Gate-based quantum simulation of Gaussian Bosonic circuits on exponentially many modes

Barthe, A.M.; Cerezo, M.; Sornborger, A.T.; Larocca, M.; Garcia-Martin, D.

Gate-Based Quantum Simulation of Gaussian Bosonic Circuits on Exponentially Many Modes

Alice Barthe,[1,2,3] M. Cerezo,[4,5,*] Andrew T. Sornborger,[4] Martín Larocca,[2,6] and Diego García-Martín[4]
[1]CERN, Meyrin, Geneva 1211, Switzerland
[2]Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
[3]Instituut-Lorentz, Universiteit Leiden, Leiden 2333CA, The Netherlands
[4]Information Sciences, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
[5]Quantum Science Center, Oak Ridge, Tennessee 37931, USA
[6]Center for Non-Linear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

(Received 22 August 2024; accepted 9 January 2025; published 20 February 2025)

We introduce a framework for simulating, on an $(n+1)$-qubit quantum computer, the action of a Gaussian bosonic (GB) circuit on a state over 2^n modes. Specifically, we encode the initial bosonic state's expectation values over quadrature operators (and their covariance matrix) as an input qubit state. This is then evolved by a quantum circuit that effectively implements the symplectic propagators induced by the GB gates. We find families of GB circuits and initial states leading to efficient quantum simulations. For this purpose, we introduce a dictionary that maps between GB and qubit gates such that particle- (non-particle-) preserving GB gates lead to real- (imaginary-) time evolutions at the qubit level. For the special case of particle-preserving circuits, we present a bounded-error-quantum-polynomial time (BQP)-complete GB decision problem, indicating that GB evolutions of Gaussian states on exponentially many modes are as powerful as universal quantum computers. We also perform numerical simulations of an interferometer on $\sim 8 \times 10^9$ modes, illustrating the power of our framework.

DOI: 10.1103/PhysRevLett.134.070604

Introduction—The study of the power of quantum computers has been a central and fundamental topic in complexity theory. While it is known that these devices can solve problems at least as fast as classical probabilistic computers, we also know that they are unable to do so more than exponentially faster [1,2]. This has raised the question: *What are the tasks that saturate this separation? In other words, what are the problems for which a quantum computer can achieve an exponential advantage over its classical counterparts?*.

To answer such questions, one starts with the class bounded-error quantum polynomial time (BQP), the set of decision problems that a quantum computer can solve in polynomial time with a small constant probability of failure. Then, one determines the subset of problems that are the hardest therein, known as BQP-complete. Several BQP-complete problems are known, such as those based on the Harrow-Hassidim-Lloyd algorithm [3], scattering in scalar quantum field theory [4], and, more recently, on the quantum simulation of exponentially many coupled classical oscillators [5]. The latter presents the intriguing perspective that simulating exponentially large linear and energy-preserving simple classical systems leads to BQP-completeness and, under reasonable complexity theory assumptions, to an exponential quantum advantage over classical methods.

In this work, we prove an analogous result to that in Ref. [5], namely, that the quantum simulation of particle-preserving Gaussian bosonic (GB) circuits (i.e., passive linear optics) [6–8] acting on Gaussian initial states on exponentially many modes also leads to BQP-completeness (see Fig. 1). At its core, our framework starts with the realization that a direct simulation of a bosonic system is intractable on a qubit-based quantum computer, as the associated Hilbert spaces are fundamentally different (with one being infinite dimensional and the other discrete). This issue can be avoided by restricting the simulation to the first and second moments of the quadrature operators. That is, we encode in a quantum state the position and momentum expectation values (and their covariance matrix) over the initial bosonic state. As such, instead of simulating the action of the GB circuit on the bosonic Hilbert space, we implement its effective action on the expectation values on a gate-based quantum computer.

In this context, we present a constructive dictionary that translates back and forth between the symplectic

*Contact author: cerezo@lanl.gov

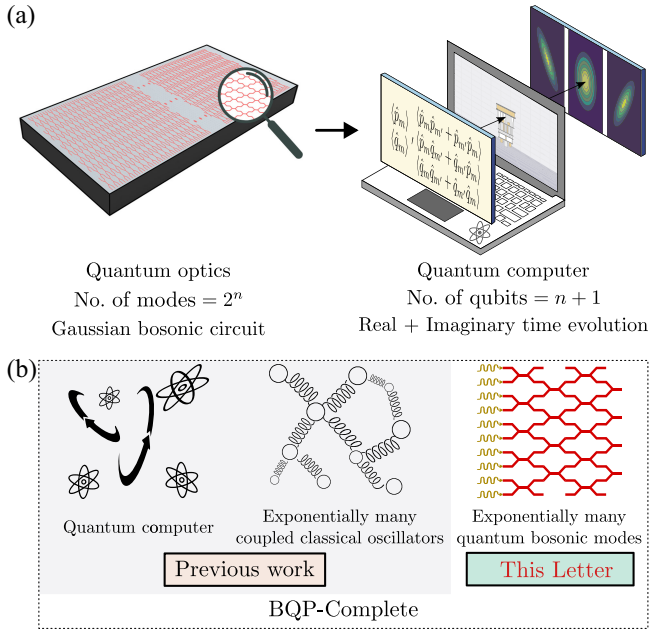

FIG. 1. Schematic representation of our main results. (a) We present a framework for simulating the action of a GB circuit on the first and second moments of quadrature operators of a bosonic state on 2^n modes on an $(n+1)$-qubit gate-based quantum computer. (b) We show that particle-preserving GB evolutions on Gaussian bosonic states are sufficient to define a problem that is BQP-complete, thus indicating that passive linear optics on exponentially many bosonic modes are as powerful as universal quantum computers.

propagator associated with a universal set of GB gates (beam splitters, phase, and squeezing gates) and qubit circuits. The efficiency of our simulation framework relies on several key conditions, such as the input qubit state being preparable in polynomial time and the quantum circuit requiring only polynomially many gates. Indeed, we present cases of interest for which these two conditions are satisfied and, therefore, for which we can achieve an exponential quantum advantage.

Background—In what follows, we will consider systems composed of M bosonic modes (with $M = 2^n$). Let $\hat{a}_m^\dagger$ and $\hat{a}_m$, with $m = 1, \ldots, M$, respectively, denote the creation and annihilation operators for the mth mode [9]. We consider the standard Hermitian *quadrature operators*, position $\hat{q}_m = (1/\sqrt{2})(\hat{a}_m + \hat{a}_m^\dagger)$ and momentum $\hat{p}_m = (i/\sqrt{2})(\hat{a}_m^\dagger - \hat{a}_m)$. They satisfy the canonical commutation relations $[\hat{q}_m, \hat{p}_{m'}] = i\delta_{mm'}$. Furthermore, we will focus on the case where an M-mode bosonic state ρ_0 evolves under the action of a GB circuit whose gates are generated by time-independent Hamiltonians that are quadratic in the position and momentum operators (a generalization to time-dependent Hamiltonians is direct using standard Hamiltonian-simulation techniques [10]).

These GB generators, also known as free-bosonic generators, are arbitrary real-valued degree-two homogeneous polynomials on the quadrature operators

$$\hat{H} = \frac{1}{2}\hat{z}^T K \hat{z}, \quad \text{with} \quad \hat{z} = (\hat{q}_1, \ldots, \hat{q}_M, \hat{p}_1, \ldots, \hat{p}_M)^T, \quad (1)$$

where K is a real $2M \times 2M$ symmetric matrix. The vector $\hat{z}$ allows us to express the commutation relations in the compact form $[\hat{z}_\alpha, \hat{z}_\beta] = i\Omega_{\alpha\beta}$, where $\Omega = iY \otimes \mathbb{1}_M$. Here, Y is the usual 2×2 Pauli matrix and $\mathbb{1}_M$ the $M \times M$ identity matrix.

Next, we collect the expectation values of the position and momentum operators in a vector $\langle\hat{z}\rangle = (\langle\hat{q}_1\rangle, \ldots, \langle\hat{q}_M\rangle, \langle\hat{p}_1\rangle, \ldots, \langle\hat{p}_M\rangle)^T \in \mathbb{R}^{2M}$, with $\langle\hat{x}\rangle$ the expectation value of $\hat{x}$ over ρ_0. As shown in Supplemental Material [11], evolving ρ_0 with a unitary generated by a GB generator $\hat{H}$ for a time t induces the evolution of $\langle\hat{z}\rangle$ as

$$\frac{\partial\langle\hat{z}\rangle}{\partial t} = \Omega K \langle\hat{z}\rangle, \quad \text{so that } \langle\hat{z}\rangle(t) = e^{t\Omega K}\langle\hat{z}\rangle(0). \quad (2)$$

Here, $\langle\hat{z}\rangle(t)$ denotes the vector containing the expectation values of positions and momenta at time t, and, thus, $\langle\hat{z}\rangle(0)$ represents the initial condition. Since the canonical commutation relations must be preserved, the propagator $e^{t\Omega K}$ is a $2M \times 2M$ symplectic matrix with real entries belonging to the Lie group $\mathbb{SP}(M, \mathbb{R})$ [9], which, in turn, implies that ΩK is an operator in the symplectic Lie algebra $\mathfrak{sp}(M, \mathbb{R})$.

Note that while, in general, $e^{t\Omega K}$ is not unitary, we characterize the quadratic Hamiltonians leading to unitary evolutions of the vector $\langle\hat{z}\rangle$, when a gate generator is of the form $\hat{H} = \sum_{m,m'=1}^{M} h_{mm'}\hat{a}_m^\dagger\hat{a}_{m'} + (\mathrm{Tr}[h]/2)\mathbb{1}_{2M}$, with h a Hermitian matrix. Then $[\Omega, K] = 0$, and the propagator $e^{t\Omega K}$ is the real-time evolution of the Hermitian $i\Omega K$. Hamiltonians of this form are known as *particle preserving* [12], since the hopping terms $\hat{a}_m^\dagger\hat{a}_{m'}$ move a boson from mode m' to mode m. We also characterize *non-particle-preserving Hamiltonians* of the form $\hat{H} = \sum_{m,m'=1}^{M} \Delta_{mm'}^\dagger \hat{a}_m\hat{a}_{m'} + \mathrm{H.c.}$, whose propagator corresponds to the imaginary-time evolution of $\langle\hat{z}\rangle$ under the effective Hamiltonian $-\Omega K$.

In addition to $\langle\hat{z}\rangle$, we also collect the expectation value of products of quadrature operators over ρ_0 in the $2M \times 2M$ positive-definite covariance matrix $\boldsymbol{\sigma}$ whose entries are given by $\boldsymbol{\sigma}_{\alpha\beta} = \frac{1}{2}\langle\hat{z}_\alpha\hat{z}_\beta + \hat{z}_\beta\hat{z}_\alpha\rangle - \langle\hat{z}_\alpha\rangle\langle\hat{z}_\beta\rangle$ [13]. Analogously to Eq. (2), we find (see Supplemental Material [11]) that

$$\frac{\partial\boldsymbol{\sigma}}{\partial t} = \Omega K\boldsymbol{\sigma} - \boldsymbol{\sigma}K\Omega, \quad \text{so that } \boldsymbol{\sigma}(t) = e^{\Omega K t}\boldsymbol{\sigma}(0)e^{\mp\Omega K t},$$

where the $-(+)$ sign corresponds to particle- (non-particle-) preserving Hamiltonians, as defined above.

The previous insights pave the way to simulate the action of GB circuits on a gate-based quantum computer.

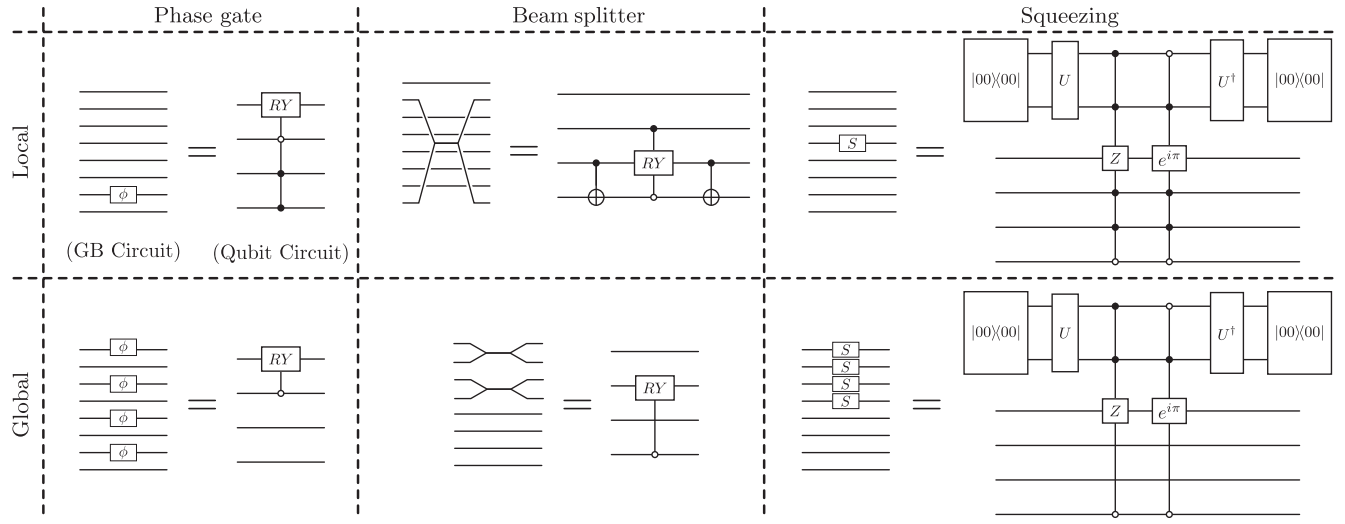

FIG. 2. Examples of GB gates in the qubit picture. We consider a bosonic system on $M = 8$ modes, leading to a circuit on four qubits. The local phase gate acts on the mode $m = 6 = 2^0 \times 0 + 2^1 \times 1 + 2^2 \times 1$. The local beam splitter acts on the modes $m = 1$ and $m' = 7$, whose binary representations share only the least significant bit. The local squeezing gate acts on mode $m = 3 = 2^0 \times 1 + 2^1 \times 1 + 2^2 \times 0$ and is represented by an imaginary-time evolution as a linear combination of unitaries with postselection on two ancillary qubits (which have been added on top). The global phase gate acts on all modes whose index is even. The global beam splitter is applied to the first half of the modes, pairing each mode with even index m to its nearest-neighbor mode with index $m' = m + 1$. The global squeezing gate is applied to the first half of the modes.

Initialization—To start the simulation on the quantum computer, we encode the normalized vector $\langle\hat{z}\rangle$ (normalized matrix $\boldsymbol{\sigma}$) into a pure (mixed) $(n+1)$-qubit quantum state $|\hat{z}\rangle \propto \langle\hat{z}\rangle$ ($\varrho_{\boldsymbol{\sigma}} \propto \boldsymbol{\sigma}$). For instance, we have

$$|\hat{z}\rangle = \frac{1}{\|\langle\hat{z}\rangle\|_2} \sum_{m=1}^{2^n} \langle\hat{q}_m\rangle|0\rangle \otimes |m\rangle + \langle\hat{p}_m\rangle|1\rangle \otimes |m\rangle. \quad (3)$$

For our scheme to be efficient, such states need to be preparable in polynomial time. Such cases occur, e.g., when there are $\mathcal{O}(\mathrm{poly}(n))$ nonzero known position and momentum expectation values. Equation (3) illustrates the privileged role of the first qubit [14], separating the Hilbert space into two subspaces, one associated with the positions and the other with the momenta. We will henceforth refer to the first qubit as the *symplectic qubit*. The remaining n qubits serve as a register for each of the 2^n modes, and we will refer to them as *register qubits*. In particular, in Eq. (3), each mode label m is encoded via its binary decomposition as $m = 2^0 m_1 + 2^1 m_2 + \cdots + 2^{n-1} m_n$, with $m_l \in \{0, 1\}$, and with the least significant qubit being the topmost register qubit.

Here, we note that, when ρ_0 is Gaussian, then it is fully characterized by the first and second moments of the quadrature operators, meaning that $|\hat{z}\rangle$ and $\varrho_{\boldsymbol{\sigma}}$ provide a full description of these states. Moreover, if the initial state is also coherent (eigenstates of $\hat{a}$), then $\varrho_{\boldsymbol{\sigma}}$ is the maximally mixed state on $(n+1)$ qubits. For other non-Gaussian states, our framework is restricted to the information contained in $\hat{\boldsymbol{z}}$ and $\boldsymbol{\sigma}$.

Evolution—Once the initial state is prepared, we can evolve it with a quantum circuit that effectively implements the symplectic propagators associated with the gates in the GB circuit [see, for instance, Eq. (2)]. In particular, we present a dictionary to efficiently translate between standard GB and qubit gates. We also refer the reader to Fig. 2 for an explicit circuit depiction of some GB gates in qubit circuit form.

The *phase gate* is a particle-preserving gate acting on a single mode m. In terms of bosonic operators, its generator is $\hat{H} = \hat{p}_m^2 + \hat{q}_m^2$, yielding $\Omega K = 2iY \otimes |m\rangle\langle m|$ in the qubit picture. This corresponds to an R_y gate (rotation about the y axis) on the symplectic qubit, conditioned on the $|m\rangle\langle m|$ state on the register.

The *beam splitter* is a particle-preserving gate acting on two modes m and m'. It is generated by $\hat{H} = \hat{q}_m \hat{p}_{m'} - \hat{q}_{m'} \hat{p}_m$ (with $m \neq m'$), which results in $\Omega K = 2i\mathbb{1} \otimes (i|m\rangle\langle m'| - i|m'\rangle\langle m|)$. This corresponds to an R_y rotation in the subspace spanned by $|m\rangle$ and $|m'\rangle$. This gate can be implemented by using controlled-not gates to transform $|m\rangle$ and $|m'\rangle$ into two computational-basis states that differ only in one qubit, then performing a controlled-R_y rotation on that qubit (conditioned on the other $n-1$ qubits in the register), and applying the same controlled-not gates to return to the original basis. We highlight the fact that this gate acts trivially on the symplectic qubit, as beam splitters conserve total momentum and total position.

The *squeezing gate* is a non-particle-preserving gate that acts on a single mode m. Its generator is

$\hat{H} = \pm(\hat{q}_m\hat{p}_m + \hat{p}_m\hat{q}_m)$, leading to $\Omega K = \pm 2X \otimes |m\rangle\langle m|$. This produces an imaginary-time evolution [15–18], which can be implemented, e.g., with two ancillary qubits and postselection. As illustrated in Fig. 2 and further discussed in Supplemental Material [11], we propose to implement it as a linear combination of unitaries [19], with the latter being multicontrolled-Z and controlled-phase gates. Notably, the fact that states cannot be arbitrarily squeezed translates in our framework as the success probability of the imaginary-time evolution for infinite time being zero.

Finally, the *displacement gate*, a common non-particle-preserving Gaussian gate, cannot be included as a linear qubit gate in the proposed framework (see Supplemental Material [11] and [20] for an additional discussion on this gate).

We stress that, while our framework requires the implementation of multicontrolled qubit operations to simulate the action of some GB gates, these can be compiled exactly using only $\mathcal{O}(n)$ local gates [21]. Moreover, if instead of one or two modes, we consider GB gates that act on several (potentially exponentially many) modes, there can be simplifications that render them much easier to implement at the qubit level. As an important example, we show in Fig. 2 (see also Supplemental Material [11]) how some global phase gates and beam splitters acting on 2^{n-1} modes simplify to two-qubit controlled-R_y rotations, as well as how the number of control qubits is reduced for some global squeezing gates. This means that, when the GB circuit is composed of polynomially many such local particle-preserving gates, the implementation of the quantum circuit is efficient. When non-particle-preserving squeezing gates are added, then the efficiency will ultimately depend on how many such gates are added, as well as on their squeezing strength parameters.

At this point, we recall that the covariance matrix of coherent states leads to a maximally mixed state $\varrho_{\boldsymbol{\sigma}}$, and, therefore, it remains invariant when evolved with a purely unitary circuit (e.g., particle-preserving GB gates in an interferometer). This result is well aligned with the bosonic picture where coherent states going through interferometers remain coherent states. Then, as squeezing gates are not particle preserving, their action on a coherent state corresponds to purifying the associated $\varrho_{\boldsymbol{\sigma}}$.

Measurements—At the output of the quantum circuit, we obtain states that represent the evolved expectation values and covariance matrix of quadrature operators (which we, respectively, denote as $|\hat{z}'\rangle$ and $\varrho_{\boldsymbol{\sigma}'}$). We now discuss how measuring these states allows us to extract useful information about the GB circuit.

There are a variety of measurements of interest for Gaussian states. As detailed in Supplemental Material [11], photon counting can be implemented for coherent states by sampling bit strings from $|\hat{z}'\rangle$. To understand why this is the case, we recall that the energy for each mode is proportional to the probability of sampling the corresponding bit string. Second, for any bosonic state, homodyne measurements correspond to retrieving the position (momentum) of a specific mode. At the qubit level, these measurements can be implemented from a swap-Hadamard test between $|\hat{z}'\rangle$ and some computational-basis state of interest. Then, our framework also allows us to estimate the fraction of total momentum and total position, as this value can be retrieved by measuring the symplectic qubit in $|\hat{z}'\rangle$. Similarly, the fractional energy of the first and second half of the modes can be estimated by measuring the bottommost (most significant) register qubit. Moreover, we note that combining computational basis measurements (or Hadamard tests) on $|\hat{z}'\rangle$ and $\varrho_{\boldsymbol{\sigma}'}$ allows us to estimate the energy in a given mode. To finish, we note that, while the total energy remains constant for particle-preserving GB circuits, this quantity can change when non-particle-preserving gates are included. In this case, while we cannot directly measure the total energy of the system (as the states need to be normalized), one can keep track of the total energy by using as a proxy the success probability of the imaginary-time evolutions.

BQP-completeness—We now show that we can leverage our framework to devise a decision problem based on a restricted class of large optical quantum interferometers and prove that it is BQP-complete. We begin by introducing a family of quantum interferometers, that we refer to as *bit structured*.

Definition 1 (bit-structured interferometer)—A bit-structured interferometer acting on 2^n nodes consists of L global beam splitters, such that each global beam splitter acts on 2^{n-1} modes. A global beam splitter is specified by two natural numbers, $k \neq l$, between 1 and n. The global beam splitter then acts on all the modes with indices $\{m\}$ such that their kth bit is equal to 0, by applying local beam splitters between modes with indices m, m' that differ only in their lth bit.

Our decision problem is then phrased as follows.

Problem 1—Consider a bit-structured interferometer (see Definition 1) acting on 2^n modes with $L \in \mathcal{O}(\mathrm{poly}(n))$ and an input state such that the first mode is displaced in position by a real constant x while the state of the remaining modes is the vacuum. Then, decide whether the expectation value of the position on the first mode at the output of the interferometer is

$$1.\ \langle \hat{q}_1 \rangle > \frac{2}{3}x \quad \text{or} \quad 2.\ \langle \hat{q}_1 \rangle < \frac{1}{3}x,$$

given the promise that either one or the other is true.

Problem 1 is illustrated in Fig. 3, where we simulate an interferometer with $\sim 8 \times 10^9$ modes (i.e., a 33-qubit circuit). There, we keep track of $\langle \hat{q}_1 \rangle / x$ (which corresponds to the overlap with the $|0\rangle^{\otimes n}$ state) as the state evolves through the beam splitters.

Our main result is the next theorem.

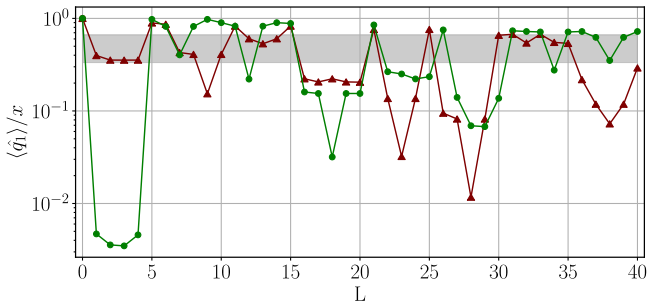

FIG. 3. Simulation of a bit-structured interferometer on $\sim 8 \times 10^9$ modes. We illustrate Problem 1 by tracking two nontrivial evolutions of $\langle \hat{q}_1 \rangle / x$ along a large bit-structured interferometer, the green (red) plot corresponding to a yes (no) instance. The gray region corresponds to $\frac{1}{3} < \langle \hat{q}_1 \rangle / x < \frac{2}{3}$. The simulations were performed with the open-source library Qibo [22,23].

Theorem 1—Problem 1 is BQP-complete.

The proof of Theorem 1 can be found in Supplemental Material [11], but we give here a summary of the key points. First, we show that Problem 1 is contained in BQP. To do this, we simply use the mapping between beam splitters and qubit gates explained previously to prove that a bit-structured interferometer with $\mathcal{O}(\mathrm{poly}(n))$ layers always results in a polynomial-size quantum circuit, which is a necessary condition to be contained in BQP.

Finally, we need to prove that Problem 1 is BQP-hard. That is, if we can solve it, we can also solve any other problem in BQP with an additional overhead that is polynomial in n. This is done by first showing that each global beam splitter gives rise to a controlled-R_y gate (acting when the control qubit is in the $|0\rangle$ state). Then, we use the result from [24], which states that controlled-R_y rotations constitute a universal gate set, in the sense that any quantum computation can be performed with these gates [25,26].

Outlook—This work contributed to the body of knowledge of schemes leading to BQP-complete tasks. In particular, along with Ref. [5], we show examples of how linear and energy-preserving evolutions of exponentially many simple physical systems can be efficiently simulated on a quantum computer. A key unique feature of our framework is that we translate a sequence of gates from one physical system to another (product of exponentials) rather than performing real-time evolution of a Hamiltonian (exponential of a sum).

Importantly, our work opens up several interesting research directions. For instance, we have shown how to simulate the evolution of the first and second moments of bosonic states. This makes the simulation complete only for Gaussian states. Hence, we envision two possible paths to extend our proposed framework to better approximate the simulation of non-Gaussian states. First, we could find ways to simulate the evolution of higher moments. To what extent this would improve the quality of a simulation for non-Gaussian states, such as Fock states, remains open. Second, because bosonic states can be written as a continuous sum over coherent states, approximating them on a grid of coherent states may also be a viable strategy to simulate non-Gaussian states. Finally, we also leave for future work a precise characterization of the computational complexity of the quantum simulation of non-particle-preserving GB circuits, which are likely outside of BQP.

Acknowledgments—We thank Samuel Slezak, Christa Zoufal, Paolo Braccia, and Nathan Killoran for insightful conversations. A. B. was initially supported by the U.S. Department of Energy through a quantum computing program sponsored by the Los Alamos National Laboratory (LANL) Information Science and Technology Institute. A. B. also acknowledges support by CERN through the CERN Quantum Technology Initiative and by the quantum computing for Earth observation (QC4EO) initiative of ESA ϕ-lab, partially funded under Contract No. 4000135723/21/I-DT-lr, in the FutureEO program. M. C. and A. T. S. were initially supported by LANL's ASC Beyond Moore's Law project. M. L. was supported by the Center for Nonlinear Studies at LANL and by the Laboratory Directed Research and Development (LDRD) program of LANL under Project No. 20230049DR. D. G.-M. was supported by LDRD program of LANL under Project No. 20230527ECR. This work was also supported by the Quantum Science Center (QSC), a National Quantum Information Science Research Center of the U.S. Department of Energy (DOE).

[1] E. Bernstein and U. Vazirani, Quantum complexity theory, SIAM J. Comput. **26**, 1411 (1997).
[2] S. Aaronson, BQP and the polynomial hierarchy, in *Proceedings of the Forty-Second ACM Symposium on Theory of Computing* (2010), pp. 141–150, 10.1145/1806689.1806711.
[3] A. W. Harrow, A. Hassidim, and S. Lloyd, Quantum algorithm for linear systems of equations, Phys. Rev. Lett. **103**, 150502 (2009).
[4] S. P. Jordan, H. Krovi, K. S. Lee, and J. Preskill, BQP-completeness of scattering in scalar quantum field theory, Quantum **2**, 44 (2018).
[5] R. Babbush, D. W. Berry, R. Kothari, R. D. Somma, and N. Wiebe, Exponential quantum speedup in simulating coupled classical oscillators, Phys. Rev. X **13**, 041041 (2023).
[6] E. Knill, R. Laflamme, and G. J. Milburn, A scheme for efficient quantum computation with linear optics, Nature (London) **409**, 46 (2001).
[7] A. Bouland and S. Aaronson, Generation of universal linear optics by any beam splitter, Phys. Rev. A **89**, 062316 (2014).
[8] A. Sawicki, Universality of beamsplitters, Quantum Inf. Comput. **16**, 291 (2016).
[9] S. L. Braunstein and P. Van Loock, Quantum information with continuous variables, Rev. Mod. Phys. **77**, 513 (2005).

[10] D. Poulin, A. Qarry, R. Somma, and F. Verstraete, Quantum simulation of time-dependent Hamiltonians and the convenient illusion of Hilbert space, Phys. Rev. Lett. **106**, 170501 (2011).
[11] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.134.070604 for the full derivation of the simulation framework, the mapping between bosonic and qubit gates, and the proof of Theorem 1.
[12] M. Oszmaniec and Z. Zimborás, Universal extensions of restricted classes of quantum operations, Phys. Rev. Lett. **119**, 220502 (2017).
[13] G. Adesso, S. Ragy, and A. R. Lee, Continuous variable quantum information: Gaussian states and beyond, Open Syst. Inf. Dyn. **21**, 1440001 (2014).
[14] D. García-Martín, P. Braccia, and M. Cerezo, Architectures and random properties of symplectic quantum circuits, arXiv:2405.10264.
[15] M. Motta, C. Sun, A. T. Tan, M. J. O'Rourke, E. Ye, A. J. Minnich, F. G. Brandao, and G. K.-L. Chan, Determining eigenstates and thermal states on a quantum computer using quantum imaginary time evolution, Nat. Phys. **16**, 205 (2020).
[16] C. Zoufal, A. Lucchi, and S. Woerner, Variational quantum Boltzmann machines, Quantum Mach. Intell. **3**, 1 (2021).
[17] C. Zoufal, D. Sutter, and S. Woerner, Error bounds for variational quantum time evolution, Phys. Rev. Appl. **20**, 044059 (2023).
[18] T. L. Silva, M. M. Taddei, S. Carrazza, and L. Aolita, Fragmented imaginary-time evolution for early-stage quantum signal processors, Sci. Rep. **13**, 18258 (2023).
[19] A. M. Childs and N. Wiebe, Hamiltonian simulation using linear combinations of unitary operations, Quantum Inf. Comput. **12**, 0901 (2012).
[20] Z. Holmes, N. Coble, A. T. Sornborger, and Y. Subaşı, On nonlinear transformations in quantum computation, Phys. Rev. Res. **5**, 013105 (2023).
[21] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
[22] S. Efthymiou, S. Ramos-Calderer, C. Bravo-Prieto, A. Pérez-Salinas, D. García-Martín, A. Garcia-Saez, J. I. Latorre, and S. Carrazza, Qibo: A framework for quantum simulation with hardware acceleration, Quantum Sci. Technol. **7**, 015018 (2021).
[23] S. Efthymiou, M. Lazzarin, A. Pasquale, and S. Carrazza, Quantum simulation with just-in-time compilation, Quantum **6**, 814 (2022).
[24] T. Rudolph and L. Grover, A 2 rebit gate universal for quantum computing, arXiv:quant-ph/0210187.
[25] Y. Shi, Both Toffoli and controlled-not need little help to do universal quantum computation, Quantum Inf. Comput. **3**, 84 (2003).
[26] D. Aharonov, A simple proof that Toffoli and Hadamard are quantum universal, arXiv:quant-ph/0301040.