



Universiteit
Leiden
The Netherlands

Luttinger liquid on a lattice

Zakharov, V.

Citation

Zakharov, V. (2025, September 23). *Luttinger liquid on a lattice*. Retrieved from <https://hdl.handle.net/1887/4261489>

Version: Publisher's Version

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/4261489>

Note: To cite this publication please use the final published version (if applicable).

Luttinger liquid on a lattice

Proefschrift

ter verkrijging van
de graad van doctor aan de Universiteit Leiden,
op gezag van rector magnificus prof.dr.ir. H. Bijl,
volgens besluit van het college voor promoties
te verdedigen op dinsdag 23 September 2025
klokke 10.00 uur

door

Vladimir Aleksandrovich Zakharov
geboren te Sint-Petersburg, Rusland
in 1997

Promotores: Prof. dr. C. W. J. Beenakker
Prof. dr. İ. Adagideli (Sabanci University, Istanbul, Turkey)

Promotiecommissie: Prof. dr. A. Boyarsky
Dr. N. Chepiga (TU Delft)
Dr. P. R. Corboz (Universiteit van Amsterdam)
Prof. dr. S. J. van der Molen
Prof. dr. K. E. Schalm

Cover:

You see water – a universal metaphor for the physics of collective behavior presented in this thesis. Water is a liquid, a state of matter where interactions play a crucial role. The surface of water serves as a good example of a disordered landscape shaped by external random forces. Finally, this landscape naturally contains many different peaks and valleys, as well as various types of saddle points connecting them.

Cover design: Maria Zakharova, original photograph: Vladimir Zakharov.

To my mother
Моей маме

Contents

1	Introduction	1
1.1	Preface	1
1.2	Topologically protected Luttinger liquid	2
1.3	Chiral fermions in QFT and SPT	4
1.4	Bosonization	5
1.5	Lattice formulation and no-go theorem	7
1.5.1	Domain wall fermion	10
1.6	This thesis	11
1.6.1	Chapter 2	11
1.6.2	Chapter 3	12
1.6.3	Chapter 4	13
1.6.4	Chapter 5	14
2	Helical Luttinger liquid on a space-time lattice	17
2.1	Introduction	17
2.2	Locally discretized Lagrangian	18
2.3	Discretized Euclidean action	21
2.4	Free-fermion propagator	22
2.5	Conclusion	25
	Appendices	26
2.A	Derivation of the Euclidean action in the tangent discretization	26
2.B	Quantum Monte Carlo calculation	29
2.B.1	Hubbard-Stratonovich transformation of the Euclidean action	29
2.B.2	Absence of a sign problem	31
2.B.3	Correlators	32
2.B.4	Monte Carlo averaging	33
2.C	Bosonization results	33

2.C.1	Bosonic form of the Hamiltonian	34
2.C.2	Spin correlator in terms of the bosonic fields	35
2.C.3	Evaluation of the thermal averages	37
2.C.4	Propagator	39
3	Luttinger liquid tensor network: sine versus tangent dispersion of massless Dirac fermions	41
3.1	Introduction	41
3.2	Matrix product operator	42
3.2.1	Free fermions	43
3.2.2	Helical Luttinger liquid	45
3.3	Correlators	46
3.3.1	Free fermions	46
3.3.2	DMRG calculation with Hubbard interaction	47
3.4	Conclusion	48
	Appendices	52
3.A	Local generalized eigenproblem allows for a scale-independent MPO	52
3.B	Jordan-Wigner transformation	53
3.C	Periodic boundary condition for MPO with sine discretization	54
3.D	Convergence of the DMRG calculations	55
3.E	Bosonization results with finite-size effects	56
3.F	Alternative tensor network representation of Ref. [52]	57
4	Majorana-metal transition in a disordered superconductor: percolation in a landscape of topological domain walls	59
4.1	Introduction	59
4.2	Topological landscape function	60
4.2.1	Lattice Hamiltonian	60
4.2.2	Spectral localizer: open boundary conditions	62
4.2.3	Spectral localizer: periodic boundary conditions	62
4.3	Phase diagram from percolation transition	63
4.3.1	Percolating domain walls	63
4.3.2	Phase diagram	65
4.3.3	Comparison with thermal conductance	66
4.4	Conclusion	67
	Appendices	67
4.A	Spectral localizer in a clean system	67

5	Landau quantization near generalized Van Hove singularities: Magnetic breakdown and orbit networks	69
5.1	Introduction	69
5.2	Magnetic breakdown at generalized Van Hove singularity	73
5.3	Coherent orbit network	76
5.4	Landau minibands	79
5.5	Conclusion	80
	Appendices	82
5.A	Magnetic breakdown near usual and high-order saddle points . .	82
5.A.1	Oscillator basis of Landau levels and formulation of problem in terms of ladder operators	82
5.A.2	Magnetic breakdown S-matrix for the usual saddle point	83
5.A.3	Monkey saddle point	87
5.A.4	High-order saddle points with different powers in effective dispersion: A_3 saddle point and regularization . . .	90
5.B	Magnetic translation operators and spectral equations for coherent networks	92
5.B.1	Magnetic translation group	92
5.B.2	The wave function dependence on eigenvalues of the magnetic translation operator	94
5.B.3	Square lattice: derivation of spectral equation for orbit network	97
5.B.4	Triangular lattice with imaginary hoppings: orbit network connected via Monkey saddle points	101
	Bibliography	109
	Summary	123
	Samenvatting	125
	Резюме	127
	Curriculum Vitæ	129
	List of Publications	131

