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Spectral analysis of inhomogeneous network models

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Spectral Analysis of Inhomogeneous Network Models

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