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Quantum computing, norms and polynomials

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Quantum computing, norms and polynomials

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Para vosotros: mamá, papá, Nacho y Raquel.

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