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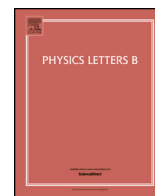
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Letter

Impact of evolving dark energy on the search for primordial gravitational waves

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ABSTRACT

Recent observational data have shown at $\gtrsim 3\sigma$ significance level that the dark energy (DE), responsible for the current accelerated expansion of our universe, is evolving. In this Letter, using the BICEP/Keck cosmic microwave background (CMB) B-mode polarization data, as well as Dark Energy Spectroscopic Instrument (DESI) baryon acoustic oscillation, the Planck 2018 CMB and Pantheon supernova data, we report the first search for the primordial gravitational waves (GWs) based on the evolving DE, which reveals a non-zero tensor-to-scalar ratio r at $\sim 2\sigma$ significance level with the bestfit $r_{0.05} \sim 0.01$. Our result can be confirmed further by upcoming CMB B-mode polarization experiments and ongoing cosmological surveys. Our work indicates that the significant signal of primordial GWs might have already emerged in CMB B-mode data, however, its detection is depending significantly on our insight into the nature of DE, opening an unexpected window for comprehending our primordial universe in new era of cosmological surveys.

1. Introduction

It is well-known that inflation is the current paradigm of very early universe [1–4]. It predicts nearly scale-invariant scalar perturbation consistent with recent cosmological and astrophysical observations, as well as the primordial gravitational waves (GWs).

The ultra-low-frequency primordial GWs at the frequency $f \sim 10^{-18} - 10^{-16}$ Hz, usually thought as the “smoking gun” of inflation, can source the B-mode polarization in the cosmic microwave background (CMB) [5–7]. In the recent years, searching for the imprint of the primordial GWs on the B-mode polarization has invigorated the cosmological community. Based on the Λ CDM model, using Planck18 CMB, baryon acoustic oscillations (BAO) and their CMB B-mode polarization dataset the BICEP/Keck collaboration has reported an improved upper bound on the tensor-to-scalar ratio, $r < 0.036$ (95% C.L.) [8], see also [9], while the models with larger Hubble constant can tighten the upper bound to $r < 0.028$ (95% C.L.) [10,11]. A joint analysis of NANOGrav 15-year data (at $f \sim 10^{-10} - 10^{-8}$ Hz) [12] and BICEP/Keck18 data has also showed $r < 0.039$ (95% C.L.) [13], see also [14–16].

The standard Λ CDM model is widely regarded as the most successful model explaining most of cosmological observations. In the past decades, identifying the nature of dark energy (DE) has been still an important challenge. Recently, combining their first year data with state-of-art CMB and supernova dataset, the DESI collaboration has showed a $\gtrsim 3\sigma$ evidence that the DE is not a constant but evolving [17–19]. The relevant issues are being intensively investigated, e.g. Refs. [20–50].

The result of the DESI collaboration about an evolving DE has astonished the scientific community, since it might alter our understanding about the universe in an unexpected way. Thus it is significant to explore the underlying impact of evolving DE on the physics of very early universe. Here, we report the first search for primordial GWs that reveals

$$r_{0.05} = 0.0153^{+0.0051}_{-0.0133} \quad (68\% \text{ CL}). \quad (1)$$

$$r_{0.05} < 0.034 \quad (95\% \text{ CL}). \quad (2)$$

The upper bound is consistent with that reported by the BICEP/Keck collaboration [8], but the lower bound is $\sim 1.9\sigma$ non-zero, which for the

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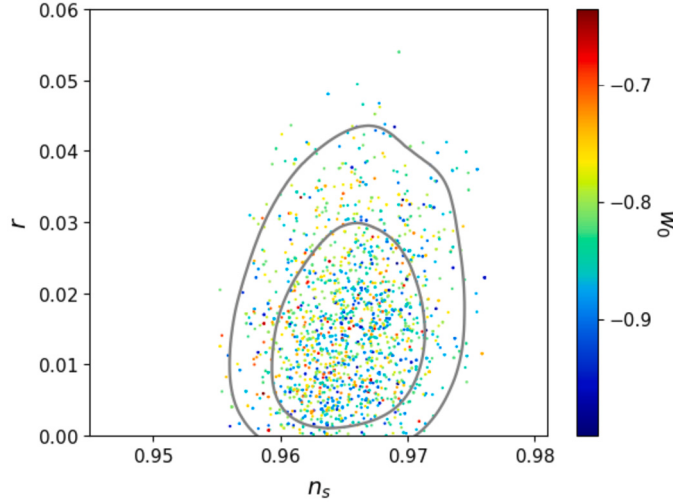


Fig. 1. The 1σ and 2σ contours of r - n_s for the evolving DE (the $w_0 w_a$ CDM model), where r is the tensor-to-scalar ratio and n_s is the spectral index of primordial scalar perturbation. And the scattered plot is for the state equation w_0 of DE at $z = 0$.

Table 1

Mean (bestfit) values and 1σ regions of the parameters of models, fitting to Planck18+BK18+DESI+Pantheon Plus datasets.

Parameters	Λ CDM	$w_0 w_a$ CDM
$100\omega_b$	$2.235(2.238)\pm 0.014$	$2.231(2.238)\pm 0.015$
ω_{cdm}	$0.119(0.119)\pm 0.001$	$0.120(0.120)\pm 0.001$
H_0	$68.17(68.43)\pm 0.41$	$68.07(68.48)\pm 0.83$
$\ln 10^{10} A_s$	$3.041(3.035)\pm 0.014$	$3.034(3.041)\pm 0.015$
n_s	$0.967(0.966)\pm 0.004$	$0.965(0.966)\pm 0.004$
τ_{reio}	$0.055(0.051)\pm 0.007$	$0.052(0.054)\pm 0.008$
w_0	-	$-0.832(-0.857)\pm 0.070$
w_a	-	$-0.667(-0.599)\pm 0.333$
r	$0.0152(0.0136)^{+0.0052}_{-0.0130}$	$0.0153(0.0091)^{+0.0051}_{-0.0133}$
Ω_m	$0.305(0.301)\pm 0.005$	$0.307(0.303)\pm 0.008$
S_8	$0.827(0.818)\pm 0.010$	$0.833(0.834)\pm 0.012$

first time highlights the potential existence of primordial GWs, and the bestfit r is $r_{0.05} \sim 0.01$, see also Fig. 1.

2. Data and methodology

Recent DESI data is based on samples of bright galaxies, LRGs, ELGs, quasars and Ly α Forest at the redshift region $0.1 < z < 4.2$ [17–19]. Here, we will use their measurements for the comoving distances $D_M(z)/r_d$ and $D_H(z)/r_d$, where

$$D_M(z) \equiv \int_0^z \frac{cdz'}{H(z')}, \quad D_H(z) \equiv \frac{c}{H(z)}, \quad (3)$$

and $r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz$ is the sound horizon with $z_d \simeq 1060$ at the baryon drag epoch and c_s the speed of sound, as well as the angle-averaged quantity D_V/r_d , where $D_V(z) \equiv (z D_M(z)^2 D_H(z))^{1/3}$.

In addition, we also use **Planck 2018 CMB** dataset (low- l and high- l TT, TE, EE spectra, and reconstructed CMB lensing spectrum [56–58]), **Pantheon Plus** data (consisting of 1701 light curves of 1550 spectroscopically confirmed Type Ia SN coming from 18 different surveys [59]), and the CMB B-mode polarization data **BK18** [8]. Throughout this paper, we work with **Planck18+BK18+DESI+Pantheon Plus** datasets.

Table 2

χ^2 of Λ CDM, $w_0 w_a$ CDM and $w_0 w_a$ CDM ($r = 0$) models, fitting to Planck18+BK18+DESI+Pantheon Plus datasets.

Models	Λ CDM	$w_0 w_a$ CDM	$w_0 w_a$ CDM($r = 0$)
χ^2_{CMB}	2772.47	2771.07	2770.05
χ^2_{DESI}	16.35	14.01	14.28
χ^2_{BK18}	536.54	537.52	538.65
$\chi^2_{\text{Pantheon+}}$	1413.83	1409.14	1409.64
χ^2_{tot}	4739.19	4731.22	4732.48

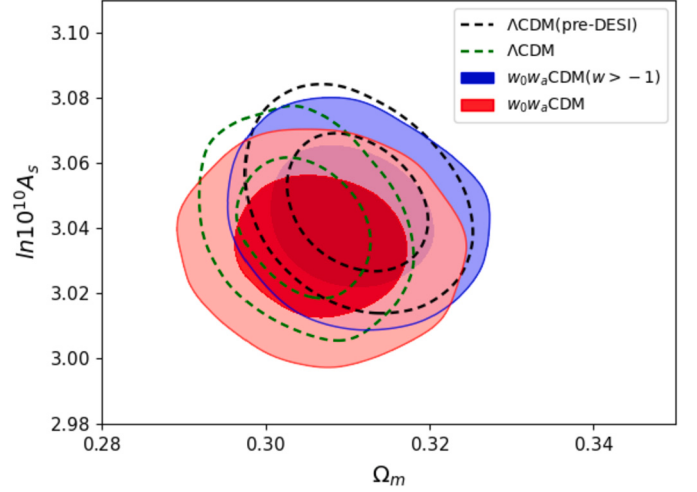


Fig. 2. The 1σ and 2σ contours of A_s - Ω_m in different models. The dataset used is the Planck18+BK18+DESI+Pantheon Plus dataset, and “pre-DESI” represents the Planck18+BK18+Pantheon Plus datasets with pre-DESI BAO data.

In our study, we consider the $w_0 w_a$ CDM model where the equation of state of DE is [60,61]

$$w_{\text{DE}} = w_0 + w_a \frac{z}{1+z}. \quad (4)$$

To perform our MCMC analysis, we used the MontePython-3.6 sampler [79,80] and CLASS codes [78,81]. The threshold of Gelman-Rubin convergence criterion is $R - 1 < 0.01$. The MCMC parameters are $\{\omega_b, \omega_{cdm}, H_0, \ln 10^{10} A_s, n_s, \tau_{reio}, w_0, w_a, r\}$ for the $w_0 w_a$ CDM model. We adopt the prior $[0, 0.5]$ on r and wide enough uniform priors on other cosmological parameters. The pivot scale k_{pivot} of r is set to $0.05(\text{Mpc})^{-1}$.

3. Results

In Table 1, we present our MCMC results. As expected, an evolving DE is preferred with $w_0 = -0.832 \pm 0.070$ and $w_a = -0.667 \pm 0.333$, consistent with those of DESI collaboration [17]. The result of tensor-to-scalar ratio r is $r = 0.0153^{+0.0051}_{-0.0133}$ at the 1σ (68%) CL. and $r = 0.0152^{+0.0188}_{-0.0155}$ at the 2σ (95%) CL., its lower bound is $\sim 1.9\sigma$ non-zero. The bestfit value is $r = 0.009$. We present the χ^2 of Λ CDM, $w_0 w_a$ CDM models in Table 2. The fit is improved with $\chi^2_{w_0 w_a \text{ CDM}} - \chi^2_{\Lambda \text{ CDM}} = -7.97$. Compared with $w_0 w_a$ CDM ($r = 0$) there is $\chi^2_{w_0 w_a \text{ CDM, DESI}} - \chi^2_{w_0 w_a \text{ CDM}(r=0), \text{DESI}} = -0.27$ accompanied by $\chi^2_{w_0 w_a \text{ CDM, BK18}} - \chi^2_{w_0 w_a \text{ CDM}(r=0), \text{BK18}} = -1.13$. This reveals the improvement coming from varying the tensor amplitude in the model and $r > 0$ is slightly preferred by the Planck18+BK18+DESI+PantheonPlus datasets.

In the searches for primordial GWs with pre-DESI data, the BICEP/Keck collaboration did not find non-zero r at $\sim 2\sigma$ significant level [8]. This can be explained as follows. Recent DESI BAO data supported the evolving DE [17], which has a significant impact on the cosmological evolution at low redshift $z \lesssim 1$. As a result, the best-

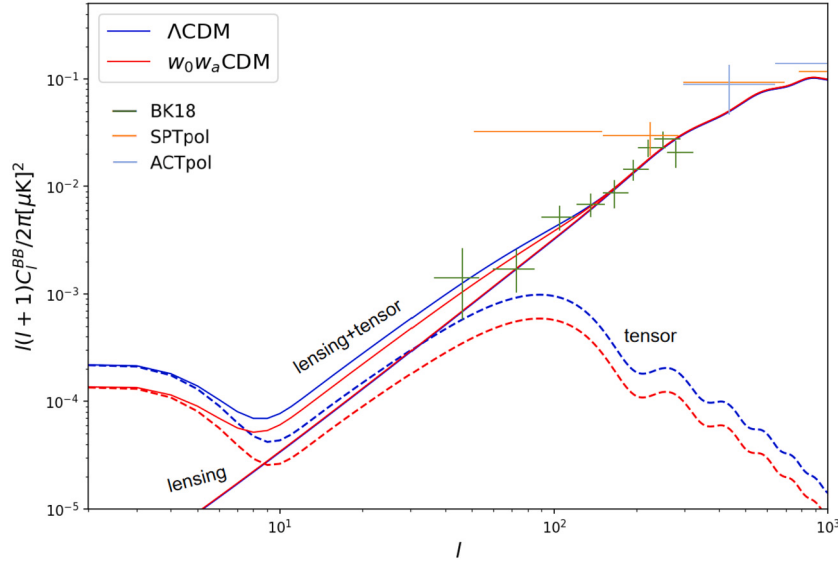


Fig. 3. The total power spectrum of B-mode $C_{l,\text{total}}^{BB}$, the power spectra of lensing B-mode $C_{l,\text{lensing}}^{BB}$ and tensor B-mode $C_{l,\text{tensor}}^{BB}$ for the bestfit values in the w_0w_a CDM model. Points with error bars are binned BK18 [8], SPT [51] and ACT [52] data points.

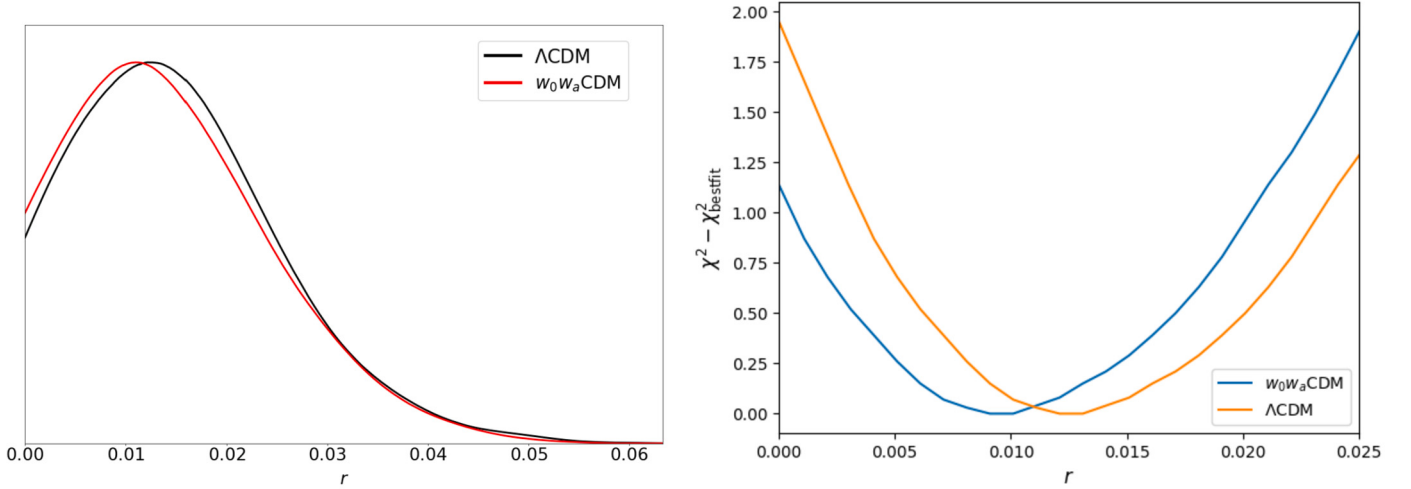


Fig. 4. The 1D posteriors and the profile likelihood of r in Λ CDM and w_0w_a CDM models, fitting to Planck18+BK18+DESI+Pantheon Plus datasets.

fit values of some Λ CDM parameters must be shifted, in particular Planck18+BK18+DESI+Pantheon Plus preferred not only a smaller matter fraction Ω_m [17], but also a smaller amplitude of scalar perturbation A_s , see Fig. 2, both are related to the constraint on r through the lensing B-modes. As pointed out in Ref. [10], the smaller A_s leads to the smaller $C_{l,\text{lensing}}^{BB}$, and thus a more significant r , since the total power spectrum of B-mode $C_{l,\text{total}}^{BB} = C_{l,\text{lensing}}^{BB} + C_{l,\text{tensor}}^{BB}$ is set by BK18 dataset, where $C_{l,\text{lensing}}^{BB}$ and $C_{l,\text{tensor}}^{BB}$ are the contributions from the lensing and tensor B-modes, respectively. As a clarification, the bestfit $C_{l,\text{total}}^{BB}$, $C_{l,\text{lensing}}^{BB}$ and $C_{l,\text{tensor}}^{BB}$ in our w_0w_a CDM model are plotted in Fig. 3.

As displayed in Fig. 4, the 1D posteriors of r in Λ CDM and w_0w_a CDM models both peak around the best-fit value $r \approx 0.01$, see also Table 1. The w_0w_a CDM prefers a smaller r as a result of a slightly larger Ω_m than Λ CDM. It should be mentioned that different from Λ CDM with pre-DESI BAO, here Λ CDM is not the best-fit model of Planck18+BK18+DESI+PantheonPlus datasets, $\chi^2_{w_0w_a\text{CDM}} - \chi^2_{\Lambda\text{CDM}} = -7.97$ according to Table 2. We also provide a profile likelihood analysis in Fig. 4 using ProCoLi (<https://github.com/tkarwal/procoli>) [82]. Here $\chi^2_{r=0} - \chi^2_{\text{bestfit}} > 1$ is clearly seen for both Λ CDM and w_0w_a CDM, which seems to prefer $r > 0$.

4. Conclusion

The search for primordial GWs has been always an unremitting pursuit in the scientific community. In this Letter, we find that recent dataset suggesting the evolving DE will inevitably bring the shifts of the bestfit values of relevant cosmological parameters, and consequently highlight the potential existence of primordial GWs, which can be confirmed by the ongoing cosmological surveys, such as DESI and Euclid [55], and the upcoming CMB B-mode polarization experiments, such as upgrading BICEP/Keck series [53] (reach $\sigma(r) \lesssim 0.003$ after delensing) and CMB-S4 [54].

Here, our approach towards r indicates that the significant signal of primordial GWs might have already emerged in CMB B-mode data, however, its detection is depending significantly on our insight into the nature of DE (the accurate insight brought by BAO data into DE is a necessary requirement for identifying the signal of primordial GWs, which has always been overlooked), opening an unexpected window for comprehending our primordial universe in new era of cosmological surveys. Currently, though it is still too early to say whether DE is evolving or not, hopefully the further data of DESI and upcoming Euclid [55] com-

Table 3

Mean (bestfit) values and 1σ regions of the parameters of models, fitting to Planck18+BK18+DESI+Pantheon Plus datasets.

Parameters	$w_0 w_a$ CDM+NCC
$100\omega_b$	$2.233(2.232)\pm 0.014$
ω_{cdm}	$0.120(0.120)\pm 0.001$
H_0	$68.03(67.95)\pm 0.75$
$\ln 10^{10} A_s$	$3.034(3.046)\pm 0.015$
n_s	$0.966(0.967)\pm 0.004$
τ_{reio}	$0.052(0.058)\pm 0.008$
w_0	$-0.902(-0.956)\pm 0.064$
w_a	$-0.544(-0.237)\pm 0.707$
Ω_L	$-1.08(-0.749)\pm 0.65$
r	$0.0160(0.0072)^{+0.0066}_{-0.0127}$
Ω_m	$0.307(0.308)\pm 0.007$
S_8	$0.833(0.842)\pm 0.012$
χ^2_{CMB}	2771.09
χ^2_{DESI}	13.78
χ^2_{BK18}	537.31
$\chi^2_{\text{Pantheon+}}$	1409.03
χ^2_{tot}	4730.71

binned with CMB and Pantheon Plus dataset would explicitly test our result, and possibly tell us the corresponding answers.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Yun-Song Piao reports financial support was provided by National Natural Science Foundation of China. Yun-Song Piao reports financial support was provided by National Key Research and Development Program of China. Yun-Song Piao reports financial support was provided by Fundamental Research Funds for the Central Universities. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. $w_0 w_a$ CDM+NCC

In order to see to what extent the results on r are model-dependent, we also consider the $w_0 w_a$ CDM+NCC model [40], where one negative cosmological constant (NCC) and one evolving DE component coexist. The negative CC, which might contribute to settle the Hubble tension (e.g. AdS-EDE [68–72] and the switch-sign CC [73–77] models), was hinted by DESI BAO data [40,41], see earlier Refs. [62–67].

The corresponding MCMC parameters are $\{\omega_b, \omega_{cdm}, H_0, \ln 10^{10} A_s, n_s, \tau_{reio}, w_0, w_a, r, \Omega_L\}$ [40] and we adapt the prior $[-2, 0.6]$ on Ω_L . In Table 3, we present the 1σ and 2σ marginalized posterior distributions of the main parameters for the $w_0 w_a$ CDM+NCC model fitting to Planck18+DESI+BK18+Pantheon Plus dataset.

The $w_0 w_a$ CDM+NCC model shows a similar result on r (its lower bound is 1.84 σ non-zero). The fit is improved by $\chi^2_{w_0 w_a \text{ CDM+NCC}} - \chi^2_{\Lambda \text{ CDM}} = -8.48$. Here, $\Omega_L = -1.08 \pm 0.65$, which suggests that the NCC, corresponding an anti-de Sitter (AdS) vacuum, might exist. It is noteworthy that in such a model with NCC the posteriors of w_0 and w_a are $w_0 = -0.902 \pm 0.064$ and $w_a = -0.544 \pm 0.707$, closer to $w_0 = -1$ and

$w_a = 0$, in particular, $w_0 + w_a \geq -1$, i.e. well-known quintessence DE, can be 1σ consistent, see also [40].

Data availability

No data was used for the research described in the article.

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