Luttinger liquid on a lattice

Zakharov, V.

Summary

Understanding interactions in quantum many-body systems remains one of the most profound and difficult challenges in condensed matter physics. While free particle systems can be solved through single-particle techniques, the introduction of interactions turns the problem essentially many-body and quickly intractable. The Hilbert space grows exponentially, and traditional approaches become useless.

To deal with this, one typically turns to numerical methods by discretizing the system on a lattice. This approach generally works well for many 1D problems, including those described by Schrödinger equation. However, when it comes to massless Dirac fermions with protected chirality, the situation becomes more complicated. Any naive discretization inevitably runs into the fermion doubling problem, formalized by the Nielsen–Ninomiya theorem, which forbids a straightforward lattice realization of a single chiral mode without either breaking symmetry or introducing unphysical degrees of freedom.

My thesis is motivated by this problem: the need to describe 1D interacting systems, and the difficulty of doing so numerically due to these fundamental obstacles. The main focus of this work is to develop and explore lattice-based numerical methods for strongly correlated chiral fermions in one dimension, with the Luttinger liquid as a central case study.

The first part of the thesis introduces and validates a novel numerical approach based on tangent fermions. In Chapter 2 we use its local action formulation in the discretized space-time, allowing for simulations via quantum Monte Carlo that are sign-problem-free at half-filling. It successfully reproduces continuum results for the helical Luttinger liquid without adjustable parameters, marking the first faithful lattice simulation of this system.

Building on that, Chapter 3 extends the tangent fermion approach to tensor network methods, particularly DMRG. This allows us to move beyond the limitations of QMC and investigate broader settings, inclusive of static potentials and non-trivial interactions, away of half-filling. We show that tangent fermions sup-

port a compact and efficient tensor network representation, accurately matching the bosonization results of the Luttinger liquid. This confirms the versatility and consistency of the method.

The second half of the thesis shifts focus to two-dimensional single-particle physics, tackling two distinct problems. Chapter 4 concerns Majorana thermal metals in disordered chiral superconductors. We use the spectral localizer, as real-space topological tools landscape function, to show that the metal–insulator transition proceeds through percolation of topological domain walls.

Finally, Chapter 5 investigates Landau quantization in systems with generalized Van Hove singularities. We develop a general method for computing magnetic breakdown in these materials, uncovering how coherent orbit networks can form minibands that enable bulk conductance under a magnetic field — potentially exceeding edge channel transport. These features offer experimentally testable signatures linked to the underlying saddle-point structure.