),f(x^{\oplus i})} = \delta_{x^{\oplus i},y} \left(\frac{f(x) - f(x^{\oplus i})}{2} \right)^2.$$

For this matrix, we can see that $\|\Gamma\|_{\mathrm{op}} \geq \overline{s}(f)$ and $\|\Gamma \circ D_i\|_{\mathrm{op}} = 1$ for all $i \in [n]$.

Indeed,

$$\begin{aligned}\|\Gamma\|_{\mathrm{op}} &\geq \sum_{x,y\in\{-1,1\}^n} \frac{1}{2^n}\Gamma_{x,y} = \sum_{x,y\in\{-1,1\}^n} \frac{1}{2^n}\delta_{x^{\oplus i},y}\left(\frac{f(x)-f(x^{\oplus i})}{2}\right)^2 \\ &= \sum_{i\in[n]} \mathbb{E}_x\left(\frac{f(x)-f(x^{\oplus i})}{2}\right)^2 = \overline{s}(f).\end{aligned}$$

On the other hand,

$$\begin{aligned}\|\Gamma\circ D_i\|_{\mathrm{op}} &= \sup_{\|u\|_2=1} \sum_{x\in\{-1,1\}^n} u_x u_{x^{\oplus i}}\delta_{f(x),f(x^{\oplus i})} \leq \sup_{\|u\|_2=1} \sum_{x\in\{-1,1\}^n} |u_x u_{x^{\oplus i}}| \\ &\leq \sup_{\|u\|_2=1} \|u\|_2^2 = 1.\end{aligned}$$

Finally, for $f = \chi_{[n]}$ we have that $\lambda(f) = \overline{s}(f) = n$. □

Corollary 8.6. *Let CM be any reasonable complexity measure. For a $1-\exp(-\exp(n))$ fraction of all Boolean functions $f : \{-1,1\}^n \to \{-1,1\}$ we have that $CM(f) = \Omega(n)$.*

Proof. If we pick a uniformly random Boolean function $f : \{-1,1\}^n \to \{-1,1\}$, then

$$\mathbb{E}_f \overline{s}(f) = \mathbb{E}_x \sum_i \mathbb{E}_f \frac{1-f(x)f(x^{\oplus i})}{2} = \mathbb{E}_x \sum_i \frac{1}{2} = \frac{n}{2}.$$

Now, note that changing the value of f on one input makes $\overline{s}(f)$ change at most $2n/2^n$. Then, by McDiarmid's inequality, Lemma 2.23, we have that

$$\Pr\left[\overline{s}(f) \leq \frac{n}{3}\right] \leq \exp(-\exp(n)).$$

Now, the statement follows from Proposition 8.5 and the fact that $\lambda(f)$ lower bounds, up to constant factors, all reasonable complexity measures [ABDK+21]. □

Bibliography

[AA09] Scott Aaronson and Andris Ambainis. The need for structure in quantum speedups. *arXiv preprint arXiv:0911.0996*, 2009.

[AA15] Scott Aaronson and Andris Ambainis. Forrelation: A problem that optimally separates quantum from classical computing. In *Proceedings of the Forty-Seventh Annual ACM Symposium on Theory of Computing*, STOC '15, page 307–316, New York, NY, USA, 2015. Association for Computing Machinery. doi:10.1145/2746539.2746547.

[AAI+16] Scott Aaronson, Andris Ambainis, Jānis Iraids, Martins Kokainis, and Juris Smotrovs. Polynomials, quantum query complexity, and Grothendieck's inequality. In *31st Conference on Computational Complexity, CCC 2016*, pages 25:1–25:19, 2016. arXiv:1511.08682. URL: https://doi.org/10.4230/LIPIcs.CCC.2016.25.

[AAKS21] Anurag Anshu, Srinivasan Arunachalam, Tomotaka Kuwahara, and Mehdi Soleimanifar. Sample-efficient learning of interacting quantum systems. *Nature Physics*, 17(8):931–935, 2021. doi:10.1038/s41567-021-01232-0.

[Aar21] Scott Aaronson. Open problems related to quantum query complexity. *ACM Transactions on Quantum Computing*, 2(4), 2021. URL: https://doi.org/10.1145/3488559.

[AB23] Andris Ambainis and Aleksandrs Belovs. An exponential separation between quantum query complexity and the polynomial degree. In *Proceedings of the Conference on Proceedings of the 38th Computational Complexity Conference*, CCC '23, Dagstuhl, DEU, 2023. Schloss

Dagstuhl–Leibniz-Zentrum fuer Informatik. URL: https://doi.org/10.4230/LIPIcs.CCC.2023.24.

[ABDK16] Scott Aaronson, Shalev Ben-David, and Robin Kothari. Separations in query complexity using cheat sheets. In *Proceedings of the forty-eighth annual ACM symposium on Theory of computing*, STOC '16, New York, NY, USA, 2016. Association for Computing Machinery. doi:10.1145/2897518.2897644.

[ABDK+21] Scott Aaronson, Shalev Ben-David, Robin Kothari, Shravas Rao, and Avishay Tal. Degree vs. approximate degree and quantum implications of huang's sensitivity theorem. In *Proceedings of the 53rd Annual ACM Symposium on Theory of Computing*, STOC 2021, New York, NY, USA, 2021. Association for Computing Machinery. doi:10.1145/3406325.3451047.

[ABP19] Srinivasan Arunachalam, Jop Briët, and Carlos Palazuelos. Quantum query algorithms are completely bounded forms. *SIAM J. Comput*, 48(3):903–925, 2019. Preliminary version in ITCS'18. URL: https://doi.org/10.1137/18M117563X.

[ABSdW13] Andris Ambainis, Arturs Backurs, Juris Smotrovs, and Ronald de Wolf. Optimal quantum query bounds for almost all Boolean functions. In Natacha Portier and Thomas Wilke, editors, *30th International Symposium on Theoretical Aspects of Computer Science (STACS 2013)*, volume 20 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 446–453, Dagstuhl, Germany, 2013. Schloss Dagstuhl – Leibniz-Zentrum für Informatik. URL: https://drops.dagstuhl.de/entities/document/10.4230/LIPIcs.STACS.2013.446, doi:10.4230/LIPIcs.STACS.2013.446.

[ACC+22] Per Austrin, Hao Chung, Kai-Min Chung, Shiuan Fu, Yao-Ting Lin, and Mohammad Mahmoody. On the impossibility of key agreements from quantum random oracles. In *Advances in Cryptology–CRYPTO 2022: 42nd Annual International Cryptology Conference, CRYPTO 2022, Santa Barbara, CA, USA, August 15–18, 2022, Proceedings, Part II*, pages 165–194. Springer, 2022.

[ACE+25] Amira Abbas, Nunzia Cerrato, Francisco Escudero Gutiérrez, Dmitry Grinko, Francesco Anna Mele, and Pulkit Sinha. Nearly optimal algorithms to learn sparse quantum hamiltonians in physically motivated distances. *arXiv preprint arXiv:2509.09813*, 2025.

[ACL+21] Srinivasan Arunachalam, Sourav Chakraborty, Troy Lee, Manaswi Paraashar, and Ronald de Wolf. Two new results about quantum exact learning. *Quantum*, 5:587, 2021.

[ACQ22] Dorit Aharonov, Jordan Cotler, and Xiao-Liang Qi. Quantum algorithmic measurement. *Nature Communications*, 13(1):1–9, 2022. doi:10.1038/s41467-021-27922-0.

[ADE25] Srinivasan Arunachalam, Arkopal Dutt, and Francisco Escudero Gutiérrez. Testing and learning structured quantum hamiltonians. In *Proceedings of the 57th Annual ACM Symposium on Theory of Computing*, STOC '25, page 1263–1270, New York, NY, USA, 2025. Association for Computing Machinery. doi:10.1145/3717823.3718289.

[ADEP24] Srinivasan Arunachalam, Arkopal Dutt, Francisco Escudero Gutiérrez, and Carlos Palazuelos. Learning low-degree quantum objects. In *51st International Colloquium on Automata, Languages, and Programming (ICALP 2024)*, pages 13–1. Schloss Dagstuhl–Leibniz-Zentrum für Informatik, 2024.

[ADEP25] Srinivasan Arunachalam, Arkopal Dutt, Francisco Escudero Gutiérrez, and Carlos Palazuelos. A cb-bohnenblust–hille inequality with constant one and its applications in learning theory. *Mathematische Annalen*, pages 1–30, 2025.

[AdW18] Srinivasan Arunachalam and Ronald de Wolf. Optimal quantum sample complexity of learning algorithms. *Journal of Machine Learning Research*, 19(71):1–36, 2018.

[AG04] Scott Aaronson and Daniel Gottesman. Improved simulation of stabilizer circuits. *Physical Review A*, 70(5):052328, 2004. doi:10.1103/PhysRevA.70.052328.

[Amb99] Andris Ambainis. A note on quantum black-box complexity of almost all boolean functions. *Information Processing Letters*, 71(1):5–7, 1999.

[Amb00] Andris Ambainis. Quantum lower bounds by quantum arguments. In *Proceedings of the thirty-second annual ACM symposium on Theory of computing*, pages 636–643, 2000.

[Amb03] A. Ambainis. Polynomial degree vs. quantum query complexity. In *44th Annual IEEE Symposium on Foundations of Computer Science, 2003. Proceedings.*, pages 230–239, 2003. doi:10.1109/SFCS.2003.1238197.

[Amb06] A. Ambainis. Polynomial degree vs. quantum query complexity. *J. Comput. System Sci.*, 72(2):220–238, 2006. Earlier version in FOCS'03. quant-ph/0305028.

[Amb07] A. Ambainis. Quantum walk algorithm for element distinctness. *SIAM Journal on Computing*, 37(1):210–239, 2007. Earlier version in FOCS'04. arXiv:quant-ph/0311001.

[Amb18] Andris Ambainis. Understanding quantum algorithms via query complexity. In *Proceedings of the International Congress of Mathematicians (ICM 2018)*, pages 3265–3285, 2018. URL: https://www.worldscientific.com/doi/abs/10.1142/9789813272880_0181, arXiv:https://www.worldscientific.com/doi/pdf/10.1142/9789813272880_0181, doi:10.1142/9789813272880_0181.

[AS04] Scott Aaronson and Yaoyun Shi. Quantum lower bounds for the collision and the element distinctness problems. *Journal of the ACM (JACM)*, 51(4):595–605, 2004.

[BB14] Arturs Backurs and Mohammad Bavarian. On the sum of l1 influences. In *2014 IEEE 29th Conference on Computational Complexity (CCC)*, pages 132–143. IEEE, 2014.

[BBB+19] Tom Bannink, Jop Briët, Harry Buhrman, Farrokh Labib, and Troy Lee. Bounding Quantum-Classical Separations for Classes of Nonlocal Games. In *36th International Symposium on Theoretical Aspects of Computer Science (STACS 2019)*, volume 126 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 12:1–12:11, 2019.

[BBC+01] Robert Beals, Harry Buhrman, Richard Cleve, Michele Mosca, and Ronald de Wolf. Quantum lower bounds by polynomials. *J. ACM*, 48(4):778–797, 2001.

[BCE+25] Andreas Bluhm, Matthias C Caro, Francisco Escudero Gutiérrez, Aadil Oufkir, and Cambyse Rouzé. Certifying and learning quantum ising hamiltonians. *arXiv preprint arXiv:2509.10239*, 2025.

[BCO24a] Andreas Bluhm, Matthias C Caro, and Aadil Oufkir. Hamiltonian property testing (version 1). *arXiv preprint 2403.02968v1*, 2024. `arXiv:2403.02968v1`.

[BCO24b] Andreas Bluhm, Matthias C Caro, and Aadil Oufkir. Hamiltonian property testing (version 2). *arXiv preprint 2403.02968v2*, 2024. `arXiv:2403.02968v2`.

[BE22] Jop Briët and Francisco Escudero Gutiérrez. On Converses to the Polynomial Method. In *17th Conference on the Theory of Quantum Computation, Communication and Cryptography (TQC 2022)*, volume 232 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 6:1–6:10. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2022. URL: `http://doi.org/10.4230/LIPIcs.TQC.2022.6`.

[BE24] Jinge Bao and Francisco Escudero Gutiérrez. Learning junta distributions, quantum junta states, and QAC^0 circuits. *arXiv preprint arXiv:2410.15822*, 2024.

[BEG24] Jop Briët, Francisco Escudero Gutiérrez, and Sander Gribling. Grothendieck inequalities characterize converses to the polynomial method. *Quantum*, 8:1526, 2024.

[Ben80] Paul Benioff. The computer as a physical system: A microscopic quantum mechanical hamiltonian model of computers as represented by turing machines. *Journal of statistical physics*, 22:563–591, 1980.

[BH31] Henri Frédéric Bohnenblust and Einar Hille. On the absolute convergence of dirichlet series. *Annals of Mathematics*, pages 600–622, 1931. URL: `https://doi.org/10.2307/1968255`.

[Bha25] Sreejata Kishor Bhattacharya. Random Restrictions of Bounded Low Degree Polynomials Are Juntas. In Raghu Meka, editor, *16th Innovations in Theoretical Computer Science Conference (ITCS 2025)*, volume 325 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 17:1–17:21, Dagstuhl, Germany, 2025. Schloss Dagstuhl –

Leibniz-Zentrum für Informatik. URL: https://drops.dagstuhl.de/entities/document/10.4230/LIPIcs.ITCS.2025.17, doi:10.4230/LIPIcs.ITCS.2025.17.

[BKT20] Mark Bun, Robin Kothari, and Justin Thaler. The polynomial method strikes back: Tight quantum query bounds via dual polynomials. *Theory of Computing*, 16(10):1–71, 2020. URL: https://doi.org/10.4086/toc.2020.v016a010.

[Ble79] Ron C Blei. Fractional cartesian products of sets. In *Annales de l'institut Fourier*, volume 29,2, pages 79–105, 1979.

[Ble01] Ron Blei. *Analysis in integer and fractional dimensions*, volume 71. Cambridge University Press, 2001.

[BLM13] S. Boucheron, G. Lugosi, and P. Massart. *Concentration inequalities: A nonasymptotic theory of independence*. Oxford university press, 2013. URL: https://doi.org/10.1093/acprof:oso/9780199535255.001.0001.

[BLMT23] Ainesh Bakshi, Allen Liu, Ankur Moitra, and Ewin Tang. Learning quantum hamiltonians at any temperature in polynomial time, 2023. arXiv:2310.02243.

[BLMT24] Ainesh Bakshi, Allen Liu, Ankur Moitra, and Ewin Tang. Structure learning of hamiltonians from real-time evolution, 2024. In FOCS'24.

[BMMN13] M. Braverman, K. Makarychev, Y. Makarychev, and A. Naor. The Grothendieck constant is strictly smaller than Krivine's bound. *Forum Math. Pi*, 1:453–462, 2013. Preliminary version in FOCS'11. arXiv:1103.6161.

[Bon70] Aline Bonami. Étude des coefficients de fourier des fonctions de $l^p(g)$. In *Annales de l'institut Fourier*, volume 20,2, pages 335–402, 1970.

[BP19] Jop Briët and Carlos Palazuelos. Failure of the trilinear operator space Grothendieck inequality. *Discrete Analysis*, 2019. Paper No. 8.

[BPSS14] Frédéric Bayart, Daniel Pellegrino, and Juan B Seoane-Sepúlveda. The Bohr radius of the n-dimensional polydisk is equivalent to $(\log n)/n$. *Advances in Mathematics*, 264:726–746, 2014.

[Bri19] Jop. Briët. Note on chebychev polynomials. *unpublished*, 2019. URL: https://zwebmail.cwi.nl/service/home/~/?auth=co&loc=en&id=462&part=2.

[BS21] Nikhil Bansal and Makrand Sinha. k-forrelation optimally separates quantum and classical query complexity. In *Proceedings of the 53rd Annual ACM SIGACT Symposium on Theory of Computing*, STOC 2021, page 1303–1316, New York, NY, USA, 2021. Association for Computing Machinery. doi:10.1145/3406325.3451040.

[BSdW22] Nikhil Bansal, Makrand Sinha, and Ronald de Wolf. Influence in Completely Bounded Block-Multilinear Forms and Classical Simulation of Quantum Algorithms. In *37th Computational Complexity Conference (CCC 2022)*, volume 234, pages 28:1–28:21, 2022. doi:10.4230/LIPIcs.CCC.2022.28.

[BSS03] H. Barnum, M. Saks, and M. Szegedy. Quantum query complexity and semi-definite programming. In *18th IEEE Annual Conference on Computational Complexity, 2003. Proceedings.*, pages 179–193, 2003. doi:10.1109/CCC.2003.1214419.

[BV93] Ethan Bernstein and Umesh Vazirani. Quantum complexity theory. In *Proceedings of the twenty-fifth annual ACM symposium on Theory of computing*, pages 11–20, 1993.

[BY23] Zongbo Bao and Penghui Yao. On Testing and Learning Quantum Junta Channels. In Gergely Neu and Lorenzo Rosasco, editors, *Proceedings of Thirty Sixth Conference on Learning Theory*, volume 195 of *Proceedings of Machine Learning Research*, pages 1064–1094. PMLR, 12–15 Jul 2023. URL: https://proceedings.mlr.press/v195/bao23b.html.

[Can20] Clément L Canonne. A short note on learning discrete distributions. *arXiv preprint arXiv:2002.11457*, 2020.

[Car23] Matthias C. Caro. Learning quantum processes and hamiltonians via the pauli transfer matrix, 2023. arXiv:2212.04471.

[CPGSV21] J. Ignacio Cirac, David Pérez-García, Norbert Schuch, and Frank Verstraete. Matrix product states and projected entangled pair states: Concepts, symmetries, theorems. *Rev. Mod. Phys.*, 93:045003, Dec 2021.

URL: https://link.aps.org/doi/10.1103/RevModPhys.93.045003, doi:10.1103/RevModPhys.93.045003.

[CS87] Erik Christensen and Allan M Sinclair. Representations of completely bounded multilinear operators. *Journal of Functional analysis*, 72(1):151–181, 1987.

[CW23] Juan Castaneda and Nathan Wiebe. Hamiltonian learning via shadow tomography of pseudo-choi states, 2023. arXiv:2308.13020.

[Dav84] A. Davie. Lower bound for K_G. Unpublished, 1984.

[Dav06] AM Davie. Matrix norms related to Grothendieck's inequality. In *Banach Spaces: Proceedings of the Missouri Conference held in Columbia, USA, June 24–29, 1984*, pages 22–26. Springer, 2006.

[DFOC+11] Andreas Defant, Leonhard Frerick, Joaquim Ortega-Cerdà, Myriam Ounaïes, and Kristian Seip. The Bohnenblust–Hille inequality for homogeneous polynomials is hypercontractive. *Annals of mathematics*, pages 485–497, 2011.

[DGMP19] Andreas Defant, Doming García, Manuel Maestre, and Pablo Peris. *Dirichlet series and holomorphic functions in high dimensions*. Cambridge Studies in Advanced Mathematics. Cambridge University Press, 07 2019.

[DJ92] David Deutsch and Richard Jozsa. Rapid solution of problems by quantum computation. *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, 439(1907):553–558, 1992. doi:10.1098/rspa.1992.0167.

[DMFPSS14] Diogo Diniz, Gustavo Muñoz-Fernández, Daniel Pellegrino, and J Seoane-Sepúlveda. Lower bounds for the constants in the Bohnenblust–Hille inequality: The case of real scalars. *Proceedings of the American Mathematical Society*, 142(2):575–580, 2014.

[DMP19] Andreas Defant, Mieczysław Mastyło, and Antonio Pérez. On the fourier spectrum of functions on boolean cubes. *Mathematische Annalen*, 374(1):653–680, 2019.

[DOS23] Alicja Dutkiewicz, Thomas E. O'Brien, and Thomas Schuster. The advantage of quantum control in many-body hamiltonian learning, 2023. arXiv:2304.07172.

[dSLCP11] Marcus P. da Silva, Olivier Landon-Cardinal, and David Poulin. Practical characterization of quantum devices without tomography. *Physical Review Letters*, 107(21):210404, 2011. doi:10.1103/PhysRevLett.107.210404.

[EFFJ+23] Francisco Escudero Gutiérrez, David Fernández-Fernández, Gabriel Jaumà, Guillermo F Peñas, and Luciano Pereira. Hardware-efficient entangled measurements for variational quantum algorithms. *Physical Review Applied*, 20(3):034044, 2023.

[EI22] Alexandros Eskenazis and Paata Ivanisvili. Learning low-degree functions from a logarithmic number of random queries. In *Proceedings of the 54th Annual ACM SIGACT Symposium on Theory of Computing*, pages 203–207, 2022.

[EIS22] Alexandros Eskenazis, Paata Ivanisvili, and Lauritz Streck. Low-degree learning and the metric entropy of polynomials. *arXiv preprint arXiv:2203.09659*, 2022.

[EM24] Francisco Escudero Gutiérrez and Garazi Muguruza. All S_p notions of quantum expansion are equivalent. *arXiv preprint arXiv:2405.03517*, 2024.

[Esc24a] Francisco Escudero Gutiérrez. Influences of fourier completely bounded polynomials and classical simulation of quantum algorithms. *Chicago Journal of Theoretical Computer Science*, 2024. URL: https://doi.org/10.48550/arXiv.2304.06713.

[Esc24b] Francisco Escudero Gutiérrez. Simple algorithms to test and learn local hamiltonians. *arXiv preprint arXiv:2404.06282*, 2024.

[Esc25] Francisco Escudero Gutiérrez. Christensen-sinclair factorization via semidefinite programming. *Linear Algebra and its Applications*, 714:28–44, 2025.

[FGG07] Edward Farhi, Jeffrey Goldstone, and Sam Gutmann. A quantum algorithm for the hamiltonian nand tree. *arXiv preprint quant-ph/0702144*, 2007.

[FHKL16] Yuval Filmus, Hamed Hatami, Nathan Keller, and Noam Lifshitz. On the sum of the l 1 influences of bounded functions. *Israel Journal of Mathematics*, 214:167–192, 2016.

[GCC24] Andi Gu, Lukasz Cincio, and Patrick J. Coles. Practical hamiltonian learning with unitary dynamics and gibbs states. *Nature Communications*, 15(1), 2024. doi:10.1038/s41467-023-44008-1.

[GJ14] Gus Gutoski and Nathaniel Johnston. Process tomography for unitary quantum channels. *Journal of Mathematical Physics*, 55(3), 2014.

[GL19] Sander Gribling and Monique Laurent. Semidefinite programming formulations for the completely bounded norm of a tensor. *arxiv,arXiv.1901.04921*, 2019. URL: https://doi.org/10.48550/arXiv.1901.04921.

[GOS+11] Parikshit Gopalan, Ryan O'Donnell, Rocco A. Servedio, Amir Shpilka, and Karl Wimmer. Testing Fourier dimensionality and sparsity. *SIAM Journal on Computing*, 40(4):1075–1100, 2011. arXiv:https://doi.org/10.1137/100785429.

[Got98] Daniel Gottesman. The heisenberg representation of quantum computers. *arXiv preprint quant-ph/9807006*, 1998.

[Gre07] Ben Green. Montréal notes on quadratic Fourier analysis. In *Additive combinatorics*, volume 43 of *CRM Proc. Lecture Notes*, pages 69–102. Amer. Math. Soc., Providence, RI, 2007. URL: https://doi.org/10.1090/crmp/043/06.

[Gro53] Alexandre Grothendieck. *Résumé de la théorie métrique des produits tensoriels topologiques*. Soc. de Matemática de São Paulo, 1953.

[Gro96] Lov K. Grover. A fast quantum mechanical algorithm for database search. In *Proceedings of the twenty-eighth annual ACM symposium on Theory of computing*, pages 212–219. ACM, 1996.

[Haa87] Uffe Haagerup. A new upper bound for the complex Grothendieck constant. *Israel Journal of Mathematics*, 60:199–224, 1987.

[Ham25] Yassine Hamoudi. A brief introduction to quantum query complexity. *In preparation*, 2025. URL: `https://yassine-hamoudi.github.io/files/publications/QueryComplexity.pdf`.

[Har72] Lawrence A Harris. Bounds on the derivatives of holomorphic functions of vectors. In *Proc. Colloq. Analysis, Rio de Janeiro*, volume 145, page 163, 1972.

[HBCP15] M Holzäpfel, T Baumgratz, M Cramer, and Martin B Plenio. Scalable reconstruction of unitary processes and hamiltonians. *Physical Review A*, 91(4):042129, 2015.

[HCP23a] Hsin-Yuan Huang, Sitan Chen, and John Preskill. Learning to predict arbitrary quantum processes. *PRX Quantum*, 4:040337, Dec 2023. URL: `https://link.aps.org/doi/10.1103/PRXQuantum.4.040337`, `doi:10.1103/PRXQuantum.4.040337`.

[HCP23b] Hsin-Yuan Huang, Sitan Chen, and John Preskill. Learning to predict arbitrary quantum processes. *PRX Quantum*, 4(4):040337, 2023. `doi:10.1103/PRXQuantum.4.040337`.

[HKP20] Hsin-Yuan Huang, Richard Kueng, and John Preskill. Predicting many properties of a quantum system from very few measurements. *Nature Physics*, 16(10):1050–1057, 2020.

[HKT22] Jeongwan Haah, Robin Kothari, and Ewin Tang. Optimal learning of quantum hamiltonians from high-temperature gibbs states. In *2022 IEEE 63rd Annual Symposium on Foundations of Computer Science (FOCS)*, pages 135–146. IEEE, 2022. `doi:10.1109/FOCS54457.2022.00020`.

[HLv07] Peter Høyer, Troy Lee, and Robert Špalek. Negative weights make adversaries stronger. In *Proceedings of the thirty-ninth annual ACM symposium on Theory of computing*, pages 526–535, 2007.

[HMG+25] Hong-Ye Hu, Muzhou Ma, Weiyuan Gong, Qi Ye, Yu Tong, Steven T Flammia, and Susanne F Yelin. Ansatz-free hamiltonian learning with heisenberg-limited scaling. *arXiv preprint arXiv:2502.11900*, 2025.

[HTFS23] Hsin-Yuan Huang, Yu Tong, Di Fang, and Yuan Su. Learning many-body hamiltonians with heisenberg-limited scaling. *Physical Review Letters*, 130(20):200403, 2023. doi:10.1103/PhysRevLett.130.200403.

[HW+79] Godfrey Harold Hardy, Edward Maitland Wright, et al. *An introduction to the theory of numbers*. Oxford university press, 1979. URL: https://doi.org/10.1126/science.90.2329.158.b.

[IBF+20] Luca Innocenti, Leonardo Banchi, Alessandro Ferraro, Sougato Bose, and Mauro Paternostro. Supervised learning of time-independent Hamiltonians for gate design. *New Journal of Physics*, 22(6):065001, 2020.

[Iva19] Paata Ivanishvili. Aaronson-ambainis conjecture. https://extremal010101.wordpress.com/2019/10/29/aaronson-ambainis-conjecture/, 2019.

[JZ11] Rahul Jain and Shengyu Zhang. The influence lower bound via query elimination. *Electronic Colloquium on Computational Complexity - ECCC*, 7, 02 2011. doi:10.4086/toc.2011.v007a010.

[KM13] Daniel M Kane and Raghu Meka. A PRG for Lipschitz functions of polynomials with applications to sparsest cut. In *Proceedings of the forty-fifth annual ACM symposium on Theory of computing*, pages 1–10, 2013. URL: https://doi.org/10.1145/2488608.2488610.

[KMY03] Hirotada Kobayashi, Keiji Matsumoto, and Tomoyuki Yamakami. Quantum merlin-arthur proof systems: Are multiple merlins more helpful to arthur? In *Algorithms and Computation: 14th International Symposium, ISAAC 2003*, pages 189–198. Springer, 2003.

[KN07] Subhash Khot and Assaf Naor. Linear equations modulo 2 and the l1 diameter of convex bodies. In *48th Annual IEEE Symposium on Foundations of Computer Science (FOCS'07)*, pages 318–328. IEEE, 2007. URL: https://doi.org/10.1109/FOCS.2007.20.

[KS19] Gil Kalai and Leonard J Schulman. Quasi-random multilinear polynomials. *Israel Journal of Mathematics*, 230:195–211, 2019.

[Lit30] John E Littlewood. On bounded bilinear forms in an infinite number of variables. *The Quarterly Journal of Mathematics*, pages 164–174, 1930.

[LMN93] Nathan Linial, Yishay Mansour, and Noam Nisan. Constant depth circuits, Fourier transform, and learnability. *Journal of the ACM (JACM)*, 40(3):607–620, 1993.

[Lov10] Shachar Lovett. An elementary proof of anti-concentration of polynomials in gaussian variables. In *Electron. Colloquium Comput. Complex.*, volume 17, page 182, 2010.

[LR05] Monique Laurent and Franz Rendl. Semidefinite programming and integer programming. *Handbooks in Operations Research and Management Science*, 12:393–514, 2005.

[LS21] Lily Li and Morgan Shirley. The general adversary bound: A survey. *arXiv preprint arXiv:2104.06380*, 2021.

[LTN+23] Haoya Li, Yu Tong, Hongkang Ni, Tuvia Gefen, and Lexing Ying. Heisenberg-limited hamiltonian learning for interacting bosons, 2023. `arXiv:2307.04690`.

[LW22] Margarite L LaBorde and Mark M Wilde. Quantum algorithms for testing hamiltonian symmetry. *Physical Review Letters*, 129(16):160503, 2022.

[LZ22] Shachar Lovett and Jiapeng Zhang. Fractional certificates for bounded functions. *Electronic Colloquium on Computational Complexity - ECCC*, 107, 2022.

[Man80] Yu I Manin. Computable and uncomputable. *Sov. Radio*, 1980.

[MFPT24] Muzhou Ma, Steven T Flammia, John Preskill, and Yu Tong. Learning k-body hamiltonians via compressed sensing. *arXiv preprint arXiv:2410.18928*, 2024.

[Mid05] Gatis Midrijanis. On randomized and quantum query complexities. *arXiv preprint quant-ph/0501142*, 2005.

[MMFPSS22] Mohammad Sal Moslehian, GA Muñoz-Fernández, AM Peralta, and JB Seoane-Sepúlveda. Similarities and differences between real and complex banach spaces: an overview and recent developments. *Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas*, 116(2):1–80, 2022. URL: `https://doi.org/10.1007/s13398-022-01222-8`.

[MMG09] Matthew McKague, Michele Mosca, and Nicolas Gisin. Simulating quantum systems using real Hilbert spaces. *Physical review letters*, 102(2):020505, 2009.

[MO08] Ashley Montanaro and Tobias J Osborne. Quantum Boolean functions. *arXiv preprint arXiv:0810.2435*, 2008.

[Mon12] Ashley Montanaro. Some applications of hypercontractive inequalities in quantum information theory. *Journal of Mathematical Physics*, 53(12):122206, 2012.

[MS69] Minsky Marvin and A Papert Seymour. Perceptrons. *Cambridge, MA: MIT Press*, 6(318-362):7, 1969.

[NC10] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information: 10th Anniversary Edition*. Cambridge University Press, Cambridge, UK, 2010. doi:10.1017/CBO9780511976667.

[NGHA15] Miguel Navascués, Yelena Guryanova, Matty J Hoban, and Antonio Acín. Almost quantum correlations. *Nature communications*, 6(1):6288, 2015.

[Nik54] Hukukane Nikaidô. On von Neumann's minimax theorem. *Pacific Journal of Mathematics*, 4:65–72, 1954.

[NPVY23] Shivam Nadimpalli, Natalie Parham, Francisca Vasconcelos, and Henry Yuen. On the Pauli spectrum of QAC0. *arXiv preprint arXiv:2311.09631*, 2023.

[NS94] Noam Nisan and Mario Szegedy. On the degree of boolean functions as real polynomials. *Computational complexity*, 4:301–313, 1994.

[NS12] Sahand Negahban and Devavrat Shah. Learning sparse boolean polynomials. In *2012 50th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pages 2032–2036. IEEE, 2012.

[O'D09] Ryan O'Donnell. *Analysis of Boolean Functions*. Cambridge University Press, 2009. doi:10.1017/cbo9781139814782.

[ORSFW23] Emilio Onorati, Cambyse Rouzé, Daniel Stilck França, and James D. Watson. Efficient learning of ground & thermal states within phases of matter, 2023. arXiv:2301.12946.

[OSSS05] Ryan O'Donnell, Michael Saks, Oded Schramm, and Rocco A Servedio. Every decision tree has an influential variable. In *46th Annual IEEE Symposium on Foundations of Computer Science (FOCS'05)*, pages 31–39. IEEE, 2005.

[OZ15] Ryan O'Donnell and Yu Zhao. Polynomial bounds for decoupling, with applications. *arXiv preprint arXiv:1512.01603*, 2015. URL: `https://doi.org/10.4230/LIPIcs.CCC.2016.24`.

[Pau03] Vern Paulsen. *Completely Bounded Maps and Operator Algebras*. arXiv.1901.04921, 02 2003. URL: `https://doi.org/10.1017/CBO9780511546631`.

[PT18] Daniel Pellegrino and Eduardo V Teixeira. Towards sharp Bohnenblust–Hille constants. *Communications in Contemporary Mathematics*, 20(03):1750029, 2018.

[Ree91] J. Reeds. A new lower bound on the real Grothendieck constant. Manuscript (`http://www.dtc.umn.edu/~reedsj/bound2.dvi`), 1991.

[RSF23] Cambyse Rouzé and Daniel Stilck França. Learning quantum many-body systems from a few copies, 2023. `arXiv:2107.03333`.

[RTW+21] Marc-Olivier Renou, David Trillo, Mirjam Weilenmann, Thinh P Le, Armin Tavakoli, Nicolas Gisin, Antonio Acín, and Miguel Navascués. Quantum theory based on real numbers can be experimentally falsified. *Nature*, 600(7890):625–629, 2021.

[RWZ24] Cambyse Rouzé, Melchior Wirth, and Haonan Zhang. Quantum talagrand, kkl and friedguts theorems and the learnability of quantum boolean functions. *Communications in Mathematical Physics*, 405(4):95, 2024.

[SFMD+24] Daniel Stilck França, Liubov A. Markovich, V. V. Dobrovitski, Albert H. Werner, and Johannes Borregaard. Efficient and robust estimation of many-qubit hamiltonians. *Nature Communications*, 15:311, 2024. `doi:10.1038/s41467-023-44012-5`.

[Sho97] Peter Shor. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM Journal of Computing*, 26(5):1484–1509, 1997. Earlier version in FOCS'94.

[Sim97] Daniel R. Simon. On the power of quantum computation. *SIAM Journal on Computing*, 26(5):1474–1483, 1997. arXiv:https://doi.org/10.1137/S0097539796298637, doi:10.1137/S0097539796298637.

[SMCG16] Sarah Sheldon, Easwar Magesan, Jerry M. Chow, and Jay M. Gambetta. Procedure for systematically tuning up cross-talk in the cross-resonance gate. *Phys. Rev. A*, 93:060302(R), Jun 2016. URL: https://link.aps.org/doi/10.1103/PhysRevA.93.060302, doi:10.1103/PhysRevA.93.060302.

[SSW21] Alexander A Sherstov, Andrey A Storozhenko, and Pei Wu. An optimal separation of randomized and quantum query complexity. In *Proceedings of the 53rd Annual ACM SIGACT Symposium on Theory of Computing*, pages 1289–1302, 2021.

[SVZ23a] Joseph Slote, Alexander Volberg, and Haonan Zhang. Bohnenblust–Hille inequality for cyclic groups. *arXiv preprint arXiv:2305.10560*, 2023.

[SVZ23b] Joseph Slote, Alexander Volberg, and Haonan Zhang. Noncommutative Bohnenblust-Hille inequality in the heisenberg-weyl and gell-mann bases with applications to fast learning. *arXiv preprint arXiv:2301.01438*, 2023.

[SY23] Adrian She and Henry Yuen. Unitary Property Testing Lower Bounds by Polynomials. In *14th Innovations in Theoretical Computer Science Conference (ITCS 2023)*, volume 251 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 96:1–96:17. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2023. doi:10.4230/LIPIcs.ITCS.2023.96.

[Tal20] Avishay Tal. Towards optimal separations between quantum and randomized query complexities. In *2020 IEEE 61st Annual Symposium on Foundations of Computer Science (FOCS)*, pages 228–239. IEEE, 2020.

[Tao12] Terence Tao. *Topics in Random Matrix Theory*. Graduate studies in mathematics. American Mathematical Society, 2012.

[Tsi80] B. S. Tsirelson. Quantum generalizations of Bell's inequality. *Letters in Mathematical Physics*, 1980. URL: https://doi.org/10.1007/BF00417500.

[TT23] Terence Tao and Joni Teräväinen. Quantitative bounds for Gowers uniformity of the Möbius and von Mangoldt functions. *Journal of the European Mathematical Society*, 2023. doi:https://doi.org/10.4171/jems/1404.

[Val84] Leslie G. Valiant. A Theory of the Learnable. *Communications of the ACM*, 27(11):1134–1142, 1984.

[Var74] N. Th. Varopoulos. On an inequality of von Neumann and an application of the metric theory of tensor products to operators theory. *J. Functional Analysis*, 16:83–100, 1974. doi:10.1016/0022-1236(74)90071-8.

[VZ23] Alexander Volberg and Haonan Zhang. Noncommutative Bohnenblust–Hille inequalities. *Mathematische Annalen*, pages 1–20, 2023.

[WKR+22] Frederik Wilde, Augustine Kshetrimayum, Ingo Roth, Dominik Hangleiter, Ryan Sweke, and Jens Eisert. Scalably learning quantum many-body hamiltonians from dynamical data, 2022. arXiv:2209.14328.

[YSHY23] Wenjun Yu, Jinzhao Sun, Zeyao Han, and Xiao Yuan. Robust and efficient hamiltonian learning. *Quantum*, 7:1045, 2023. doi:10.22331/q-2023-06-29-1045.

[YZ20] Grigory Yaroslavtsev and Samson Zhou. Fast Fourier Sparsity Testing. In *Symposium on Simplicity in Algorithms*, pages 57–68. SIAM, 2020.

[YZ22] Takashi Yamakawa and Mark Zhandry. Verifiable quantum advantage without structure. In *2022 IEEE 63rd Annual Symposium on Foundations of Computer Science (FOCS)*, pages 69–74. IEEE, 2022.

[Zha24] Andrew Zhao. Learning the structure of any hamiltonian from minimal assumptions. *arXiv preprint arXiv:2410.21635*, 2024.

[ZYLB21] Assaf Zubida, Elad Yitzhaki, Netanel H. Lindner, and Eyal Bairey. Optimal short-time measurements for hamiltonian learning, 2021. arXiv:2108.08824.