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Going Dutch: Jan Zaanen and the strange metals consortium

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Under the visionary guidance of Jan Zaanen, a group of researchers within The Netherlands formed a consortium in 2018 to explore the physics of strange metals; its core objective to determine whether strange metals represent a novel quantum critical phase and whether this phase can be described by holographic emergence principles. The consortium itself brought together theorists working on, or at the boundaries of, the Anti-de Sitter/ Conformal Field Theory (AdS/CFT) correspondence and experimentalists with collective expertise in optical conductivity, high-field magnetotransport, scanning tunneling spectroscopy (STS) and angle-resolved photoemission spectroscopy (ARPES). For reasons that will become apparent, the experimental team chose to perform their spectroscopy and transport studies on the same crystals of a single cuprate family – $(Pb,Bi)_2Sr_{2-x}La_xCuO_{6+\delta}$ (Bi2201). Holographic signatures were indeed found in the nodal self energies observed by ARPES. Optical conductivity and magnetotransport also found evidence for the dual character of the strange metal phase, manifest in STS as a real-space differentiation into superconducting and non-superconducting regions. The evolution of the superconducting state with temperature and doping was found to be at odds with a conventional BCS picture. This compendium of the output of that consortium serves as both a tribute to Jan's vision and perhaps, a signpost for how progress in such a complex field can be made through multiple experiments on the same material.

1. Introduction

In the seminal 2015 review of the high-T_c cuprates [1], for which Jan Zaanen was corresponding author, the overdoped regime was afforded a single paragraph, perhaps reflecting the view of the community at that time that the real mysteries of the cuprates lay elsewhere. Indeed, the wording of the paragraph hinted that beyond optimal doping and the pseudogap phase (but still inside the superconducting dome), a conventional, albeit correlated Fermi-liquid (FL) regime was recovered, at least at low temperatures. The strange metal – recognized as one of the most mysterious aspects of the cuprate phase diagram and deemed worthy of a full page of dedicated text in the review – was a distinct phase, characterized by a robust linear-in-temperature resistivity that extended up to the melting point of the relevant cuprate [2]. Viewed through a conventional lens, it would imply that the momentum-relaxing mean-free-path ℓ became significantly shorter than the interatomic distance – a clear violation of the standard notion of what constitutes metallic behaviour and compelling evidence for the complete absence of quasiparticles.

Over time, T-linear resistivity extending up to anomalously high temperatures became associated with 'bad' rather than 'strange' metallicity (though confusingly, these descriptors continue to be interchanged within the community). Earlier optical studies revealed that as the temperature is raised and ℓ becomes shorter, low-energy (Drude) spectral weight is progressively transferred to a higher energy scale, of order the Mott-Hubbard gap [3–5]. This in turn leads to a reduction in the dc conductivity at elevated temperatures and the preservation of a metallic-like dc resistivity even in the absence of a well-defined momentum relaxation rate [6].

While the bad metal refers to the anomalous transport behaviour manifest at high T, the strange metal (SM), at least in the cuprates, became synonymous with the T-linear component of the low-T

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resistivity that extends over a wide doping range between the end of the pseudogap regime ($p = p^*$) and the edge of the superconducting (SC) dome ($p = p_{sc}$) [7,8]. In this way, the SM and overdoped regimes became inextricably entwined. Dominant T-linear behaviour at low T across a finite doping range beyond p^* is clearly at odds with expectations for a recovery of a conventional FL at these doping values (as intimated in Ref [1]) or even of a characteristic quantum critical fan emanating from a single quantum critical point (QCP) at p^* [9]. Rather, it was more suggestive of some form of quantum critical 'phase', defined as the appearance of quantum critical-like behaviour over an extended region along the zero-temperature axis, rather than at a singular QCP, the region in question being populated by a dominant non-FL sector.

A wide variety of models, based on both extensions of conventional FL (Boltzmann transport) theory and non-FL but still field-theoretic approaches, have since been explored [10–19] but as yet, a widely accepted description of the SM state has failed to materialize. This striking failure ultimately led to the notion that strange metals are fundamentally new states of matter. Jan Zaanen was at the forefront of this realization, advocating the idea that strange metals were in fact finite density fermion systems controlled by long-range quantum entanglement [20] – a central theme of his lecture notes [21]. Experimental evidence for this may be beginning to emerge [22].

A new window into the SM phenomenology arose from the realization that the mathematical machinery associated with theories of quantum gravity, more specifically the so-called AdS/CFT correspondence, provides a genuine framework to describe interacting electron systems at finite density [20,23,24]. The correspondence implies that certain correlated systems have an equivalent *holographic* description in terms of an extension of Einstein's theory of general relativity in one more dimension. Remarkably, when applied to finite density systems, holography displays many of the gross physical properties of strange metals. Moreover, with the established equivalence between the low-T fixed point of the Sachdev-Ye-Kitaev (SYK) Model and these holographic methods [25–27], we can now confirm that these are indeed long-range entangled systems without quasiparticles. Zaanen was among a select group of condensed matter theorists who recognised at that time the potential breakthrough in our understanding that could emerge from this holographic approach [23] and in 2016, he reached out to the Dutch strongly correlated electron community and persuaded them to form a consortium with the goal of determining whether the strange metal is a genuinely novel long-range entangled quantum state of matter described in detail by holographic emergence principles.

The experimental arm of the consortium comprised experts in angle-resolved photoemission spectroscopy (ARPES), scanning tunnelling spectroscopy (STS), optical conductivity and high-field magnetotransport. They were joined by theorists working at the interface between holography and quantum field theoretic approaches and a programme of research was duly conceived around an accompanying grant proposal. The combination of experimental techniques was both accidental, in the sense that it required the relevant experts to be working concurrently in what is comparatively a small community in The Netherlands, but also deliberate, having deemed it essential from the outset to probe simultaneously both the evolution of the single-particle and collective properties across the SM regime. Crucial for the program was the realization that the single electron properties of holographic strange metals, as measured by ARPES and STS, are *a-priori* detached from the collective properties such as the conductivity; the single electron spectra parameterized as FL quasiparticles that decay into a quantum critical continuum – defined here as a spectrum of multiparticle (entangled) states whose spectral function is characterised by, e.g. a power-law, rather than discrete quasiparticle peaks – while the collective properties were determined almost exclusively by the quantum critical continuum itself.

In choosing a system for such a comprehensive study, it needed to satisfy a number of key criteria. Firstly, it needed to cover the relevant doping range from optimal doping $p = p_{opt}$ (≈ 0.16) to p_{sc} (≈ 0.27 – 0.30), and preferably beyond. Of all the cuprate families, only $La_{2-x}Sr_xCuO_4$ (LSCO), $Tl_2Ba_2CuO_{6+\delta}$ (Tl2201) and Bi2201 can be hole-doped beyond p_{sc}. These systems also have the advantage of being single-layered and hence, their electronic structure is relatively simple, comprising as it does a single warped cylinder.

Secondly, the system needed to be accessible to the whole gamut of experimental techniques proffered by the consortium. While ARPES measurements have been performed on LSCO [28–30] and Tl2201 [31], it has proved much harder to prepare cleaved surfaces for systematic STS studies across the doping series of both these families, with only sporadic reports in LSCO [32] and none to date in Tl2201. By contrast, with Bi2201, like its bilayer cousin $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi2212), it is possible, in principle, to explore the evolution of the electronic state across the entire SM regime with both ARPES and STS. Nevertheless, before the advent of the present programme, spectroscopic studies that extended to the edge of the SC dome in Bi2201 were rather limited. The clear choice was therefore Bi2201, and a distinct added value of this consortium was the opportunity to share crystals (i.e. to 'go Dutch') and to perform multiple measurements on the same single set of Bi2201 samples.

Over the past 6 years, the consortium, in collaboration with partners across Europe and Japan, has generated a number of new results that reveal further aspects of the SM phenomenology, its link to high-temperature superconductivity and the efficacy of (semi-)holography to capture quantitatively key experimental findings. This article is essentially a status report of the output of that consortium, which we hereby dedicate to Jan's memory.

2. Single- vs collective-particle dichotomy

The anchor of this programme was the juxtaposition of two different types of experimental probes to approach the SM phase, namely the 'single-particle' short-distance/short-timescale probes, such as ARPES and STS, and the 'two-particle' long-distance/long-time probes, such as magnetotransport and (optical) conductivity. In standard theoretical approaches – i.e., in standard physics models built on long-lived weakly interacting quasiparticles – these probes are connected and in principle it should be possible to obtain dynamic 'two-particle' properties by combining single-particle dispersive properties for two particles (band velocities etc..) with two-to-two particle state scattering rates within a Boltzmann approach.

It is in fact only within such a Boltzmann quasiparticle theory that the long-distance probes are 'two-particle' properties. Early computations in holography showed that its predictions depart from this notion in a spectacular fashion: the single-particle states can still be parameterized as quasiparticle-like states decaying into a quantum critical continuum, but the pull of the continuum is so strong that their lifetimes can be anomalously short and characterized by a non-FL power-law self-energy $\Sigma(\omega) \propto \omega^{2\alpha}$ with $\alpha \neq 1$ [33–35]. The crucial difference with standard theory is that the two-particle or dynamic response is completely determined by the quantum critical continuum, and not by the Boltzmann summation of interactions of these quasiparticle like excitations. The link between macroscopic transport and the microscopic quasiparticle excitations probed in ARPES or STS experiments is ultimately lost.

What served as a significant second prompt for this programme was a surprising preprint by Reber et al. in 2015 indicating that high-precision ARPES indeed detects such anomalous power-law self-energies at nodal points in the Brillouin zone of the cuprates [36] (see caption of Fig. 1 for details). A prime goal of this programme was thus to verify this finding and to determine the extent of its applicability. An important detail to this story is that several of the response functions in the cuprates show signs of energy-temperature ($\hbar\omega/k_BT$) scaling. The most notable one is the mid-infrared (mid-IR) regime in the optical conductivity [37]. The absence of a simultaneous rescaling of the Fermi-momentum gave rise to the concept of *local quantum criticality* [38]. Self-energies in Fermi-surface excitations that behave as a simple power-law in frequency with no momentum-dependence are a theoretical example of this notion.

Our ARPES experiments performed on Bi2201 indeed confirmed the power-law scaling in the nodal self-energies observed originally in Bi2212, and established its validity across the overdoped regime of the phase diagram [39]. The results are summarised in Figure 1, where the red dots and plusses indicate the doping dependent exponent α (see Figure caption for its definition) for Bi2201 and Bi2212, respectively. In addition, the new experiments uncovered a subtle asymmetry in the lineshapes along the nodal *k*-space direction, signaling an additional momentum dependence in the self-energy. While the power-law scaling proposed by Reber was purely phenomenological, holographic models had previously shown that there was a weak remnant of spatial structure. This emerges as a momentum-dependent exponent $\alpha(k) = \alpha_0 + \alpha_1(k - k_F) + \ldots$ and detailed analysis revealed that including this captures the experimentally observed asymmetry [39,40]. Within this holographic framework, the doping dependence of the exponent is governed by a single, dimensionless parameter which suggests that the SM phase comprises of a line of critical points, where at each doping level a unique ground state exists with its own ($\hbar\omega/k_BT$) scaling exponent. This is, in essence, one of the key features of the quantum critical phase that characterises the cuprate strange metal.

3. Optical response within the SM regime – one or two components?

An interesting experimental bridge between the macroscopic and microscopic regime is provided by the optical conductivity. At low frequency, and especially in the dc limit, the response can be understood in terms of relaxation of the collective momentum current. Here the response is determined from hydrodynamic principles without reference

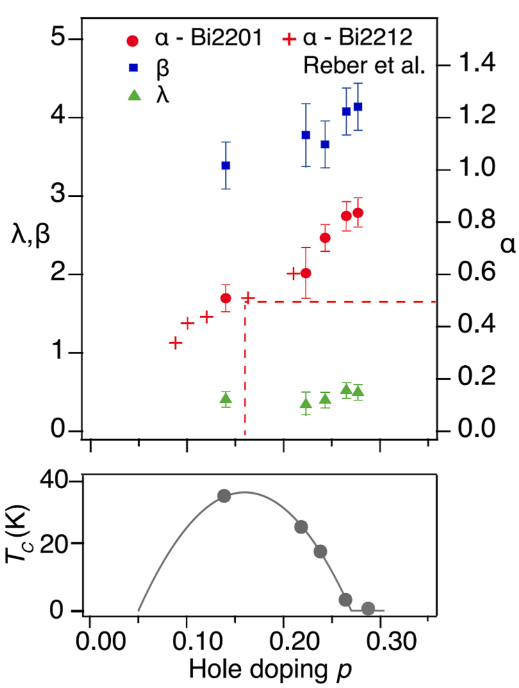

Fig 1. Upper panel: Doping dependence of the various parameters of the power-law scaling analysis. In this analysis, the imaginary part of the self-energy extracted from ARPES MDC widths was defined by the following expression: $\Gamma = 2\mathrm{Im}\Sigma''(\omega, T)/v_F = G_0(\omega, T) + \lambda[(\hbar\omega)^2 + (\beta k_BT)^2]^{\alpha}/(\hbar\omega_N)^{2\alpha-1}$. While the effective coupling λ and temperature prefactor β vary only slightly with doping, the exponent α shows a strong doping dependence. Here, ω_N is an energy scale (= 0.5 eV for all dopings) that normalises λ and $G_0(\omega, T)$ is an extra term, combining a self-energy contribution from disorder and electron-phonon coupling. **Lower panel:** Doping dependence of T_c and the samples measured by the ARPES and STS teams. Adapted from Ref. [39].

to the nature of the charge carriers. At the same time, in the mid-IR and higher frequency ranges, one starts to probe more microscopic dynamics, where signatures of strong correlations and long-range entanglement should be more manifest.

The earlier 2003 discovery of a power-law regime in the mid-IR of the optical conductivity of the cuprates [37], in which Zaanen also played a crucial role, emphasised the dual character of the optical response. The two component description runs counter to the prevalent narrative that below 1 eV the data could be fully understood from dressed quasiparticle excitations [41,42]. This pointed to a simple empirical question: is the optical response characterized by one or two separate components?

Reconciling these opposing worldviews remains an outstanding question, as evidenced by the near simultaneous posting of three optical papers on the preprint server with different interpretations of nominally the same experimental data [43–45]. With experiments spanning the entire SC dome and beyond p_{sc}, the optical data we gathered on Bi2201 [43] offers a unique view on the evolution of the low energy electron dynamics. A key observation is that empirically, $\sigma_1(\omega) \propto \omega^{-2}$ for small ω, T suggesting the presence of a Drude response with an ω-independent momentum relaxation rate. At higher energy, a second component, dubbed $\sigma_{BC}(\omega, T)$, takes the form of a generalized interband transition with an asymptotic power-law behaviour $\sigma_{BC}(\omega) \sim \omega^{-\beta}$ at high energy. Note that the exponent β that appears here is not the power-law exponent in the self-energies mentioned above. Ref. [44] interpreted their spectra with two components, but assigned a FL phenomenology to the 'Drude' component. In Ref. [45], data measured on LSCO was described using a single component, computed with the Kubo formula and a model for the self-energy. The collective response is supported by the full critical spectrum and not just from scattering of the quasiparticle peaks near k_F. These authors argued for a single component interpretation and suggested that there was only an *apparent* power-law dependence. Although there is a difference between these two works in estimating the interband contribution (ε_∞), it appears that both interpretations of the data could be valid.

In the following, we propose a way forward to distinguish between a single- and two-particle interpretation of the optical conductivity of strange metals. We do so by modelling the expected temperature evolution of the full conductivity at elevated temperatures for two cases: (i) a continuously evolving Drude dc response combined with a generalised interband transition and (ii) the evolution of quasiparticles coupled to a spectrum of bosonic excitations.

In Figure 2(a), we obtain the mid-IR response $\sigma_{BC}(\omega, T)$ by subtracting the Drude response, $\sigma_D(\omega, T)$, from $\sigma_1(\omega, T)$. In Ref. [44], a strongly T-dependent momentum relaxation rate (Γ_D) was found for the Drude piece, which is shown in the inset of Fig. 2(a). Assuming simple power-law behaviour, we can extrapolate Γ_D to high temperature and estimate the Drude response at 600 K. Since $\sigma_{BC}(\omega, T)$ shown in Fig. 2(a) is only weakly temperature dependent, we can construct $\sigma_1(\omega, 600\ \mathrm{K})$ by assuming $\sigma_{BC}(\omega, T)$ to be T-independent. The dark red symbols in Fig. 2 (a) then represent the full conductivity at 600 K, obtained by summing $\sigma_D(\omega, 600\ \mathrm{K})$ and $\sigma_{BC}(\omega, 300\ \mathrm{K})$. For a system of quasiparticles coupled to a bosonic spectrum, we use the Kubo formula and the bosonic spectrum $\alpha^2F(\omega)$ (inset in panel 2(b); see Ref. [42]) to compute $\sigma_1(\omega, T)$. Here the temperature evolution is fully governed by the Bose and Fermi factors appearing in the Kubo formula. Figure 2(b) shows the computed optical conductivity at several temperatures up to 600 K.

Comparing the data in panel 2(a) and 2(b), we can now contrast between the one- and two-component interpretation. The continuously increasing momentum relaxation rate results in the collapse of the low energy Drude response. At high enough temperature, this should reveal the maximum associated with the mid-IR band as Fig. 2(a) shows. This is distinctly different from the Kubo prediction where the two-component structure only emerges at low T. At elevated temperature, the response becomes increasingly dominated by a single component that resembles a single Drude form (Fig. 2(b), 600 K calculation). While not yet

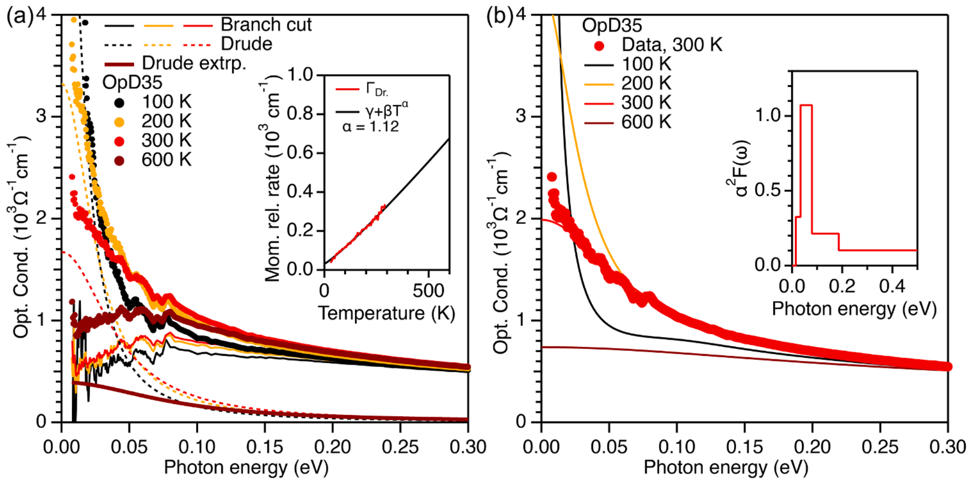

Fig 2. (a): Decomposition of experimental data (symbols) in optimally doped Bi2201 in a Drude (dashed line) and branch cut mid-IR response (solid line) for 100 K, 200 K and 300 K. Also shown is the estimated $\boldsymbol{\sigma_D}(\boldsymbol{\omega}, 600\ \boldsymbol{K})$, based on extrapolated Drude parameters. The 600 K "data" curve is obtained from $\boldsymbol{\sigma}(\boldsymbol{\omega}, 600\boldsymbol{K}) = \boldsymbol{\sigma_D}(\boldsymbol{\omega}, 600\ \boldsymbol{K}) + \boldsymbol{\sigma_{BC}}(\boldsymbol{\omega}, 300\ \boldsymbol{K})$. The inset shows the extrapolated Drude momentum relaxation rate to elevated temperature. (b): The optimized $\alpha^2F(\omega)$ at 300 K (see inset) is used to compute $\sigma(\omega, T)$ at various temperatures (solid lines).

conclusive, the observation of a maximum at elevated temperature across the overdoped side of the phase diagram could thus be a key signature of two components and importantly of a 'running exponent' in the generalized interband transition as proposed in Ref. [41]. Confirmation of this mid-IR maximum in the optical response would provide important new insight into the physics of the SM phase.

The simultaneous theoretical effort of this proposal tried to address the same question of dissecting the mechanisms underlying the optical response. In such theoretical modeling, the Drude response is easy: it is tied to weak momentum relaxation and thus can be captured in a fully phenomenological hydrodynamical description. For strongly coupled states of electronic matter with an effective momentum-preserving mean free path that is shorter than the lattice spacing, such a hydrodynamic approach should be valid. A surprise is that in the presence of a lattice, charged hydrodynamics possessses such a mid-IR band because of hydrodynamical rather than electronic Umklapp processes [46].[1] Further exploration of the optical response with its ability to probe various energy scales is likely to remain one of the key steps in enabling the community to distinguish between normal, bad and strange metals.

4. Weak vs strong momentum relaxation and Planckian dissipation

One such exploration is related to the origin of the mid-IR peak in the optical conductivity. The Umklapp hydrodynamics explanation is specifically tied to a weakly-coupled lattice. The other requires spectral weight depletion in the presence of a second component that is independent of the spatial structure. Strong lattice potentials or strong disorder could give rise to this second component (a strong lattice potential is in fact theoretically indistinguishable from strong disorder). The same theoretical effort that proffered an explanation of the mid-IR peak in terms of lattice hydrodynamics also confirmed earlier findings [47] that in the strong momentum-relaxation regime, the thermal diffusivity saturates at a quantum bound, known as 'Planckian dissipation' that was postulated long before by Zaanen [48]. Originally this was hoped to apply to charge transport as a universal explanation of the linear-in-T dc resistivity. This turns out to not be the case. In the regime of strong momentum relaxation, the mechanisms of thermal transport and electrical transport become decoupled at low temperatures.

Such Planckian thermal transport is seen in cuprates [20,49], but this raises more questions than it answers. The notable presence of a Drude peak at low and intermediate temperatures does suggest, though, that one is in a weakly to moderate strength momentum-relaxation regime. Theoretically, the Planckian thermal diffusivity, however, is only expected to emerge close to the strong relaxation regime. The theoretical holographic modeling is at the same time still quite detached from details of momentum-relaxation in actual samples and their composition and specifically the role of disorder.

5. The role of disorder and the breakdown of superconductivity

This previous statement becomes abundantly clear when these models are compared with input from other experimental approaches. One such technique is scanning tunneling spectroscopy that has the distinct advantage of probing the local landscape. The STS group within the consortium measured the same concentrations of Bi2201 that were studied by ARPES (see bottom panel of Fig. 1) [50]. We demonstrated that superconductivity does not, as perhaps expected, break down because of a vanishing pairing interaction. This can be seen by the fact that the gap persists over the whole doping range (see panel e) of Fig. 3), even beyond the superconducting dome. Instead, superconductivity breaks down via puddle formation driven by gap filling. Calculations by Zaanen and his student Miguel Sulangi showed that an inverse proximity effect can lead to strong filling, even at the Fermi level, if superconducting puddles are immersed in a metallic matrix [51].

This underscores (likely intrinsic) disorder as a key driver of the physics of these materials and certainly as the driver in the breakdown of superconductivity. In addition, Zaanen's calculations showed that the relation between gap and filling is exactly the opposite to what one would expect from BCS theory (more pairs lead to a larger gap), emphasizing the non-BCS nature of the breakdown.

An observation that is still part of our ongoing investigation is that disorder in the van-Hove singularity energy and the broadness of the quasiparticle interference (QPI) peaks outside of the gap seem difficult to reconcile with the near-Lorentzian quasiparticle-like peaks seen with photoemission (note the different behaviour inside the gap in Bi2212, where sharp octet peaks are indeed visible). The discrepancy we measure here in Bi2201 is surprising; in a weakly correlated 'drosophila' test case, ARPES and QPI agree well [58]. There might be a way out of the discrepancy: as Zaanen and others showed, there is additional information in QPI data beyond Fermiology. An example is the shape of the scatterers in Bi2212, where Zaanen's calculations showed that this

[1] Note that this is a distinct Umklapp hydrodynamic effect that differs from a pinned Goldstone mode as seen in charge density waves at low T [77–79].

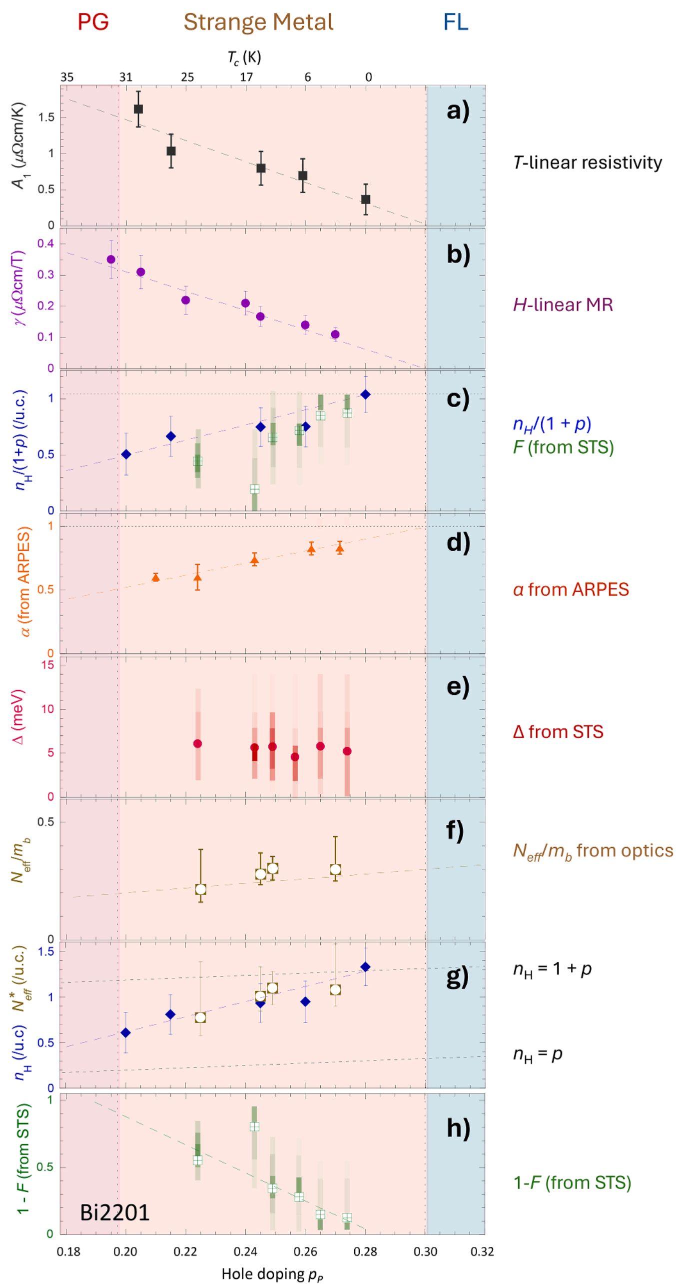

Fig 3. Summary of results obtained on Bi2201 by the strange metal consortium during the course of this program, plotted as a function of hole doping p_P, as deduced from T_c and the Presland formula [52] with $T_c^{max} = 36$ K. (a): Coefficient (A_1) of the low-T-linear resistivity [53]. (b) Coefficient (γ) of the high-field H-linear MR deduced from the saturated value of $d\rho/dH$ as $T \to 0$ [54]. (c) (Blue diamonds) Ratio of the Hall number (n_H) to the expected carrier density $(1 + p)$ [53]. (Crossed squares) Fraction (F) of residual density of states at zero energy deduced from STS [50]. (d) Exponent α deduced from nodal ARPES MDCs using the power-law-liquid formalism [39] (see caption of Figure 1 for definition). (e) Superconducting gap energy (Δ) deduced from STS measurements. The bars indicate the spread in Δ over a 400 Å x 400 Å window [50]. (f) Effective Drude weight (N_{eff}/m_b) deduced from fitting the real part of the low-frequency optical conductivity $\sigma_1(\omega)$ to a Drude response [43]. (g) Comparison of the effective carrier densities deduced from high-field Hall effect (n_H, blue diamonds) [60] and $\sigma_1(\omega)$ studies ($N^*_{eff} = 3.6\ N_{eff}/m_b$) [43]. (h) The fraction (1-F) of states that are gapped out below T_c, as deduced from STS [50]. This parameter serves as an estimate of the superfluid density n_s – essentially the ratio of carriers that enter into the SC state. A similar suppression of n_s has previously been reported in Tl2201 [55,56] and LSCO [57].

shape is indeed far from point-like, more like the slowly varying Coulomb potential stemming from the charge transfer layer [51].

Could it be that what we see in QPI in overdoped Bi2201 are echoes from strange electrons? This is a question Zaanen would certainly have loved to discuss for hours during a smoking break. Or he would have known the answer instantly, using his signature mixing of down-to-earth knowledge of chemistry, his encyclopaedic knowledge of the cuprate literature and his boundless creativity. Whatever the outcome, his advice would likely be 'to stop following the herd'. In the Dutch SM consortium, we have tried to do just that.

6. Dual character of the (magneto)transport

Prior to the inception of the Dutch SM consortium, evidence for extended criticality in the overdoped region was limited to the form of the in-plane dc resistivity – $\rho_{ab}(T) = \rho_0 + A_1T + O(T^2)$ – and in particular the omnipresent T-linear component at low-T whose coefficient A_1 decreases as doping increases [6] but which crucially, does not vanish until all vestiges of superconductivity have been extinguished [59]. While this phenomenology challenged the notion of a singular quantum critical point in cuprates [8], it was not perhaps enough on its own, to convince the community that a non-FL ground state persisted beyond p^* [1]. Moreover, the fact that high fields were required to suppress the superconductivity in order to reveal the form of the low-T resistivity led to some uncertainty as to whether the magnetic field itself was modifying the ground state or whether extrapolation of the magnetoresistance (MR) to lower fields influenced the form of the effective zero-field resistivity. Given that superconductivity and strange metallicity appeared to go hand-in-hand, such extrapolation was unavoidable and thus some uncertainty prevailed.

In the course of our programme, however, several new aspects of the SM transport phenomenology were revealed that broadened the characterisation of strange metallicity beyond dc resistivity to include both the Hall coefficient R_H and the MR itself. In principle, the low-T Hall number $n_H = 1/(R_He)$ can provide information about the evolution of the planar carrier density n_{pl} across the phase diagram. Badoux *et al.* had reported a sharp rise in n_H from p (the number of doped holes per Cu atom) to $1 + p$ (corresponding to the full Luttinger volume) across p^* in $YBa_2Cu_3O_{7-\delta}$ (Y123) [60] that was interpreted as evidence for a quantum phase transition associated with the closing of the pseudogap at p^* [61, 62]. In Y123, however, the assumed relationship $n_{pl} = n_H$ is complicated by the shorting effect of the quasi-one-dimensional conducting CuO chains oriented along the b-axis. In the single-layered cuprates Bi2201 and $Tl_2Ba_2CuO_{6+\delta}$ (Tl2201), where these CuO chains are absent, n_H was found to exhibit a much more gradual p to $1 + p$ crossover extending from close to optimal doping p_{opt} to the edge of the SC dome at $p = p_{sc}$ [53]. Intriguingly, the field dependence of the Hall resistivity $\rho_{yx}(H)$ (see Fig. 3c) could be captured by a Boltzmann transport model assuming an anisotropic scattering rate, of the form previously determined in overdoped Tl2201 [63] suggesting that the number density inferred from n_H was associated with the normal (FL-like) carriers present within the SM regime. The departure from the expected Luttinger volume $(1 + p)$ below p_{sc}, however, was unexpected.

Further insights into the nature of the SM regime were subsequently provided by a series of high-field MR studies on Bi2201 and Tl2201 [54, 64]. In conventional single-band metals, the fractional MR $\Delta\rho(H,T)/\rho(0,T)$ is quadratic at low field strengths (more precisely, at low values of $\omega_c\tau$, where ω_c is the cyclotron frequency and τ the transport lifetime) and obeys Kohler scaling in which $\Delta\rho(H,T)/\rho(0,T) \propto (H/\rho(0,T))^2$. This contrasts with the MR scaling found in the iron-based superconductors P-doped $BaFe_2As_2$ [65] and S-doped FeSe [66] tuned to their putative QCP, where the MR follows an unusual quadrature form $\Delta\rho(H,T) \propto ((\alpha k_BT)^2 + (\gamma\mu_0H)^2)^{1/2}$ which displays a robust H-linear MR at high fields (with a slope of magnitude γ) and exhibits H/T scaling rather than the conventional $H/\rho(0,T)$ scaling.

Similar 'quantum critical' scaling was also observed in both Bi2201 and Tl2201 [64], although just as with the T-linear resistivity, this scaling was not confined to a (putative) QCP at p^*, but was found to persist across the entire SM regime. Furthermore, it was not possible to model $\Delta\rho(H,T)$ with any variant of Boltzmann theory, and certainly not the variant that could account for the Hall response. A similar magnitude of the longitudinal MR (Lorentz-force-free configuration) also suggested that the MR response was not orbital in character [64]. Moreover, the marked similarities between the evolution of the magnetotransport properties of Bi2201 and Tl2201 revealed a universality in the charge response that does not depend on details of the Fermiology of the various cuprate families nor on the degree of disorder or inhomogeneity (note that the residual resistivity ρ_0 in Bi2201 is an order of magnitude larger than in Tl2201).

This dichotomy in the magnetotransport properties of overdoped cuprates – a Hall response resembling that seen in conventional metals (albeit in the presence of anisotropic scattering) and an MR response resembling that seen in the non-FL regime of quantum critical metals – is most naturally interpreted as arising from two distinct types of charge carriers, though whether these are differentiated in real-space (i.e. as patches) or in k-space (as with the nodal-antinodal dichotomy) was not clear from those initial experiments. Support for real-space differentiation was provided through the observation of a correlation between the slopes (coefficients) of the T-linear (A_1) and H-linear (γ) resistivities across the SM regime [54]. As with the form and T-dependence of the MR, it proved difficult to account for such a correlation within a Boltzmann (momentum-differentiated) approach, even one incorporating additive conductivities. Moreover, as shown by our STS studies [50], real-space patchiness has been directly imaged for all dopings within the SM regime. Accordingly, we applied effective medium theory (for which the granularity of the model is scale-invariant) to a network of distinct FL and non-FL patches. The correlation between A_1 and γ, coupled with the tendency for the magnetotransport to interpolate smoothly from pure FL-like behaviour (quadratic resistivity and negligible MR) at high dopings to pure SM behaviour (T-linear resistivity and large MR) near p^* led us to propose a scenario in which the fraction f of the SM component decreases uniformly from around unity to 0 across the SM regime. How such a scenario might relate to the rest of the consortium's research will be discussed in the following section.

7. Summary and discussion

A summary of our broad experimental investigation of Bi2201 is presented in Figure 3. The totality of the data highlight several aspects of the overdoped regime that are in conflict with expectations for a FL metal hosting a BCS-like superconducting state. Panel a) shows that, as with LSCO and Tl2201, the low-T resistivity is dominated by a non-FL T-linear component that only vanishes beyond a doping level of around $p = 0.31$ ($> p_{sc}$). Likewise, panel d) reveals that the exponent α of the power-law liquid analysis of the nodal spectral response in ARPES is always less than the value of unity that would reflect a recovery of a FL. Indeed, extrapolation of the linear dependence of $\alpha(p)$ suggests that α will only reach unity at around the same doping level ($p = 0.31$). These features of the collective- and single-particle responses are likely to reflect the nature of the quantum critical phase and more specifically, the degree of scattering of the quasiparticles from the quantum critical continuum. The corresponding scattering rates may well have a momentum dependence to them, though this will require additional investigation.

By contrast, the inability to describe the in-plane MR response in overdoped Bi2201 (or Tl2201) with any variant of Boltzmann transport theory implies that the charge response in a magnetic field has a distinct character, possibly reflecting the dynamics of the quantum critical continuum itself. The correlation between γ – the H-linear slope of the MR $\Delta\rho(H) \approx \gamma\mu_B\mu_0 H$ (panel b)) – and A_1 – the coefficient of the T-linear resistivity $\Delta\rho(T) \approx A_1 T$ (panel a)) – coupled with the fact that both H and T appear in quadrature in $\Delta\rho(H, T)$ informs us that the H- and T-linear resistivities share a common origin. The modelling presented in Ref. [54] assumes that both γ and A_1 are proportional to the fraction of 'Planckian' carriers, i.e. those carriers whose relaxation rate is pinned at the Planckian bound ($\hbar/\tau \sim \pi k_B T$), that are present at any doping level. It has been argued that the transport scattering rate in iron-based superconductors such as $BaFe_2(As_{1-x}P_x)_2$ [67] and $FeSe_{1-x}S_x$ [68] at their respective QCPs is also tied to the Planckian bound, the inference being that all carriers are subject to the same dissipative process. In the cuprates, the quantum critical *phase* is thus characterised by a variable concentration of such critical carriers across the SM regime.

The anti-correlation between $n_H(0)$ and $n_s(0)$ across the SM regime shown in panels g) and h) is particularly striking and non-intuitive. In a clean BCS superconductor, all carriers are expected to participate in the condensate. The prevailing explanation for the reduction in $n_s(0)$ with overdoping [55–57] is pair breaking induced by disorder [69,70] coupled with a rapidly diminishing pairing amplitude Δ. According to our STS study, however, the spread in Δ at each doping level is essentially the same (see panel e)); only the mean value is shifted downwards with doping though crucially, never vanishes. Even in nominally non-SC samples, gap features were observed extending up to a comparable scale to those found at optimal doping. Moreover, the dirty d-wave scenario predicts a rapid reduction in T_c as the normal state scattering rate increases [71] in marked contrast with recent findings [72]. Bi2201 typically exhibits a large residual resistivity $\rho_0 \approx 100$ $\mu\Omega$cm that equates to a normal state transport scattering rate $\Gamma_{tr} \approx 200$ K. According to dirty d-wave theory, scattering rates that exceed $T_c(0)$ – the highest T_c value that can be attained in the absence of disorder – should result in a complete suppression of superconductivity, yet in our Bi2201 crystals, T_c was found to be constant for a spread in ρ_0 between 40 and 290 $\mu\Omega$cm, equivalent to a variation in Γ_{tr} from 70 K to 500 K [72].

A second striking correlation, shown in panel c), is also found between $n_H(0)$ and F – the fraction of residual (zero-energy) states in the SC state deduced from our STS study [50] and confirmed in a subsequent study by another group [73]. The quantitative agreement between the two parameters $n_H(0)$ and F in the same cuprate family (Bi2201) suggests that the states contributing to $n_H(0)$ in the field-induced normal state do not contribute to the SC condensate [74]. Conversely, those states that emerge from the SC condensate upon application of a large magnetic field do not appear to exhibit an intrinsic Hall response of their own, due to them being localized, bosonic (particle-hole symmetric) or, if still fermionic, fundamentally incoherent. Projecting this onto Zaanen's conjecture about the nature of the two components within the SM regime, this raises the tantalizing prospect that the SC condensate itself emerges from the quantum critical continuum. Surely, this prospect will motivate further concerted investigations on the same cuprate family, be that Bi2201 or some other family, in order to answer this question categorically.

In closing, we highlight here one outstanding issue in Bi2201, namely the striking inconsistency that exists in the effective carrier density in Bi2201 deduced by transport, optics and ARPES experiments. The difficulty in determining the precise doping level in Bi2201 is well known and is due in part to its complex stoichiometry and the role of the interstitial oxygen, as well as the multiple valence states that certain constituents like bismuth possess. This is compounded by the fact that different p values have been inferred from different experimental techniques. Our combined studies have uncovered new insights into this controversy. Firstly, we observed that the magnitude $\rho_{ab}(T)$ of Bi2201 and LSCO crystals with comparable T_c values (normalised to the spacing between the CuO_2 planes) as well as its T-dependence are essentially identical [75], implying that their carrier densities are also the same. In LSCO, p is believed to be tied to x (the Sr content) and the $T_c(x)$ dome follows the so-called Presland formula [52]. Since the $T_c^{max} = 37 \pm 1$ K in both LSCO and Bi2201, we have used the T_c value for each Bi2201 crystal together with the Presland formula to specify p ($= p_P$) in Figure 3.

Our complementary ARPES study indicated the presence of a single Fermi cylinder whose area corresponds to a Luttinger count p_L that is invariably larger than p_p [75]. Taking into account the strong (6–10%) asymmetry in the nodal Fermi wave vectors in orthorhombic Bi2201 [76], we were able to quantify this difference as a constant offset across the SC dome; $p_L \approx p_p + 0.1$. The origin of this offset has not yet been identified, but this is only part of the story. The most striking aspect of the carrier density controversy in Bi2201 is that while ARPES and STS (through QPI) reveal a large Fermi cylinder containing $1 + p_L$ holes, the carrier density deduced from the (high-field) Hall effect (panel c)) [53] and optical conductivity (panel f)) [43] is found to be consistently lower. This discrepancy is present across the entire SM regime and is largest near p^* (≈ 0.20). Note that this discrepancy appears *outside* of the pseudogap regime and is thus a property of the strange metal, not of the pseudogapped metal. Understanding the origin of this discrepancy will likely be key to unravelling the dichotomy in the single-particle and collective properties of the cuprates as well as the origin of the dual character of the cuprate strange metal and further studies to resolve these outstanding issues are ongoing. Indeed, Jan Zaanen would have expected nothing less.

CRediT authorship contribution statement

M.P. Allan: Writing – review & editing, Validation, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **M.S. Golden:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **N.E. Hussey:** Writing – review & editing, Writing – original draft, Validation, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **K. Schalm:** Writing – review & editing, Writing – original draft, Visualization, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **H.T.C. Stoof:** Writing – review & editing, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **E. van Heumen:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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