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Tautological relations and double ramification cycles with spin parity

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**Tautological relations and double ramification cycles
with spin parity**

**Relations tautologiques et cycle de double ramification
avec parité de spin**

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Tautological relations and double ramification cycles with spin parity

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