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We treat privacy in a network of quantum sensors where accessible information is limited to specific functions of the network parameters, and all other information remains private. We develop an analysis of privacy in terms of a manipulation of the quantum Fisher information matrix, and find the optimal state achieving maximum privacy in the estimation of linear combination of the unknown parameters in a network of quantum sensors. We also discuss the effect of uncorrelated noise on the privacy of the network. Moreover, we illustrate our results with an example where the goal is to estimate the average value of the unknown parameters in the network. In this example, we also introduce the notion of quasiprivacy (ϵ privacy), quantifying how close the state is to being private.

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Simultaneously estimating spatially distributed parameters via a quantum network, or networked quantum sensing, has numerous applications, including clock synchronization [1,2] and phase imaging [3–5]. Alongside experimental progress [6,7], theoretical studies are advancing to address practical challenges in the field. A major concern is the presence of adversaries who may eavesdrop on quantum channels [8–10]. In this context, the goal is not only to achieve optimal precision in parameter estimation but also to ensure security. While security in single-parameter quantum estimation has been explored [11–14], networked quantum sensing requires a distinct focus on security concepts [15,16].

In this Letter, we develop the notion of *privacy* introduced in [15] and its relation to standard multiparamater estimation tools, notably the quantum Fisher information matrix. The goal of a private network of quantum sensors is to ensure optimal precision *and* that all parties have access only to the allowed information, and not more—so that it remains private. To set the stage, let us consider a statistical model made of nodes, where at each node an unknown parameter θ_μ is encoded locally on a global quantum state via a given quantum channel $\Lambda_\mu(\theta_\mu)$. The overall channel is given by

$$\mathbf{\Lambda}_\Theta = \bigotimes_{\mu=1}^{d} \Lambda_\mu(\theta_\mu), \tag{1}$$

where $\Theta = \{\theta_1, \theta_2, \ldots, \theta_d\}$ denotes the set of unknown parameters. After the encoding stage, local measurements are performed at each node and the results are announced publicly. The conditional probability distribution of the outcomes is given by the Born rule $p(x|\Theta) = \mathrm{Tr}[\rho_\Theta \Pi_x]$, in which ρ_Θ is the quantum state of the probe after the encoding and $\{\Pi_x\}$ represents a (factorized) positive operator-valued measure acting on the global Hilbert space describing the overall state at all the nodes. After collecting results x from repeated (local) measurements, one can estimate the value of unknown parameter θ_μ by an estimator function $\tilde{\theta}_\mu(x)$. The general scheme of the protocol is depicted as in Fig. 1.

In local estimation theory, the classical Fisher information matrix (CFIM) quantifies the amount of information that may be extracted about the set of unknown parameters given the state of the probe (also known as the statistical model) and a specific measurement. The entries of the CFIM are given by

$$\mathcal{F}_{\mu\nu}(\Theta) = \int \mathrm{d}x\, p(x|\Theta)\partial_\mu \ln p(x|\Theta)\partial_\nu \ln p(x|\Theta), \tag{2}$$

where $\partial_\mu = (\partial/\partial\theta_\mu)$. In turn, the CFIM determines a lower bound on the precision of estimation through the so-called multiparameter Cramér-Rao bound [17–24]

$$\mathrm{Cov}(\Theta) \geq \frac{1}{\mathcal{F}}, \tag{3}$$

in which

$$\mathrm{Cov}_{\mu\nu}(\Theta) = \int \mathrm{d}x p(x|\Theta)\big(\tilde{\theta}_\mu(x) - \theta_\mu\big)\big(\tilde{\theta}_\nu(x) - \theta_\nu\big). \tag{4}$$

The metrological problem that we pose in this Letter is that of estimating a linear combination of unknown

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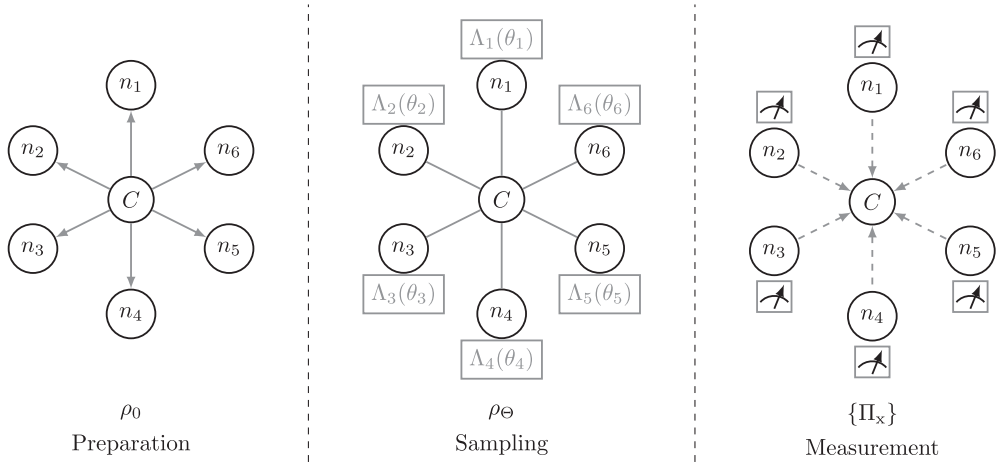

FIG. 1. Schematic of a network of quantum sensors with $d = 6$. After preparing and sharing the quantum probe ρ_0 by the central node (C) (preparation stage; in general it will be an entangled state), the μth unknown parameter (θ_μ) is encoded by local quantum operations $[\Lambda_\mu(\theta_\mu)]$, overall described by the factorized channel $\mathbf{\Lambda}_\Theta$ (sampling stage). In order to estimate the values Θ, the set of parameters, the quantum probe is locally measured (measurement stage) at each node. Measurement results are sent publicly to the central node.

parameters, namely, $f(\Theta)$. In this setting, privacy was introduced in [15] and means that each party μ can only access $f(\Theta)$ and their own parameter θ_μ and no other information [for example, they are not allowed to know the parameters of other parties unless it is equal to $f(\Theta)$]. However, that work focused on one particular function (the average of the parameters), and lacked a general way of addressing different combinations of unknown parameters. This Letter develops a more detailed account of privacy for any linear combinations, which also allows a more detailed analysis of optimality and noise.

Such a privacy quantifier in the network of quantum sensors should capture the idea that only the information about $f(\Theta)$ can be extracted from the network of quantum sensors, but the individual values of each parameter should remain hidden.

The CFIM actually depends on both the quantum statistical model ρ_Θ and the particular set of measurement operators $\{\Pi_x\}$. One can set an upper bound on the CFIM, by optimizing over all possible measurements (including joint entangled measurements across the nodes). Such an upper bound may be derived by introducing the symmetric logarithmic derivative (SLD) operator for each parameter, denoted by L_μ [19] as

$$\partial_\mu \rho_\Theta = \frac{1}{2}\{L_\mu, \rho_\Theta\}, \tag{5}$$

where $\{\cdot\}$ denotes the anticommutator. By substituting Eq. (5) in Eq. (2) and employing the Cauchy-Schwarz inequality [25–28], one obtains the following upper bound on the CFIM $\mathcal{F}_{\mu\nu} \leq \mathbf{Q}_{\mu\nu}[\Theta]$, where the quantum Fisher information matrix (QFIM) is defined as

$$\mathbf{Q}_{\mu\nu}[\Theta] = \frac{1}{2}\mathrm{Tr}\rho_\Theta\{L_\mu, L_\nu\}. \tag{6}$$

The QFIM is a symmetric matrix with real elements, which quantifies the maximum amount of extractable information about different unknown parameters over *all* possible measurements. In particular, the off-diagonal entries of the QFIM imply that the different unknown parameters are statistically correlated to each other. If the different SLDs do not commute, the different parameters cannot be estimated independently without the addition of intrinsic noise of quantum origin.

If the aim is to estimate some function(s) of unknown parameters, $\Theta' = f(\Theta)$, the corresponding CFIM and QFIM may be obtained by reparametrization,

$$\mathcal{F}' = B^T \mathcal{F} B, \tag{7}$$

$$\mathbf{Q}[\Theta'] = B^T \mathbf{Q}[\Theta] B, \tag{8}$$

where the elements of the transformation matrix B are defined as $B_{\mu\nu} = \partial\theta_\mu/\partial\theta'_\nu$ [26,29].

We will now see how the notion of privacy puts constraints on the form of the QFIM, that will allow us to state conditions for privacy and lead to its quantification in our example (the average of local parameters). The starting point is to first ask that the reparametrized QFIM, $\mathbf{Q}[\Theta']$, is a diagonal matrix. The diagonal form of the QFIM implies that there is no statistical correlation between the different linear functions of the unknown parameters (different θ's). Since the QFIM is a real symmetric positive definite matrix, it can be diagonalized by a similarity transformation. In the diagonal representation, the eigenvectors of $\mathbf{Q}[\Theta]$ correspond to the coefficients of the linear combination of the unknown parameters which can be

estimated in private. In particular, if the diagonal representation of $\mathbf{Q}[\Theta]$ is a one-rank matrix, only a single linear combination of the unknown parameters can be estimated privately. This is the requirement we should impose.

Let us assume that, in fact, the aim of the network is to share an estimate of a (single) linear combination of Θ; $\theta_1' = \mathbf{w}^T\Theta$ for some $\mathbf{w} \in \mathbb{R}^d$ [8,9,30]. In order to ensure privacy of this shared estimation protocol, the QFIM must be a one-rank matrix, i.e., $\mathbf{Q}[\Theta] \propto \mathbf{w}\mathbf{w}^T$ (or $\mathbf{Q}[\Theta] = a\mathbf{w}\mathbf{w}^T$, where a is a real positive constant). This fact implies that the only extractable information from the network is about θ_1' and the local information about the parameters is kept private. For a given vector of interest like $\mathbf{w}$, one can construct $W = \mathbf{w}\mathbf{w}^T$. In order to get the privacy, the QFIM of the statistical model should be proportional to W.

Since the concept of privacy in quantum networks is highly sensitive to the relationships between the different entries of the QFIM, the definition of privacy can be linked to the continuity relations among them [31–34]. Without any specific assumption about the initial states and how quantum states acquire their parameter dependence, we may arrive at the following theorem, which is the generalization of results in [33] for the entries of the QFIM.

Theorem—Given the generic statistical model ρ_Θ, the following inequality holds true:

$$\left|\mathbf{Q}_{\mu\nu}[\Theta] - \mathbf{Q}_{\mu'\nu'}[\Theta]\right| \leq \frac{1}{2}\xi\left[\|\partial_\mu\rho_\Theta - \partial_{\mu'}\rho_\Theta\|_1(\|\partial_\nu\rho_\Theta\|_1 + \|\partial_{\nu'}\rho_\Theta\|_1) + \|\partial_\nu\rho_\Theta - \partial_{\nu'}\rho_\Theta\|_1(\|\partial_\mu\rho_\Theta\|_1 + \|\partial_{\mu'}\rho_\Theta\|_1)\right], \tag{9}$$

where

$$\xi = \frac{1}{\lambda_{\min}(\tilde{\rho})}\left(1 + \frac{32}{\lambda_{\min}(\tilde{\rho})}\right), \tag{10}$$

and $\tilde{\rho}$ is the (invertible) restriction of ρ onto the support subspace of the quantum state.

Proof—See the Supplemental Material for the complete proof [35]. ∎

Such a continuity relation not only can help to find a proper initial state which provides privacy in the networked sensing but also paves the way to define quasiprivacy or ϵ privacy, which will be considered later in this Letter.

In order to gain better insight about the applications of the above results, let us consider the case where $\mathbf{w}^T = (\omega_1, \omega_2, \ldots, \omega_d)$, $\forall\ \omega_\mu \in \mathbb{R}$. This yields

$$W = \mathbf{w}\mathbf{w}^T = \begin{pmatrix} \omega_1\omega_1 & \omega_1\omega_2 & \cdots & \omega_1\omega_d \\ \omega_2\omega_1 & \omega_2\omega_2 & \cdots & \omega_2\omega_d \\ \vdots & \vdots & \ddots & \vdots \\ \omega_d\omega_1 & \omega_d\omega_2 & \cdots & \omega_d\omega_d \end{pmatrix}. \tag{11}$$

To obtain the privacy in the estimation of $\theta_1' = \mathbf{w}^T\Theta$, the QFIM should be proportional to W:

$$\mathbf{Q}_{\mu\nu}[\Theta] \propto W_{\mu\nu} \Rightarrow \mathbf{Q}_{\mu\nu}[\Theta] \propto \omega_\mu\omega_\nu, \quad \forall\ \mu,\nu. \tag{12}$$

For the purpose of finding proper quantum states where their corresponding QFIMs satisfy Eq. (12), the continuity relation, Eq. (9), can be recast as follows:

$$\left|\mathbf{Q}_{\mu\mu}[\Theta] - \mathbf{Q}_{\mu\nu}[\Theta]\right| \leq \xi'\|\partial_\mu\rho_\Theta - \partial_\nu\rho_\Theta\|_1, \quad \forall\ \mu \neq \nu, \tag{13}$$

where ξ' includes all other terms that are not pertinent to the rest of the derivation. Substituting Eq. (12) in Eq. (13) gives

$$|\omega_\mu - \omega_\nu| \leq \zeta\|\partial_\mu\rho_\Theta - \partial_\nu\rho_\Theta\|_1, \quad \forall\ \mu \neq \nu, \tag{14}$$

in which $\zeta = \xi'/|\omega_\mu|$. Since the proportionality is crucial here, without loss of generality, Eq. (14) can be rephrased as follows:

$$\|\partial_\mu\rho_\Theta - \partial_\nu\rho_\Theta\|_1 \propto |\omega_\mu - \omega_\nu|, \quad \forall\ \mu \neq \nu. \tag{15}$$

Hence, any set of quantum states which satisfies the above condition [Eq. (15)] can estimate θ_1' in private, irrespective of how it acquires the parameter dependence. Among the quantum states in this private set, the one that maximizes the single diagonal entry of the QFIM is the optimal state for precision. In the following, we specify our Letter to the case where the unknown parameters are encoded via local unitary evolutions, $U(\theta_\mu) = \mathrm{e}^{-iH_\mu(\theta_\mu)}$ onto a shared quantum state. Here $H_\mu(\theta_\mu)$ is a Hermitian operator that acts nontrivially on the Hilbert space of each quantum sensor. Hence, the sampling operator can be presented by

$$\mathbf{U}_\Theta = \bigotimes_{\mu=1}^{d} U(\theta_\mu) = \mathrm{e}^{-i\sum_\mu H_\mu}, \tag{16}$$

where $H_\mu = \mathbb{1} \otimes \mathbb{1} \otimes \cdots \otimes [H_\mu(\theta_\mu)]^{\otimes\omega_\mu} \otimes \cdots \otimes \mathbb{1} \otimes \mathbb{1}$. The first derivative of the density matrix in the case of unitary evolution is derived as follows:

$$\partial_\mu\rho_\Theta = -i[H_\mu', \rho_\Theta], \tag{17}$$

where $[\cdot]$ denotes the commutator and $H_\mu' = \partial_\mu H_\mu$. From whence the condition (15) can be cast in this form:

$$\|[H'_\mu - H'_\nu, \rho_\Theta]\|_1 \propto |\omega_\mu - \omega_\nu|, \quad \forall\ \mu \neq \nu. \tag{18}$$

For the unitary evolutions where their associated generators satisfy

$$[\partial_\mu H_\mu(\theta_\mu), H_\mu(\theta_\mu)] = 0, \quad \forall\ \mu, \tag{19}$$

Eq. (18) can be simplified more. Using the fact that $\rho_\Theta = \mathbf{U}_\Theta \rho_0 \mathbf{U}_\Theta^\dagger$ and $[\mathcal{A}, \mathcal{BCD}] = [\mathcal{A}, \mathcal{B}]\mathcal{CD} + \mathcal{BC}[\mathcal{A}, \mathcal{D}] + \mathcal{B}[\mathcal{A}, \mathcal{C}]\mathcal{D}$ for any arbitrary operators $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$, Eq. (18) yields

$$\|[H'_\mu - H'_\nu, \rho_0]\|_1 \propto |\omega_\mu - \omega_\nu|, \quad \forall\ \mu \neq \nu. \tag{20}$$

In order to estimate the linear combination of spatially distributed unknown parameters [which are encoded via local unitary operations where their generators satisfy Eq. (19)], the initial state of quantum probe should satisfy Eq. (20). Let us consider the case of multiplicative unknown parameter in which $H_\mu(\theta_\mu) = \theta_\mu H$ [which satisfies Eq. (19)]. Ergo,

$$H'_\mu = \mathbb{1} \otimes \mathbb{1} \otimes \cdots \otimes [\omega_\mu H \otimes (\theta_\mu H)^{\otimes \omega_\mu - 1}] \otimes \cdots \otimes \mathbb{1} \otimes \mathbb{1}. \tag{21}$$

In this case any pure states in the form of

$$|\Psi\rangle = \sum_{i=1}^{n} \alpha_i \bigotimes_{\mu=1}^{d} |\lambda_i\rangle^{\otimes \omega_\mu}, \tag{22}$$

where $\alpha_i \in \mathbb{C}$ and $\{|\lambda_i\rangle\}$ are the eigenvectors of n-dimensional H, satisfy condition (20) and provide privacy in the estimation of the linear combination with integer coefficients in the networked sensing.

Noise model—We now analyze the effect of noise. Generally, noise can affect any metrological schemes after or before the sampling stage. Let us consider the case where the quantum probe satisfies condition (15) and the noise affects the probe state after the sampling stage,

$$\rho'_\Theta = \Lambda_\Theta(\rho_0) = \sum_{\mathbf{k}=1}^{q^d} \mathbf{A}_\mathbf{k} \mathbf{U}(\Theta) \rho_0 \mathbf{U}(\Theta)^\dagger \mathbf{A}_\mathbf{k}^\dagger = \sum_{\mathbf{k}} \mathbf{A}_\mathbf{k} \rho_\Theta \mathbf{A}_\mathbf{k}^\dagger, \tag{23}$$

where $\mathbf{A}_\mathbf{k} = A_{k_1} \otimes A_{k_2} \otimes \cdots \otimes A_{k_d}$ in which $\mathbf{k} = \{k_1, k_2, \ldots, k_d\}$. In this notation $k_i \in \{1, 2, \ldots, q\}$ denotes the k_ith Kraus operator of the noise model which satisfies $\sum_{k=1}^{q} A_k^\dagger A_k = \mathbb{1}$ and acts on the ith node of the network [39]. Without loss of generality, one can consider the case where the Kraus operators do not depend on the set of unknown parameters. Hence,

$$\|\partial_\mu \rho'_\Theta - \partial_\nu \rho'_\Theta\|_1 = \left\| \sum_{\mathbf{k}=1}^{q^d} \mathbf{A}_\mathbf{k} (\partial_\mu \rho_\Theta - \partial_\nu \rho_\Theta) \mathbf{A}_\mathbf{k}^\dagger \right\|_1 \propto |\omega_\mu - \omega_\nu|, \quad \forall\ \mu \neq \nu, \tag{24}$$

which shows that the probe state remains private.

We explore privacy in cases where noise affects the quantum probe between preparation and sampling stages. Suppose the quantum states that ensure privacy in the ideal case are shared across the network. If all Kraus operators of the noise model commute with the sampling operators, the parameter of interest can still be privately estimated, as the noise model and sampling operators are separable, allowing relation (24) to hold.

Example—We now consider the specific case in which the aim is to estimate the average value of the spatially distributed unknown parameters which are encoded via local evolutions, Eq. (1). In this case our parameter of interest is $\bar{\theta} = \mathbf{w}^T \Theta$, where $\mathbf{w}^T = 1/d(1, 1, \ldots, 1)$. Hence, $|\omega_\mu - \omega_\nu| = 0, \ \forall\ \mu, \nu$. This implies that all entries of the QFIM should be equal to each other. From Eq. (15), if all first derivatives of the probe state (after the sampling stage) with respect to the different unknown parameters are equal, then all entries of the QFIM are equal to each other. Thus, any quantum states which satisfy the following condition,

$$\partial_\mu \rho_\Theta = \partial_\nu \rho_\Theta. \quad \forall\ \mu, \nu, \tag{25}$$

can be used in the private estimation of the average value irrespective of how they acquire the parameter dependence. Once more, we can consider the case of unitary evolution with multiplicative unknown parameter where $H = \sigma_z/2$ Therefore, the unitary evolution reads

$$\mathbf{U}(\Theta) = \bigotimes_{\mu=1}^{d} (|0\rangle\langle 0| + \mathrm{e}^{i\theta_\mu} |1\rangle\langle 1|). \tag{26}$$

From whence, the privacy condition in Eq. (25) can be written as

$$[H'_\mu - H'_\nu, \rho_\Theta] = 0, \quad \forall\ \mu, \nu. \tag{27}$$

Now, by substituting the eigenvectors of σ_z in Eq. (22), one can find the private states in the form of

$$|\Phi\rangle = \alpha |0\rangle^{\otimes d} + \beta |1\rangle^{\otimes d} \equiv |\text{GHZ-like}\rangle, \tag{28}$$

where $\alpha^2 + \beta^2 = 1$ (can be named as GHZ-like state) or mixed states like

$$\gamma_0 |\Phi\rangle\langle\Phi| + \sum_i \gamma_i |\phi_i\rangle\langle\phi_i|, \tag{29}$$

where $|\phi_i\rangle = |l_1, l_2, \ldots, l_d\rangle$, $l_j \in \{0, 1\}$, and $\sum_{i=0} \gamma_i = 1$. Such states satisfy condition (27) and get the privacy in the

estimation of the average value. Among these private states, Eqs. (28) and (29), the GHZ state with $\alpha = \beta = 1/\sqrt{2}$ is the only one that maximizes the single diagonal entry of the QFIM. Practically speaking, one can distribute a GHZ state, $|\psi_0\rangle = (1/\sqrt{2})(|0\rangle^{\otimes d} + |1\rangle^{\otimes d})$, throughout the network. Ergo, we can ask each node to perform the measurement in the X basis and announce the result in public [1], Fig. 1. Regarding the result of the measurement, the conditional probability distribution can be derived as $p(\pm|\Theta) = 2^{-d}[1 \pm \cos(d\bar{\theta})]$, where $\pm$ represents the result of the parity measurement. One can easily calculate the entries of the CFIM [Eq. (2)] for the given conditional probability distribution and show that it is equal to the QFIM:

$$\mathcal{F}_{\mu\nu}(\Theta) = \mathbf{Q}_{\mu\nu}(\Theta) = 1, \quad \text{for } \mu \neq \nu. \tag{30}$$

This form illustrates that the information about all unknown parameters is distributed equally throughout the network and that the GHZ state is the appropriate initial state to estimate the average value in the network of quantum sensors privately. Since any quantum state in the form of $\varrho = |\Psi\rangle\langle\Psi|$ is a private state (in the estimation of average function), we can define ϵ privacy in the sense of the closeness of an arbitrary state to the ideal state which provides the (perfect) privacy, e.g., ϱ. Given σ, the ϵ privacy may be quantified:

$$\epsilon = \|[H'_\mu - H'_\nu, \sigma]\|_1 = \|[H'_\mu - H'_\nu, \sigma - \varrho]\|_1 \leq 4\|H'_\mu\|_\infty \|\sigma - \varrho\|_1 \leq 4\|H\|_\infty \|\sigma - \varrho\|_1 \leq 8\|H\|_\infty \sqrt{1 - F^2(\sigma, \varrho)}, \tag{31}$$

where $F(\sigma, \varrho)$ denotes the fidelity of two quantum states $F(\sigma, \varrho) = \mathrm{Tr}[\sqrt{\sqrt{\varrho}\sigma\sqrt{\varrho}}]$. The last inequality follows from $1 - F(\sigma, \varrho) \leq \frac{1}{2}\|\sigma - \varrho\|_1 \leq \sqrt{1 - F^2(\sigma, \varrho)}$. Equation (31) shows that the privacy of the network is a continuous function of fidelity, which in turn implies the robustness of our protocol against noise. In other words, some form of privacy may be achieved also for suboptimal states in a neighborhood of the optimal one. Note that when the sampling operator is given by Eq. (26), the corresponding dephasing and erasure noise Kraus operators [35] commute with the encoding unitary $U(\theta_\mu)$. On the other hand, if the Kraus operators of the noise model do not commute with the sampling operators, the presence of noise before the sampling stage may affect privacy. For example, the Kraus operators of the depolarizing and amplitude damping noises do not commute with the unitary evolution. Nevertheless, privacy of the initial GHZ-like state still holds, as we prove in the Supplemental Material [35].

Conclusion—We have given a quantitative definition of privacy in the estimation of linear combination of unknown parameters that are spatially distributed in a network, in the sense that specific information can be extracted from the network of quantum sensors. Regarding the function (linear combination of unknown parameters) of interest to be estimated and the continuity relation between different entries of the QFIM, one can find the proper set of initial states which estimate the function privately. Any quantum state in the private set that maximizes the relevant entries of the QFIM is optimal for precision. The effect of uncorrelated noise in the private estimation has been studied.

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