Tautological relations and double ramification cycles with spin parity

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Stellingen

Behorende bij het proefschrift
Tautological relations and double ramification cycles with spin parity

1. The $(g-i)$-th Chern class of the Hodge bundle $\overline{\mathcal{H}}_{g,n}$, denoted by λ_{g-i}, can be expressed in terms of decorated strata classes with at most i loops (Theorem 2.1.2).

2. The parity of the dimension of the space of sections of a spin structure remains constant under degenerations. In particular, this result yields a decomposition of the moduli space of spin structures $\mathcal{M}_{g,n}^{1/2}$ according to the parity of the dimension of the space of sections at each point.

3. Points on stacks may have non-trivial automorphisms and the degree of a finite flat morphism $f\colon \mathcal{X} \to \mathcal{Y}$ takes them into account. This is why, even though there are 2^{2g} spin structures over a smooth curve C of genus g, the degree of the morphism

$$\epsilon\colon \mathcal{M}_{g,n}^{1/2} \to \mathcal{M}_{g,n}$$

is 2^{2g-1}. In particular, every spin structure possesses a non-trivial automorphism given by multiplication by -1 in its fibers.

4. Compared to the case of $\overline{\mathcal{M}}_{g,n}$, the parametrization of the boundary strata of $\overline{\mathcal{M}}_{g,n}^{1/2}$ requires the additional combinatorial data of 2-weightings.

5. Let x be a geometric point of $\overline{\mathcal{M}}_{g,n}$ whose dual graph is given by Γ, and let

$$\epsilon\colon \overline{\mathcal{M}}_{g,n}^{1/2} \to \overline{\mathcal{M}}_{g,n}$$

denote the morphism forgetting the spin structure. In the fiber of ϵ over x, restricted to those points whose associated 2-weightings on Γ admit an even value, there are an an equal number of even and odd spin structures (Proposition 3.1.16).

6. Given a smooth curve C, a line bundle $\mathcal{L}$ on C is trivial if and only if one of its roots is trivial. Therefore, over the locus of smooth curves, the double ramification locus of the universal spin structure $\mathrm{DRL}^{1/2}$ maps bijectively

onto DRL in $\mathcal{M}_{g,n}$. However, this morphism is not an isomorphism of stacks, as it has a μ_2-gerbe structure (Corollary 3.2.40).

7. As in the non-spin case, the spin double ramification cycle $\mathsf{DR}_g^{\pm}(a,k)$ admits both a star-graph expression and a spin analogue of Pixton's formula (Theorems 3.4.5 and 3.5.12).

8. Both the double ramification locus and the double ramification cycle naturally live on log-blow ups of $\overline{\mathcal{M}}_{g,n}$ rather than on $\overline{\mathcal{M}}_{g,n}$ itself.

9. Viewing $\overline{\mathcal{M}}_{g,n}$ as an algebraic stack equipped with a log structure enables us to study this space using tools from both logarithmic and tropical geometry.

10. Currently, universities operate in a highly hierarchical way. Students are considered customers and therefore expendable, even though they constitute a vital component of a university. Reinforcing horizontal structures and including students in the decision-making processes regarding the functioning of a university can greatly enhance the quality of both research and teaching.

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