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Toward primordial gravitational waves and $n_s = 1$ in light of BICEP/Keck and DESI BAO data and the Hubble tension

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(Received 19 December 2024; accepted 16 May 2025; published 6 June 2025)

Recent observational data seem to show $\gtrsim 3\sigma$ evidence for an evolving dark energy (DE) against the cosmological constant, so the standard Λ Cold Dark Matter (ΛCDM) model. In this paper, we perform the search for the primordial gravitational waves with the potential prerecombination solutions to the Hubble tension, using recent DESI baryon acoustic oscillation measurements combined with BICEP/Keck cosmic microwave background (CMB) B-mode polarization, Planck CMB, and Pantheon supernova data, which reveal that the low bound of the tensor-to-scalar ratio r is $> 1.5\sigma$ nonzero with the best fit $r_{0.05} \sim 0.01$ and the scalar spectral index $n_s = 1$ (both $|r_{0.05} - 0.01|$ and $|n_s - 1| \sim \mathcal{O}(0.001)$). In particular, we observe the unnoticed impact of CMB B-mode polarization data for constraining the nature of DE, which together with early dark energy solutions to the Hubble tension is calling for the return of postrecombination ΛCDM.

DOI: 10.1103/w19x-trrq

I. INTRODUCTION

It is well known that inflation is the current paradigm of the very early Universe [1–4]. It predicts nearly scale-invariant scalar perturbation consistent with recent cosmological and astrophysical observations, as well as primordial gravitational waves (GWs).

The primordial GWs, usually thought as the "smoking gun" of inflation, can source the B-mode polarization in the cosmic microwave background (CMB) [5–7]. Recently, based on the Λ Cold Dark Matter (ΛCDM) model the BICEP/Keck Collaboration has reported the tensor-to-scalar ratio $r < 0.036$ (95% CL) [8] (see also [9]), while the scalar spectral index $n_s \sim 0.965$ [10].

Though the concordant ΛCDM model is the most successful model to explain the cosmological observations, it suffered from the Hubble tension; see, e.g., [11–14]. The early dark energy (EDE) solutions to the Hubble tension, in which an EDE component is not negligible just a while before recombination [15–17], have been widely investigated, and it has been also found that an anti–de Sitter (AdS) phase around recombination can efficiently strengthen the EDE contribution (AdS-EDE) [18,19], so that $H_0 \sim 73$ km/s/Mpc. In the cosmologies with prerecombination EDE, the upper bound on r can be further tightened [20] and $n_s = 1$ since n_s scales as [21,22]

$$\delta n_s \simeq 0.4 \frac{\delta H_0}{H_0}. \qquad (1)$$

In the concordant ΛCDM model, the simplest possibility of dark energy (DE) is the cosmological constant (CC). However, recently the baryon acoustic oscillation (BAO) data of DESI [23] combined with Planck CMB and supernova data have shown a $\gtrsim 3\sigma$ evidence that DE is evolving; see also [24,25]. The relevant issues have been also intensively investigated, e.g., [26–69]. Inevitably, such an evolving DE will bring the shifts of the best fit values of relevant cosmological parameters, and an unexpected result is that the tensor-to-scalar ratio r is 2σ nonzero [52].

However, the prerecombination EDE solutions to the Hubble tension would suppress the support of DESI for the evolving DE [26]. In recent years, the search for the primordial GWs has been always an unremitting pursuit in the cosmological community; thus it is significant to explore the underlying impact of both DESI and prerecombination solutions to the Hubble tension on it. In this paper, we perform such a search for the primordial GWs, using recent DESI BAO data combined with BICEP/Keck CMB B-mode polarization, Planck CMB, and Pantheon Plus (with Cepheid calibration) data, and find that the lower

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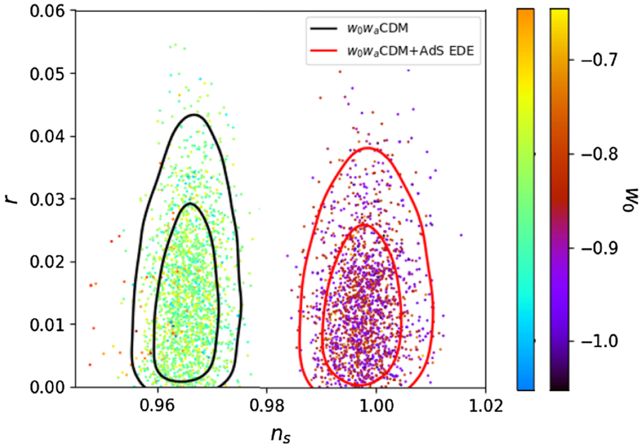

FIG. 1. 1σ and 2σ contours of $r - n_s$ for the w_0w_aCDM model, without (left color map) or with AdS-EDE (right color map) respectively. The dataset used is the Planck18 + BK18 + DESI + Pantheon Plus (with Cepheid calibration) datasets. The scatter colors of w_0w_aCDM for w_0 are relatively consistent ($w_0 = -0.9 \sim -0.8$), while the scatter colors with AdS-EDE are more mottled, which actually corresponds to more possibilities of DE, such as the CC and quintessence ($w_0 + w_a \geq -1$), and is a natural reflection of the results in Fig. 2. In the w_0w_aCDM model $H_0 \sim 70$ km/s/Mpc, and $H_0 \sim 72$ km/s/Mpc with AdS-EDE which resolves the Hubble tension.

bound of the tensor-to-scalar ratio r is $> 1.5\sigma$ nonzero, and still $n_s = 1$ ($|n_s - 1| \sim \mathcal{O}(0.001)$); see Fig. 1.

II. METHODOLOGY

A. Datasets

Here, we will use recent BAO measurements (see Table I) reported by DESI [23] for the comoving distances $D_M(z)/r_d$ and $D_H(z)/r_d$, where

$$D_M(z) \equiv \int_0^z \frac{cdz'}{H(z')}, \quad D_H(z) \equiv \frac{c}{H(z)}, \tag{2}$$

and $r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)}$ is the sound horizon with $z_d \simeq 1060$ at the baryon drag epoch and c_s the speed of sound, as well as the angle-averaged quantity D_V/r_d, where $D_V(z) \equiv (zD_M(z)^2 D_H(z))^{1/3}$.

In addition, we also use the Planck 2018 CMB dataset (low-l and high-l Temperature anisotropy spectrum (TT), TE, E-mode polarization power spectrum (EE) spectra, and reconstructed CMB lensing spectrum [10,70,71]), BK18 CMB B-mode polarization data, and Pantheon Plus data (consisting of 1701 light curves of 1550 spectroscopically confirmed Type Ia SN coming from 18 different surveys [72], using additional Cepheid distances as a calibrator of the SN1a magnitude [73]). Throughout this paper, we work with Planck18 + BK18 + DESI + Pantheon Plus datasets.

TABLE I. Statistics for the DESI samples of the DESI DR1 BAO measurements used in this paper.

Tracer	$z_{\rm eff}$	D_M/r_d	D_H/r_d	D_V/r_d
BGS	0.30	⋯	⋯	7.93 ± 0.15
LRG	0.51	13.62 ± 0.25	20.98 ± 0.61	⋯
LRG	0.71	16.85 ± 0.32	20.08 ± 0.60	⋯
LRG + ELG	0.93	21.71 ± 0.28	17.88 ± 0.35	⋯
ELG	1.32	27.79 ± 0.69	13.82 ± 0.42	⋯
QSO	1.49	⋯	⋯	26.07 ± 0.67
Lya QSO	2.33	39.71 ± 0.94	8.52 ± 0.17	⋯

B. Models

Here, to observe the joint impact of DESI and the Hubble tension on $r - n_s$, we model the evolution after matter-radiation equality "w_0w_aCDM + EDE" (see [26,74]), and modified the MontePython-3.6 sampler [75,76] and CLASS codes [77,78] to perform our Markov Chain Monte Carlo (MCMC) analysis. In the corresponding model, the state equation of DE is Chevalier-Polarski-Linder (CPL)-like [79,80], while only in the prerecombination epoch the axionlike EDE [16] or AdS-EDE [18] is non-negligible.[1]

In the axionlike EDE model, the potential of an axion field is considered as Ref. [16]:

$$V(\phi) = \Lambda^4 (1 - \cos[\phi/f])^n. \tag{3}$$

The field is frozen before the matter-radiation equality and acts as a cosmological constant, but when the Hubble parameter drops below a certain value, $H^2 \simeq \partial_\phi^2 V$, at a critical redshift $z_c = a^{-1} - 1$, the field begins to oscillate and then behaves as a fluid with an equation of state $w = (n-1)/(n+1)$. The homogeneous EDE energy density dilutes like matter for $n = 1$, like radiation for $n = 2$, and faster than radiation whenever $n \leq 3$. It is noted that $n = 3$ is better for a higher best fit H_0. We perform the MCMC analysis on the parameter set $\{\omega_b, \omega_{\rm cdm}, H_0, \ln(10^{10}A_s), n_s, \tau_{\rm reio}, \log_{10}a_c, f_{ede}, \theta_i, w_0, w_a, r\}$.

In the AdS-EDE model, we consider the potential as Ref. [18]:

$$V(\phi) = V_0 \left(\frac{\phi}{M_p}\right)^4 - V_{\rm ads}, \tag{4}$$

which is glued to $V(\phi) = 0$ at $\phi = (V_{\rm ads}/V_0)^{1/4} M_p$ by interpolation, where $V_{\rm ads}$ is the depth of the AdS well. Initially, the scalar field ϕ sits at the hillside of its potential, and its energy density ρ_ϕ is negligible, but when the Hubble parameter drops below a certain value $H^2 \simeq \partial_\phi^2 V$ at z_c, which occurred before recombination, the field starts

[1]The models with prerecombination EDE are usually accompanied with the exacerbation of the S_8 tension [81–87]; however, it might be cured by mechanisms independent of EDE, e.g., [88–98].

TABLE II. Mean (best fit) values and 1σ regions of the parameters of models. It should be mentioned that in Ref. [52] the results for $w_0 w_a$CDM is without Cepheid calibration in the Pantheon Plus dataset; however, its $r - n_s$ contour is consistent with that here for $w_0 w_a$CDM; see Fig. 1 here and Fig. 1 in Ref. [52].

Parameters	$w_0 w_a$CDM	$w_0 w_a$CDM + AdS EDE	$w_0 w_a$CDM + axion-like EDE
$100\omega_b$	$2.241(2.227) \pm 0.016$	$2.341(2.338) \pm 0.018$	$2.284(2.250) \pm 0.024$
$\omega_{\rm cdm}$	$0.119(0.120) \pm 0.001$	$0.134(0.134) \pm 0.002$	$0.131(0.128) \pm 0.003$
H_0	$70.39(70.26) \pm 0.68$	$72.65(71.88) \pm 0.66$	$72.13(71.62) \pm 0.77$
$\ln 10^{10} A_s$	$3.035(3.027) \pm 0.017$	$3.064(3.057) \pm 0.015$	$3.056(3.059) \pm 0.015$
n_s	$0.967(0.962) \pm 0.003$	$0.998(0.991) \pm 0.005$	$0.990(0.984) \pm 0.007$
$\tau_{\rm reio}$	$0.053(0.047) \pm 0.008$	$0.052(0.048) \pm 0.008$	$0.055(0.055) \pm 0.007$
f_{ede}	…	$0.113(0.106) \pm 0.007$	$0.104(0.095) \pm 0.015$
$\log_{10} a_c / \ln(1 + z_c)$	…	$8.189(8.186) \pm 0.072$	$-3.70(-3.57) \pm 0.15$
θ_i	…	…	$2.78(2.65) \pm 0.09$
w_0	$-0.881(-0.833) \pm 0.053$	$-0.890(-0.853) \pm 0.064$	$-0.846(-0.824) \pm 0.058$
w_a	$-0.737(-0.665) \pm 0.176$	$-0.351(-0.509) \pm 0.254$	$-0.627(-0.728) \pm 0.258$
r	$0.0153(0.0090)^{+0.0049}_{-0.0136}$	$0.0134(0.0131)^{+0.0045}_{-0.0118}$	$0.0150(0.0145)^{+0.0061}_{-0.0127}$
Ω_m	$0.286(0.292) \pm 0.006$	$0.298(0.304) \pm 0.006$	$0.296(0.293) \pm 0.006$
S_8	$0.821(0.857) \pm 0.010$	$0.857(0.866) \pm 0.012$	$0.848(0.842) \pm 0.011$

rolling down the potential; meanwhile ρ_ϕ becomes nonnegligible. Then the field will roll over an AdS phase, and during this period ρ_ϕ quickly redshifts away. Hereafter, the field rapidly climbs up to the $\Lambda > 0$ region, so that the Universe eventually settles down in the ΛCDM-like phase until now. $\alpha_{\rm ads}$ corresponds to $V_{\rm ads}$ by $V_{\rm ads} = \alpha_{\rm ads}(\rho_m(z_c) + \rho_r(z_c))$, which is fixed to 3.79×10^{-4}. We perform the MCMC analysis on the parameter set $\{\omega_b, \omega_{\rm cdm}, H_0, \ln(10^{10}A_s), n_s, \tau_{\rm reio}, \ln(1+z_c), f_{ede}, w_0, w_a, r\}$.

In the ΛCDM model, the dark energy activates as a cosmological constant with the equation of state $w = -1$. However, in scalar field models of dark energy w may be evolving or oscillating. Here, we work with the CPL parametrization [79,80]

$$w(z) = w_0 + w_a \frac{z}{1+z}, \tag{5}$$

which is well behaved at high z.

III. RESULTS

In Table II, we present our MCMC results. It can be seen that for the $w_0 w_a$CDM with AdS-EDE, $r_{0.05} = 0.0134^{+0.0045}_{-0.0118}$ (the lower bound is $\sim 1.5\sigma$ nonzero), while with axionlike EDE, $r_{0.05} = 0.0150^{+0.0061}_{-0.0127}$ (the lower bound is $\sim 1.6\sigma$ nonzero). Here, we still have $n_s = 1$ (see also Fig. 1), which is actually the inevitable result of the $n_s - H_0$ scaling relation (1) [18,21,22,99–101] noting $H_0 \gtrsim 72$ km/s/Mpc; see also [102,103].

In corresponding $w_0 w_a$CDM models, we have $w_0 = -0.890^{+0.059}_{-0.069}$, $w_a = -0.351^{+0.273}_{-0.235}$ with AdS-EDE, and $w_0 = -0.846^{+0.060}_{-0.058}$, $w_a = -0.627^{+0.280}_{-0.241}$ with axionlike EDE; see also Fig. 2. It has been observed in Ref. [26] that the prerecombination resolutions of the Hubble tension significantly suppress the support of DESI for the evolving DE. Here, since the evolution of DE at low redshift will bring the shifts of the posteriors of relevant cosmological parameters (specially A_s and Ω_m) and consequently result in larger lensing B modes than ΛCDM (see also Fig. 3), which is constrained strictly by the BK18 data (setting the upper bound for the lensing B-mode spectrum), the corresponding suppression is further strengthened; see also Table III.

It is also significant to check the correlations between r and the AdS-EDE model parameters z_c and f_{ede}, so that one can see whether AdS-EDE is responsible for the detection of r. In Fig. 4, we show the corresponding

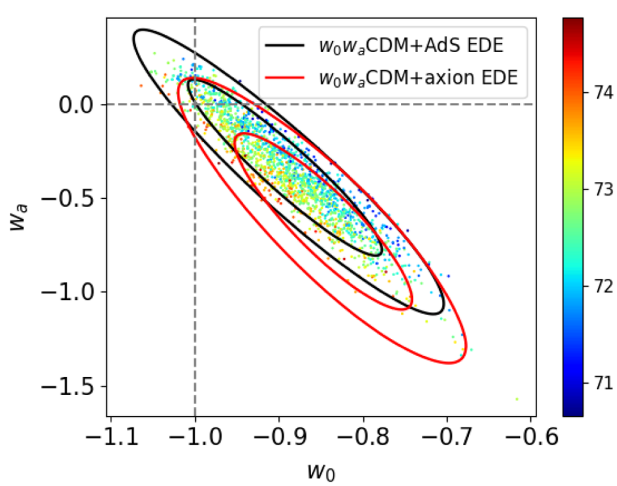

FIG. 2. 1σ and 2σ contours of $w_0 - w_a$ for the $w_0 w_a$CDM + axionlike EDE and $w_0 w_a$CDM + AdS EDE models, respectively, scattered by H_0 for that with AdS-EDE.

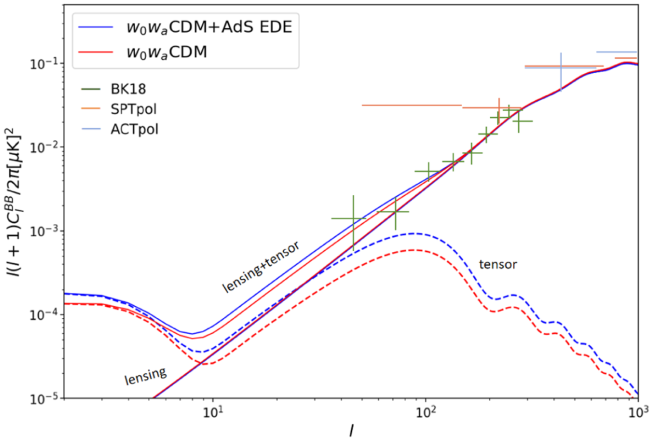

FIG. 3. Power spectra of total B mode $C^{\mathrm{BB}}_{l,\mathrm{total}}$, lensing B mode $C^{\mathrm{BB}}_{l,\mathrm{lensing}}$, and tensor B mode $C^{\mathrm{BB}}_{l,\mathrm{tensor}}$ for the best fit values $r = 0.009$ and $r = 0.013$ (see Table II) in the w_0w_aCDM and w_0w_aCDM + AdS EDE models, respectively, where $C^{\mathrm{BB}}_{l,\mathrm{total}} = C^{\mathrm{BB}}_{l,\mathrm{lensing}} + C^{\mathrm{BB}}_{l,\mathrm{tensor}}$. Points with error bars are binned BK18 [8], SPT [104], and ACT [105] data points.

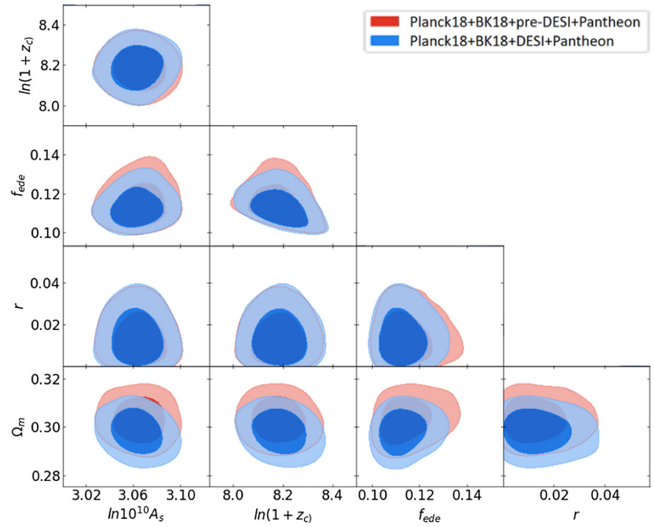

FIG. 4. 1σ and 2σ contours for AdS-EDE parameters $\{f_{ede}, \ln(1+z_c), r\}$ and with Planck18 + BK18 + pre-DESI + Pantheon Plus and Planck18 + BK18 + DESI + Pantheon Plus datasets respectively.

results. There is no correlation between r and $\ln(1+z_c)$. However, compared with the pre-DESI BAO result, f_{ede} is slightly suppressed in the AdS-EDE model, which is accompanied with a lower Ω_m and thus slightly enhances the significance of $r > 0$. This result can be explained as follows. The power spectrum of the total CMB B mode is

$$C^{\mathrm{BB}}_{l,\mathrm{total}} = C^{\mathrm{BB}}_{l,\mathrm{lensing}} + C^{\mathrm{BB}}_{l,\mathrm{tensor}}, \tag{6}$$

where $C^{\mathrm{BB}}_{l,\mathrm{lensing}}$ and $C^{\mathrm{BB}}_{l,\mathrm{tensor}}$ are the lensing B mode and tensor B mode, respectively. In the Limber approximation [106], the lensing convergence spectrum $C_l^{\kappa\kappa} = l^2(l+1)^2 C_l^{\phi\phi}/4$ is

$$C_l^{\kappa\kappa} = \frac{9}{4}\Omega_m^2 h \int_0^\infty dy a^{-2}(y)\hat{g}_L^2(y) P_m\left(k = \frac{l}{\chi}h, \eta(y)\right), \tag{7}$$

where the variable $y = h\chi$ ($H_0 = 100h$ km/s/Mpc) and χ is the angular diameter distance, and $\hat{g}_L = \int_y^\infty dy'(1 - \frac{y}{y'})W(y')$ is the lensing kernel with the window function $W(y)$. It is obvious that $\hat{g}_L$ is independent of Ω_m; thus the peak height of $C_l^{\kappa\kappa}$ is only relevant to the prefactor $\Omega_m^2 h$. A lower Ω_m preferred by DESI partly suppresses the lensing spectrum and enlarges the significance of $r > 0$.

Though the evolving DE after the recombination seems to be still best fit, since the postrecombination CC has fewer parameters, it is required to evaluate the Bayesian evidence $\mathcal{Z}_{\mathrm{model}}$ for different models and the corresponding Bayesian ratio $B = \frac{\mathcal{Z}_{\Lambda\mathrm{CDM+EDE}}}{\mathcal{Z}_{w_0w_a\mathrm{CDM+EDE}}}$. Here, considering AdS-EDE, we have

$$\ln B = \ln \frac{\mathcal{Z}_{\Lambda\mathrm{CDM+EDE}}}{\mathcal{Z}_{w_0w_a\mathrm{CDM+EDE}}} = 0.54, \tag{8}$$

where $\mathcal{Z}_{\mathrm{model}}$ is calculated using MCEvidence [107] for a set of MCMC chains. The Bayesian ratio suggests that the Universe after the recombination is slightly more

TABLE III. The significance level of postrecombination ΛCDM for w_0w_aCDM, with and without the corresponding EDE, with and without BK18 respectively. The baseline denotes the Planck18 + DESI + Pantheon Plus dataset.

Datasets	w_0w_aCDM	+AdS EDE	+axionlike EDE
Baseline	2.31σ	1.87σ	1.95σ
Baseline + BK18	2.08σ	1.10σ	1.73σ

TABLE IV. Mean (best fit) values and 1σ regions of the parameters of the ΛCDM + AdS EDE model.

Parameters	ΛCDM + AdS EDE
$100\omega_b$	$2.338(2.326) \pm 0.018$
ω_{cdm}	$0.134(0.134) \pm 0.002$
H_0	$73.09(73.33) \pm 0.48$
$\ln 10^{10}A_s$	$3.064(3.072) \pm 0.016$
n_s	$0.997(0.999) \pm 0.004$
τ_{reio}	$0.051(0.056) \pm 0.008$
r	$0.0139(0.0096)^{+0.0046}_{-0.0119}$
Ω_m	$0.295(0.292) \pm 0.005$
S_8	$0.856(0.857) \pm 0.011$

ΛCDM-like. In the corresponding $\Lambda\mathrm{CDM} + \mathrm{EDE}$ model the best fit $H_0 = 73.3$ km/s/Mpc; see Table IV.

IV. DISCUSSION

In this paper, we perform the search for the primordial GWs with the potential prerecombination solutions to the Hubble tension and recent DESI BAO data, which reveals that the low bound of r is $\gtrsim 1.5\sigma$ nonzero, marking the potential existence of primordial GWs with the bestfit $r_{0.05} \sim 0.01$, and $n_s = 1$ (both $|r_{0.05} - 0.01|$ and $|n_s - 1| \sim \mathcal{O}(0.001)$).

It will be interesting to revisit inflation models in light of our results on $r_{0.05} \sim 0.01$ and $n_s = 1$ (see recent Refs. [108–117]) for its implications for inflation models; see also Refs. [118,119]. According to our results, different DE models might bias our detection for the primordial GWs; thus it is also necessary to investigate the effects of different resolutions on the Hubble tension (see, e.g., [11–14]) on the search for the primordial GWs. Recently, the combination of Planck ($\ell < 1000$), recent SPT-3G [120], and the ACT dataset [105] has been applied to the study on prerecombination solutions to the Hubble tension; it is also significant to reassess the corresponding models with the combinated Planck + SPT + ACT dataset.

It is possible that recent DESI or BICEP/Keck data have some unknown systematics. However, in the forthcoming years if current data are confirmed to be consistent, our work not only highlights the potential possibility of $r_{0.05} \sim 0.01$, but also presents the unnoticed impact of CMB B-mode polarization data for constraining the nature of DE, which together with EDE is calling for the return of the postrecombination ΛCDM.

We acknowledge the use of publicly available codes AxiCLASS ([121]) and CLASSMULTISCF ([122]).

ACKNOWLEDGMENTS

Y. S. P. is supported by National Key Research and Development Program of China (Grant No. 2021YFC2203004), NSFC (Grant No. 12075246), and the Fundamental Research Funds for the Central Universities. G. Y. is supported by NWO and the Dutch Ministry of Education, Culture and Science (OCW) (Grant No. VI.Vidi.192.069).

DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: 2D CONTOUR RESULTS OF w_0w_aCDM + EDE MODELS WITH PRE-DESI DATASETS

In this appendix, for comparison we also use pre-DESI BAO datasets (the BOSS DR12 [123] with its full covariant matrix for BAO as well as the 6dFGS [124] and MGS of SDSS [125] for low-z BAO). We work with Planck18 + BK18 + pre-DESI + Pantheon Plus datasets. We display the 1σ and 2σ contours for the w_0w_aCDM, w_0w_aCDM + axionlike EDE and w_0w_aCDM + AdS-EDE models with Planck18 + BK18 + pre-DESI + Pantheon Plus and Planck18 + BK18 + DESI + Pantheon Plus datasets in Figs. 5–7, respectively. The results of Planck18 + BK18 + pre-DESI + Pantheon Plus are shown in Fig. V.

It can be observed that when DESI substitutes for pre-DESI data the evolving DE is preferred more significantly, while A_s is slightly lower. Thus larger best fits r are preferred by DESI for all three models (with total B mode constrained by CMB B-mode data), especially for AdS-EDE and axion-like EDE, which enlarges the hint of $r > 0$. Compared with the results with pre-DESI, the preference for smaller Ω_m by DESI results in a slightly larger H_0 for w_0w_aCDM + EDE models while there are not large changes of f_{ede} and z_c.

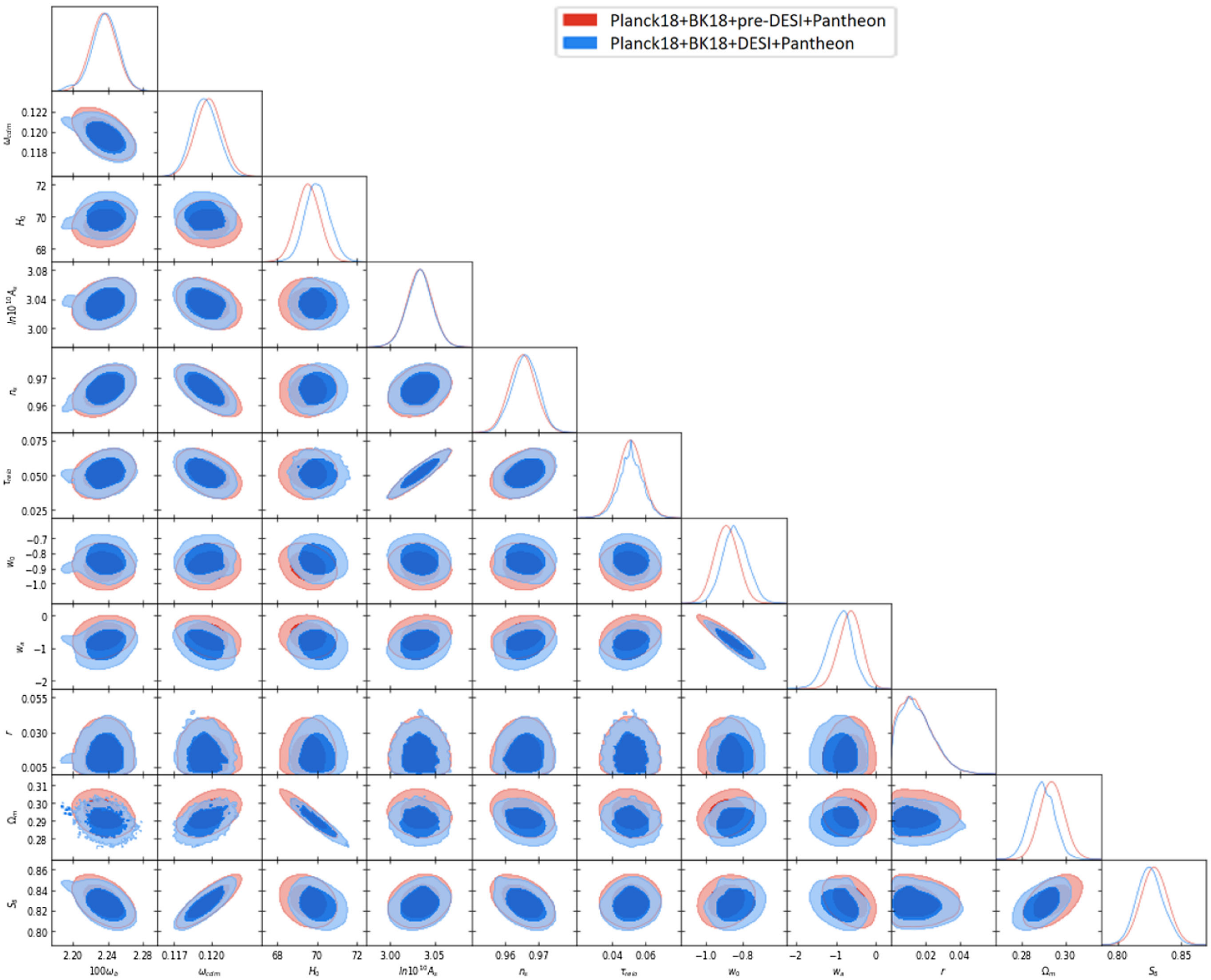

FIG. 5. 1σ and 2σ contours for the $w_0 w_a$CDM model with Planck18 + BK18 + pre-DESI + Pantheon Plus and Planck18 + BK18 + DESI + Pantheon Plus datasets respectively.

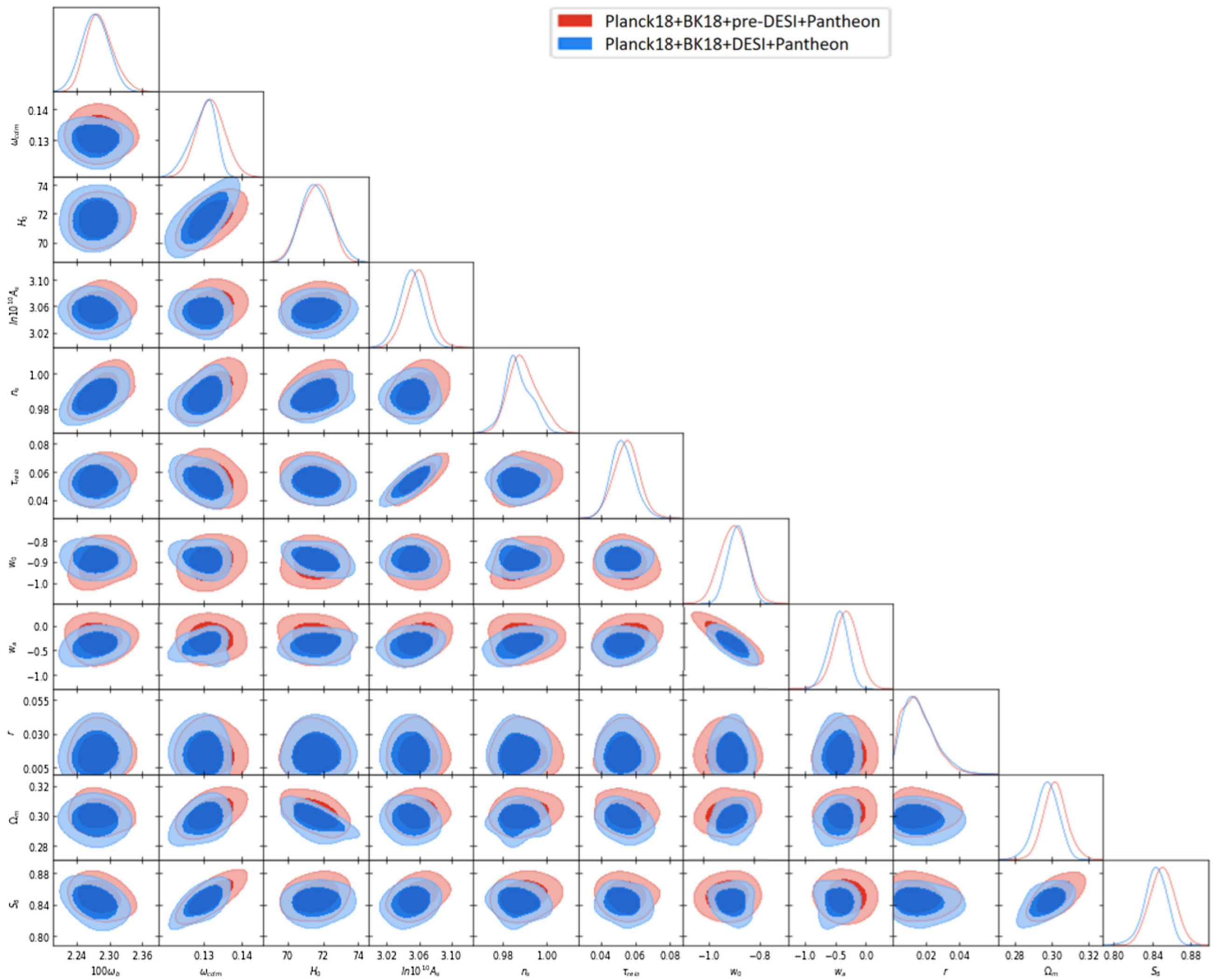

FIG. 6. 1σ and 2σ contours for the w_0w_aCDM + axionlike EDE model with Planck18 + BK18 + pre-DESI + Pantheon Plus and Planck18 + BK18 + DESI + Pantheon Plus datasets respectively.

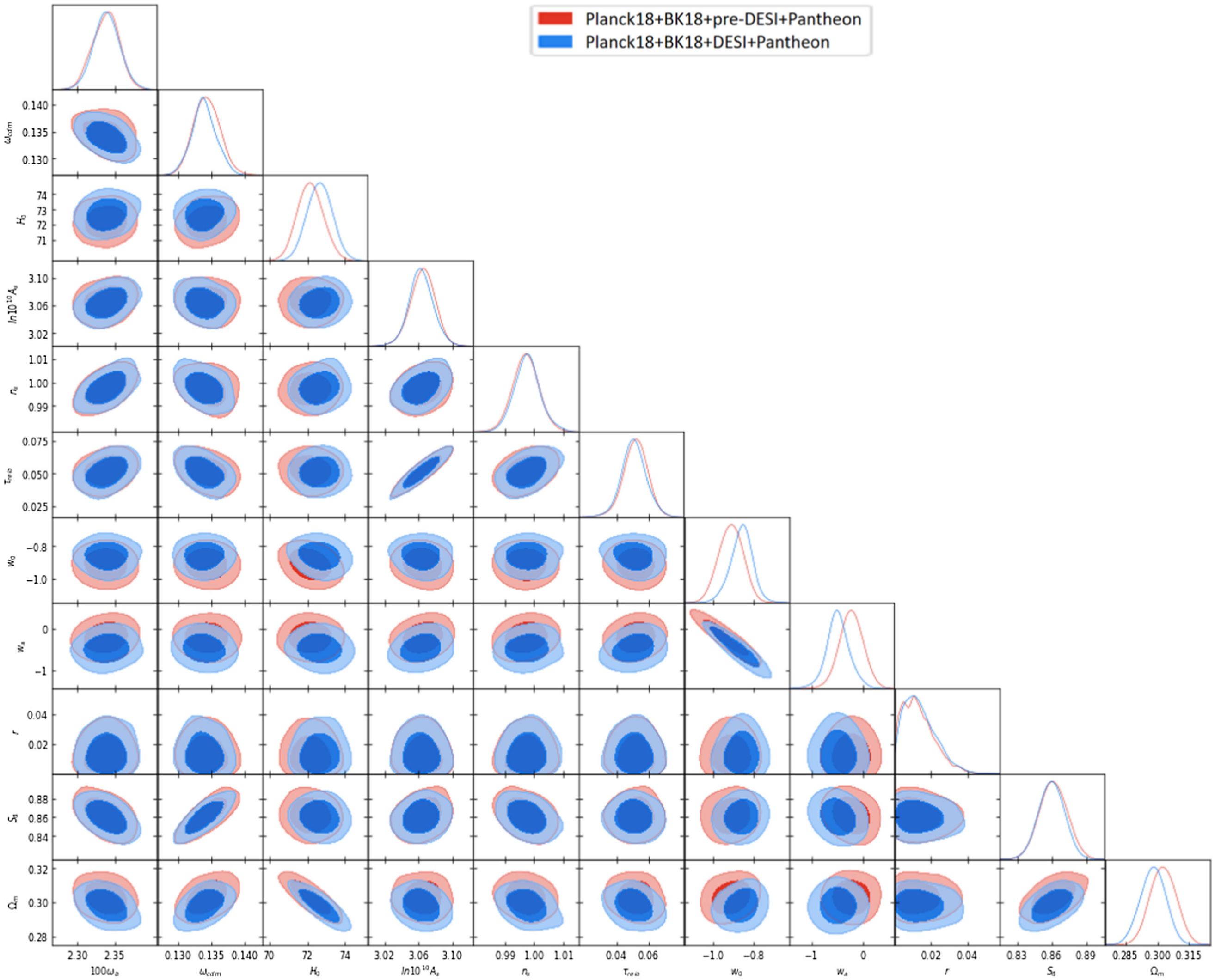

FIG. 7. 1σ and 2σ contours for the w_0w_aCDM + AdS EDE model with Planck18 + BK18 + pre-DESI + Pantheon Plus and Planck18 + BK18 + DESI + Pantheon Plus datasets respectively.

TABLE V. Mean (best fit) values and 1σ regions of the parameters of models with Planck18 + BK18 + pre-DESI + Pantheon Plus.

Parameters	w_0w_aCDM	w_0w_aCDM + AdS EDE	w_0w_aCDM + axion-like EDE
$100\omega_b$	$2.235(2.232) \pm 0.015$	$2.338(2.339) \pm 0.018$	$2.282(2.272) \pm 0.025$
$\omega_{\rm cdm}$	$0.120(0.120) \pm 0.001$	$0.134(0.134) \pm 0.002$	$0.132(0.134) \pm 0.004$
H_0	$69.54(69.89) \pm 0.60$	$72.17(72.45) \pm 0.69$	$71.60(71.51) \pm 0.80$
$\ln 10^{10}A_s$	$3.034(3.032) \pm 0.014$	$3.065(3.065) \pm 0.015$	$3.059(3.068) \pm 0.015$
n_s	$0.965(0.966) \pm 0.004$	$0.997(0.997) \pm 0.005$	$0.990(0.989) \pm 0.007$
$\tau_{\rm reio}$	$0.051(0.051) \pm 0.007$	$0.052(0.052) \pm 0.008$	$0.055(0.053) \pm 0.007$
f_{ede}	...	$0.115(0.112) \pm 0.008$	$0.120(0.132) \pm 0.020$
$\log_{10} a_c/\ln(1+z_c)$	...	$8.184(8.205) \pm 0.072$	$-3.68(-3.57) \pm 0.14$
θ_i	...	...	$2.76(2.74) \pm 0.09$
w_0	$-0.888(-0.910) \pm 0.065$	$-0.913(-0.921) \pm 0.060$	$-0.902(-0.889) \pm 0.051$
w_a	$-0.645(-0.585) \pm 0.275$	$-0.219(-0.257) \pm 0.226$	$-0.324(-0.334) \pm 0.193$
r	$0.0151(0.0056)^{+0.0052}_{-0.0130}$	$0.0133(0.0032)^{+0.0045}_{-0.0118}$	$0.0152(0.0074)^{+0.0057}_{-0.0122}$
Ω_m	$0.294(0.290) \pm 0.006$	$0.303(0.304) \pm 0.006$	$0.302(0.307) \pm 0.006$
S_8	$0.829(0.825) \pm 0.011$	$0.862(0.866) \pm 0.012$	$0.850(0.864) \pm 0.011$

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