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Tautological relations and double ramification cycles with spin parity

Politopoulos, G.

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Curriculum Vitae

Georgios Politopoulos was born on the 1st of August 1995, in Athens, Greece. After finishing high school in 2013, he started studying mathematics at the National and Kapodistrian University of Athens, from which he graduated in 2018.

In January 2019, Georgios moved to Leiden to pursue a MSc in pure mathematics at Leiden University. He wrote his thesis under the supervision of Adrien Sauvaget, titled “A Relation of Psi Integrals via the Localization Formula on the Moduli of Stable Maps to \mathbb{P}^1 ”. He completed his studies in July 2021.

In October 2021, Georgios began a joint PhD program between Cergy-Pontoise University and Leiden University, under the co-supervision of Adrien Sauvaget and David Holmes. The contents of this thesis are the outcome of that joint PhD program.

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