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Opinion dynamics on random graphs

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Citation

Capannoli, F. (2025, November 19). *Opinion dynamics on random graphs*.
Retrieved from <https://hdl.handle.net/1887/4283502>

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Opinion Dynamics on Random Graphs

Proefschrift

ter verkrijging van
de graad van doctor aan de Universiteit Leiden,
op gezag van rector magnificus prof.dr.ir. H. Bijl,
volgens besluit van het college voor promoties
te verdedigen op woensdag 19 november 2025
klokke 13:00 uur

door

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geboren te Siena, Italië
in 1996

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The research presented in this thesis was supported in part by the Netherlands Organisation for Scientific Research (NWO) through the Gravitation NETWORKS grant 024.002.003 and European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement no. 945045.



Cover: Francesca Leonardi

Ai miei nonni

To my grandparents

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