Opinion dynamics on random graphs

Capannoli, F.

Summary

Suppose that you draw some points on a piece of paper, with arrows connecting them. The points represent the people living in a country and the arrows represent the acquaintance relationships between them. Since the people we encounter each day are subject to change, we assume that the acquaintances of each person are randomly chosen from a given distribution. Now suppose that political elections will take place in the country, with only two parties participating. Each person holds an opinion over time about who to vote for, represented on the piece of paper by a coloring of the corresponding point either blue or red. Each person changes color randomly over time, based on the influence received from its acquaintances. Such a simplified description gives rise to the so-called voter model, evolving on random graphs. The aim of this thesis is to analyze the evolution of such a process over time, determining asymptotic behaviors of certain observables as the number of people becomes large and capturing interesting emergent phenomena.

In Chapter 2, we study the standard voter model on a class of sparse directed configuration models. The main result concerns the asymptotic behavior of the consensus time distribution on a typical realization of the graph as the size of the graph grows to infinity. This analysis is carried out by studying the meeting times of random walks starting from stationarity. Recent results by Oliveira make it possible to relate these quantities when the underlying sequence of graphs satisfies certain mean-field conditions. As a consequence of the complete characterization of the distribution of the meeting time, we provide the first-order approximation of its expectation, showing that it is linear in the size of the graph. The constant is an explicit quantity that depends on simple statistics of the underlying degree sequence, and its explicit knowledge makes it possible to examine how the regularity or variability of the degree sequence plays a role in the diffusion of the opinions.

In Chapter 3 we exploit the results obtained in Chapter 2 to extend them to a deeper level of analysis. More precisely, we investigate the evolution of the density of discordant edges, that is, edges connecting vertices with opposite opinions. The main result of this chapter includes the derivation of the precise asymptotics for the expected density of discordant edges across different time scales, ranging from constant order with respect to the size of the network to the consensus time scale. On the first time scale, the initial density of discordances quickly drops to a given value. For any time scale in between this initial drop and the consensus time, the process stabilizes around

an explicit limiting value for a long time. Finally, on the consensus time scale, the density of discordant edges exhibits a sharp descent from this plateau, approaching zero at a rate that coincides with the functional found in Chapter 2 for the expected consensus time.

In Chapter 4, we analyze a variant of the voter model considered in Chapters 2 and 3, consisting of a nonlinear opinion dynamics model that captures the interaction between an external disruptive bias favoring the adoption of a novel opinion and individual stubbornness favoring the adherence to the initial opinion. This interplay is modeled by two competing parameters. We provide an initial result on random out-regular graphs where, exploiting a mean-field approximation where the underlying graph is independently resampled at each step, we identify a phase transition phenomenon occurring with high probability. For fixed out-degree and stubbornness parameter, we establish the existence of a critical threshold of the disruptive bias such that, above this threshold, the dynamics quickly reaches consensus on the blue opinion, while below the threshold, the system rapidly settles into a metastable state characterized by a stable non-consensus proportion of agents adopting the novel opinion. This metastable regime persists on a time scale that is exponential in the network size.

In Chapter 5, we investigate several questions related to the voter model on both directed and undirected random graph models, focusing on heavy-tailed degree distributions. We conduct a systematic numerical study and propose multiple conjectures based on mathematical insights, exploring how the network topology, especially heterogeneity in the vertices degrees, affects the opinion dynamic evolution. The main contribution is an extension of the analysis of consensus times to directed random graphs with arbitrary given degree sequences. Additionally, we evaluate the validity of mean-field approximations within these graph ensembles. Numerical simulations demonstrate that these approximations accurately describe the distribution of the consensus time, provided the underlying directed ensemble has finite mean degrees. Conversely, in scenarios with infinite mean degrees, significant deviations occur between the empirical distribution of consensus times and the theoretical predictions.