Quantum computing, norms and polynomials
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Part III

Bonus

Chapter 8

Cute remarks

In this chapter, we gather new proofs of known results that we find elegant and concise. All of them relate to functions defined on the Boolean cube.[1]

8.1 Generalizing a work of Kalai and Schulman

Kalai and Schulman studied the influences of multilinear polynomials with $\{-1, 0, 1\}$-valued coefficients [KS19] (we refer to their work for motivation). They showed an upper bound of $\sum_{i\in[n]} \sqrt{\mathrm{Inf}_i[p]}$ in terms of $\|p\|_\infty$. They proved that

$$\sum_{i\in[n]} \sqrt{\mathrm{Inf}_i[p]} \leq 3^d d^{5/2} \|p\|_\infty$$

for unimodular polynomials. We can improve this bound, generalize it to arbitrary polynomials (not necessarily unimodular), and show that the exponential dependence on d is necessary. Our proof is simple, short and based on hypercontractivity [Bon70] and a bound on the sum of L_1 influences [BB14, FHKL16].

Before diving into the proof of the main result of this section, Proposition 8.3, we need to the define the L_q influences and state two results that we use as lemmas. The L_q influence is defined by

$$\mathrm{Inf}_i^q[p] = \mathbb{E}_x \left[\left| \frac{p(x^{i\to 1}) - p(x^{i\to -1})}{2} \right|^q \right],$$

[1]The results of Section 8.1 were derived in a conversation with Miquel Saucedo during a research stay in Hausdorff Institute for Mathematics, in Bonn, Germany.

where the expectation is taken with respect to the uniform distribution on $\{-1,1\}^n$. Note that $\mathrm{Inf}^2[p]$ equals the $\mathrm{Inf}[p]$ defined in Section 2.5. In the Boolean case, $\mathrm{Inf}_i^2[p] = \mathrm{Inf}_i^q[p]$ for every $q \in [1,\infty)$.

Theorem 8.1 (Hypercontractivity)**.** *Let $p : \{-1,1\}^n \to \mathbb{R}$ be a polynomial of degree at most d. Then,*

$$\sqrt{\mathbb{E}_x\,|p(x)|^2} \leq e^d \mathbb{E}_x |p(x)|.$$

Theorem 8.2 (Bound on sum of L_1 influences)**.** *Let $p : \{-1,1\}^n \to \mathbb{R}$ be a polynomial of degree at most d. Then,*

$$\sum_{i\in[n]} \mathrm{Inf}_i^1[p] \leq d^2 \|p\|_\infty.$$

Proposition 8.3. *Let $p : \{-1,1\}^n \to \mathbb{R}$ be a polynomial of degree d. Then,*

$$\sum_{i\in[n]} \sqrt{\mathrm{Inf}_i^2[p]} \leq e^d d^2 \|p\|_\infty.$$

In addition, there is a unimodular degree d polynomial p such that

$$\sum_{i\in[n]} \sqrt{\mathrm{Inf}_i^2[p]} \geq \sqrt{2}^{d-2} \|p\|_\infty.$$

Proof. By Theorem 8.1 it follows that for every $i \in [n]$

$$\sqrt{\mathrm{Inf}_i^2[p]} \leq e^d \mathrm{Inf}_i^1[p].$$

Now, taking the sum over $i \in [n]$ and applying Theorem 8.2 we arrive at the claimed result.

Let $n = 2^{d-1}$. The (unnormalized) address function of $p : (\{-1,1\})^n)^d \to \mathbb{R}$ of degree d is defined as

$$p(x) = \sum_{a\in\{-1,1\}^{d-1}} \underbrace{(x_1(1) - a_1x_1(2))\ldots(x_{d-1}(1) - a_{d-1}x_{d-1}(2))}_{g_a(x_1,\ldots,x_{d-1})} x_d(a), \tag{8.1}$$

where we identify $\{-1,1\}^{d-1}$ with $[2^{d-1}]$. It is satisfied that $\|p\|_\infty = 2^{d-1}$, because given $(x_1,\ldots,x_{d-1}) \in (\{-1,1\}^n)^{d-1}$ there is only one $a \in \{-1,1\}^{d-1}$ such that $g_a(x_1,\ldots,x_{d-1})$ is not 0, in which case it takes the value $\pm 2^{d-1}$. For every of the 2^{d-1} variables $x_d(a)$ we have that $\mathrm{Inf}_{d,a}^2[p] = 2^{d-2}$, so $\sum \sqrt{\mathrm{Inf}_i^2[p]} \geq (2^{d-2})^{3/2}$. □

8.2 The adversary method via Grothendieck's inequality

In the first part of this thesis, we have focused on the polynomial method. Here, we will revisit the other main method to prove quantum query lower bounds: the adversary method [Amb00, HLv07] (see [LS21] for a survey). To define the adversary bound we must introduce some notation. Let $f : \{-1,1\}^n \to \{-1,1\}$. A matrix $\Gamma \in M_{2^n}(\mathbb{C})$ is an *adversary matrix for* f if it is Hermitian and for every $x, y \in \{-1,1\}^n$ such that $f(x) = f(y)$ we have that

$$\langle x|\Gamma|y\rangle = 0,$$

where $\{|x\rangle\}$ is an orthonormal basis of $\mathbb{C}^{2^n}$. Given $i \in [n]$, $D_i \in M_{2^n}$ is the matrix defined by

$$\langle x|D_i|y\rangle = \begin{cases} 0 & \text{if } x_i = y_i, \\ 1 & \text{if } x_i \neq y_i. \end{cases}$$

The *adversary bound of* f is defined by

$$\mathrm{Adv}(f) := \sup_{\Gamma} \frac{\|\Gamma\|_{\mathrm{op}}}{\max_{i\in[n]} \|\Gamma \circ D_i\|_{\mathrm{op}}}, \tag{8.2}$$

where the supremum runs over all adversary matrices Γ and $\circ$ denotes the entry-wise matrix product, namely $(A \circ B)_{ij} = A_{ij}B_{ij}$. In this section, we will give a, to the best of our knowledge, novel proof of the following result via Grothendieck's inequality (see Section 2.7.1).

Proposition 8.4. *Let* $f : \{-1,1\}^n \to \{-1,1\}$. *Then,* $Q(f) = \Omega(\mathrm{Adv}(f))$.

Before diving into the proof, we give an intuition of why such a result holds. Consider an algorithm whose bias approximates f with high probability. Before making any query, the algorithm prepares a state that does not depend on the input x. In terms of adversary matrices Γ, this will mean that some closeness measure, to be defined below, will have value $\|\Gamma\|_{\mathrm{op}}$ before making any query. Also, at the end of the algorithm, the state prepared on a pair of inputs x and y such that $f(x) \neq f(y)$ must be *far away*, so the algorithm can distinguish them with a measurement. In terms of Γ, this will be formalized via Grothendieck's inequality and will mean that the closeness measure will have value $\leq K_G/5 \cdot \|\Gamma\|_{\mathrm{op}}$ at the end of the algorithm. Finally, it will follow from a simple argument that the algorithm can only separate the states prepared when querying x and y a bounded amount per query. In terms of

Γ, this will mean that the closeness measure can only decrease $2\max_i \|\Gamma \circ D_i\|_{\mathrm{op}}$ per query. Putting everything together we have that the algorithm must make at least

$$\frac{\|\Gamma\|_{\mathrm{op}} - \frac{K_G}{5}\|\Gamma\|_{\mathrm{op}}}{2\max_{i\in[n]}\|\Gamma \circ D_i\|_{\mathrm{op}}}$$

queries.

Proof. We will show that $Q_{2/100}(f) = \Omega(\mathrm{Adv}(f))$. Let $\mathcal{A}$ be an algorithm that makes t queries and whose bias $2/100$ approximates $f(x)$. This means, that $\mathcal{A}$ fails with probability $\|\Pi_{-f(x)}|\psi_x^t\rangle\|_2^2 \leq 1/100$. Let Γ be an adversary matrix for f. Let $|\delta\rangle$ be such that $|\langle\delta|\Gamma|\delta\rangle| = \|\Gamma\|_{\mathrm{op}}$. We define the closeness measure at step $s \in \{0,\dots,t\}$ as

$$\mathcal{C}^s := |\sum_{x,y}\Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^s|\psi_y^s\rangle|,$$

where $|\psi_x^s\rangle$ is the state prepared by the algorithm on input x just after the sth query. We divide the rest of the proof in three steps. First, we note that

$$\mathcal{C}^0 = |\sum_{x,y}\Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^0|\psi_y^0\rangle| = |\sum_{x,y}\Gamma_{xy}\delta_x^*\delta_y| = |\langle\delta|\Gamma|\delta\rangle| = \|\Gamma\|_{\mathrm{op}},$$

where we have used that $|\psi_x^0\rangle$ does not depend on x because no queries have been made.

Second, we claim that

$$\mathcal{C}^t \leq \frac{K_G}{5}\|\Gamma\|_{\mathrm{op}}$$

where K_G is the (complex) Grothendieck's constant, which is strictly smaller than 5. Indeed, let Π_{-1},Π_1 be the measurement performed by the algorithm, then

$$\begin{aligned}
\mathcal{C}^t &= |\sum_{x,y}\Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^t|\psi_y^t\rangle| = |\sum_{x,y:f(x)\neq f(y)}\Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^t|(\Pi_{-1}+\Pi_1)|\psi_y^t\rangle| \\
&\leq |\sum_{x,y:f(x)\neq f(y)}\Gamma_{xy}\delta_x^*\delta_y(\langle\psi_x^t|\Pi_{-f(x)})|\psi_y^t\rangle| + |\sum_{x,y:f(x)\neq f(y)}\Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^t|(\Pi_{-f(y)}|\psi_y^t\rangle)| \\
&\leq \frac{2}{10}\sup_{\|u_x\|_2,\|v_y\|_2\leq 1}|\sum_{x,y}\Gamma_{xy}\delta_x^*\delta_y\langle u_x,v_y\rangle| \\
&\leq \frac{2K_G}{10}\sup_{\alpha_x,\beta_y\in\{-1,1\}}|\sum_{x,y}\Gamma_{xy}\delta_x^*\delta_y\alpha_x\beta_y| \\
&= \frac{K_G}{5}\|\Gamma\|_{\mathrm{op}},
\end{aligned}$$

where in the first line we have used that Γ is an adversary matrix; in the third line that $\|\Pi_{-f(x)}|\psi_x^t\rangle\|_2^2$ is the failure probability, so it is smaller than $1/100$; and in the fourth line we have used Grothendieck's inequality, Theorem 2.19.

Finally, we claim that

$$\mathcal{C}^s - \mathcal{C}^{s-1} \leq 2\max \|\Gamma \circ D_i\|_{\mathrm{op}}$$

for $s \in [t]$. Let U^s be the unitary in between the $(s-1)$th and the sth queries. Recall that O_x acts as the controlled version of $|i\rangle \to x_i|i\rangle$, so $O_x|0\rangle|i\rangle = |0\rangle|i\rangle$ and $O_x|1\rangle|i\rangle = x_i|1\rangle|i\rangle$. Let d be the extra dimensions of the algorithm as in Eq. (2.3). Then,

$$\begin{aligned}
\mathcal{C}^s - \mathcal{C}^{s-1} &\leq |\sum_{x,y}\Gamma_{xy}\delta_x^*\delta_y(\langle\psi_x^s|\psi_y^s\rangle - \langle\psi_x^{s-1}|\psi_y^{s-1}\rangle)| \\
&= |\sum_{x,y}\Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^{s-1}|(O_x\otimes \mathrm{Id}_d)\underbrace{U_s^\dagger U_s}_{\mathrm{Id}_{2nd}}(O_y\otimes\mathrm{Id}_d) - (\mathrm{Id}_{2nd})|\psi_y^{s-1}\rangle| \\
&= |\sum_{ij}\sum_{x,y}\Gamma_{xy}\delta_x^*\delta_y\langle\psi_x^{s-1}|(|1i\rangle\langle 1i|\otimes\mathrm{Id}_d)\underbrace{(x_iy_j-1)}_{-2(D_i)_{xy}}(|1j\rangle\langle 1j|\otimes\mathrm{Id}_d)|\psi_y^{s-1}\rangle| \\
&= 2|\sum_i\sum_{x,y}(\Gamma\circ D_i)_{xy}\delta_x^*\langle\psi_x^{s-1}|(|1i\rangle\otimes\mathrm{Id}_d)(\langle 1i|\otimes\mathrm{Id}_d)|\psi_y^{s-1}\rangle\delta_y|.
\end{aligned}$$

Now, if we define $\widetilde{\Gamma}$ as the block diagonal matrix with $\Gamma\circ D_i$ as diagonal blocks for $i\in[n]$, and G as the block diagonal matrix whose blocks are the Gram matrices of $\{\delta_x(\langle 1i|\otimes\mathrm{Id}_d)|\psi_x^{s-1}\rangle\}_x$, we have that

$$\begin{aligned}
\mathcal{C}^s - \mathcal{C}^{s-1} &= 2|\langle\widetilde{\Gamma}, G\rangle| \leq 2\|\widetilde{\Gamma}\|_{\mathrm{op}}\|G\|_{\mathrm{tr}} \\
&= 2\max_{i\in[n]}\|\Gamma\circ D_i\|_{\mathrm{op}}\sum_i \mathrm{tr}[\mathrm{Gram}(\{\delta_x(\langle 1i|\otimes\mathrm{Id}_d)|\psi_x^{s-1}\rangle\}_x)] \\
&= 2\max_{i\in[n]}\|\Gamma\circ D_i\|_{\mathrm{op}}\underbrace{\sum_x|\delta_x|^2}_{=\langle\delta,\delta\rangle=1}\underbrace{\sum_i\langle\psi_x^{s-1}|(|1i\rangle\langle 1i|\otimes\mathrm{Id}_d)|\psi_x^{s-1}\rangle}_{\leq\langle\psi_x^{s-1}|\psi_x^{s-1}\rangle=1} \\
&= 2\max_{i\in[n]}\|\Gamma\circ D_i\|_{\mathrm{op}}.
\end{aligned}$$

Putting everything together, it follows that

$$t \geq \frac{\mathcal{C}^t - \mathcal{C}^0}{2\max_{i\in[n]}\|\Gamma\circ D_i\|_{\mathrm{op}}} = \Omega\left(\frac{\|\Gamma\|_{\mathrm{op}}}{\max_{i\in[n]}\|\Gamma\circ D_i\|_{\mathrm{op}}}\right). \qquad \square$$

8.3 Average sensitivity lower bounds all reasonable complexity measures

We will show that the average sensitivity $\overline{s}(f)$ of a Boolean function lower bounds all the *reasonable* complexity measures of a Boolean function, which is the list of well-studied complexity measures considered in [ABDK$^+$21]. For total Boolean functions, all of these measures are polynomially related to classical and quantum query complexity. In particular, we will show that the average sensitivity lower bounds the spectral sensitivity of a Boolean function $\lambda(f)$. This is enough, as $\lambda(f)$ lower bounds, up to constant factors, all the reasonable complexity measures. From there, we can easily show that all reasonable complexity measures are $\Omega(n)$ for almost all Boolean functions, concisely reproving previous results such as $Q(f) = \Omega(n)$ for almost all total Boolean functions [Amb99, ABSdW13]. More formally, given $f : \{-1, 1\}^n \to \{-1, 1\}$ its average sensitivity is defined by

$$\overline{s}(f) := \mathbb{E}_x \sum_{i\in[n]} \left[\left(\frac{f(x) - f(x^{\oplus i})}{2} \right)^2 \right],$$

which also equals the sum of the influences, $\sum_{i\in[n]} \mathrm{Inf}_i^2[f]$. Its spectral sensitivity is given by

$$\lambda(f) := \sup_{\Gamma} \frac{\|\Gamma\|}{\max_{i\in[n]} \|\Gamma \circ D_i\|},$$

where the supremum runs over all adversary matrices that satisfy $\Gamma[x, y] = 0$ if the Hamming distance between x and y is not 1 (see Section 8.2 for the definitions of adversary matrix and D_i).

Proposition 8.5. *Let $f : \{-1, 1\}^n \to \{-1, 1\}$. Then, $\overline{s}(f) \leq \lambda(f)$. Furthermore, the inequality is tight for $f = \chi_{[n]}$.*

Proof. Let Γ be the adversary matrix such that $\Gamma_{x,y} = 1$ if the Hamming distance between x and y is exactly one and $f(x) \neq f(y)$ and 0 in the other case. Note that Γ can be written as

$$\Gamma_{x,y} = \delta_{x^{\oplus i},y}\delta_{f(x),f(x^{\oplus i})} = \delta_{x^{\oplus i},y} \left(\frac{f(x) - f(x^{\oplus i})}{2} \right)^2.$$

For this matrix, we can see that $\|\Gamma\|_{\mathrm{op}} \geq \overline{s}(f)$ and $\|\Gamma \circ D_i\|_{\mathrm{op}} = 1$ for all $i \in [n]$.

Indeed,

$$\begin{aligned}\|\Gamma\|_{\mathrm{op}} &\geq \sum_{x,y\in\{-1,1\}^n} \frac{1}{2^n}\Gamma_{x,y} = \sum_{x,y\in\{-1,1\}^n} \frac{1}{2^n}\delta_{x^{\oplus i},y}\left(\frac{f(x)-f(x^{\oplus i})}{2}\right)^2 \\ &= \sum_{i\in[n]} \mathbb{E}_x\left(\frac{f(x)-f(x^{\oplus i})}{2}\right)^2 = \overline{s}(f).\end{aligned}$$

On the other hand,

$$\begin{aligned}\|\Gamma\circ D_i\|_{\mathrm{op}} &= \sup_{\|u\|_2=1} \sum_{x\in\{-1,1\}^n} u_x u_{x^{\oplus i}}\delta_{f(x),f(x^{\oplus i})} \leq \sup_{\|u\|_2=1} \sum_{x\in\{-1,1\}^n} |u_x u_{x^{\oplus i}}| \\ &\leq \sup_{\|u\|_2=1} \|u\|_2^2 = 1.\end{aligned}$$

Finally, for $f=\chi_{[n]}$ we have that $\lambda(f)=\overline{s}(f)=n$. □

Corollary 8.6. *Let CM be any reasonable complexity measure. For a $1-\exp(-\exp(n))$ fraction of all Boolean functions $f:\{-1,1\}^n\to\{-1,1\}$ we have that $CM(f)=\Omega(n)$.*

Proof. If we pick a uniformly random Boolean function $f:\{-1,1\}^n\to\{-1,1\}$, then

$$\mathbb{E}_f\overline{s}(f) = \mathbb{E}_x\sum_i \mathbb{E}_f\frac{1-f(x)f(x^{\oplus i})}{2} = \mathbb{E}_x\sum_i\frac{1}{2} = \frac{n}{2}.$$

Now, note that changing the value of f on one input makes $\overline{s}(f)$ change at most $2n/2^n$. Then, by McDiarmid's inequality, Lemma 2.23, we have that

$$\Pr\left[\overline{s}(f)\leq\frac{n}{3}\right]\leq\exp(-\exp(n)).$$

Now, the statement follows from Proposition 8.5 and the fact that $\lambda(f)$ lower bounds, up to constant factors, all reasonable complexity measures [ABDK+21]. □

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