Hot off the press: parallel multi-objective optimization for expensive and Inexpensive objectives and constraints

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Hot off the Press: Parallel Multi-Objective Optimization for Expensive and Inexpensive Objectives and Constraints

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ABSTRACT

This paper presents the Inexpensive Objectives and Constraints Self-Adapting Multi-Objective Constraint Optimization algorithm using Radial Basis Function Approximations (IOC-SAMO-COBRA). This algorithm is introduced to address constraint multi-objective optimization problems that involve a mix of computationally expensive and inexpensive evaluation functions. The IOC-SAMO-COBRA algorithm iteratively learns the expensive functions with radial basis function surrogates, while the inexpensive functions are directly used in the search for promising solutions. Two benefits of using the inexpensive functions directly include: (1) The inexpensive functions do not introduce approximation errors like surrogates do. (2) Time can be saved as there is no need to fit and interpolate surrogate models for the inexpensive functions. Results on 22 test functions indicate that exploiting the inexpensive functions can be beneficial as this results in better Pareto front approximations compared to using surrogates for both expensive and inexpensive functions.

This paper serves as an extended abstract for the Hot-off-the-Press track at GECCO 2024, building upon the paper *Parallel Multi-Objective Optimization for Expensive and Inexpensive Objectives and Constraints* by de Winter et al., published in *Swarm and Evolutionary Computation*, Volume 86 (2024) [4].

CCS CONCEPTS

• **Theory of computation** → **Continuous optimization**; **Online learning algorithms**.

KEYWORDS

Surrogate Assisted Optimization, Multi-Objective Optimization, Constraint Optimization, Parallel Optimization

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1 INTRODUCTION

This work focuses on optimizing time-consuming ("expensive") constraint multi-objective problems with continuous parameters. The formal notation for these problems is provided in Equation 1.

$$\begin{aligned} &\text{minimize: } \mathbf{f} : \Omega \to \mathbb{R}^k \ , \ \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_k(\mathbf{x}))^\top \\ &\text{subject to: } g_i(\mathbf{x}) \leq 0 \ \forall i \in \{1, \ldots, m\} \\ &\qquad \mathbf{x} \in \Omega \subset \mathbb{R}^d. \end{aligned} \tag{1}$$

For such problems, the constraint and objective functions are not always computationally equally expensive. When the evaluation of constraints and objectives can be done independently, it can be beneficial to first check the inexpensive functions before evaluating an expensive function. In cases where the evaluation of one of the constraint functions ($g(\mathbf{x})$) or one of the objective functions ($f(\mathbf{x})$) is computationally less expensive compared to fitting and interpolating a surrogate model, the inexpensive functions can be directly used in the optimization process.

One optimization algorithm that exploits the inexpensive functions in the search for promising Pareto-efficient solutions is the Inexpensive Objectives and Constraints Self-Adapting Multi-Objective Constraint Optimization algorithm using Radial Basis Function Approximations (IOC-SAMO-COBRA) [4].

2 METHODOLOGY

IOC-SAMO-COBRA initiates the optimization process like most Bayesian optimization algorithms with a design of experiments (DoE). The recommended DoE methodology for SAMO-COBRA is a small as possible Halton sample [6] as this allows for more adaptive sampling steps. In each iteration, the algorithm fits a set of Radial Basis Functions (RBFs) [3] with various kernels to act as surrogates for the time-consuming objective and constraint functions. Upon fitting the surrogates, the algorithm uses the inexpensive

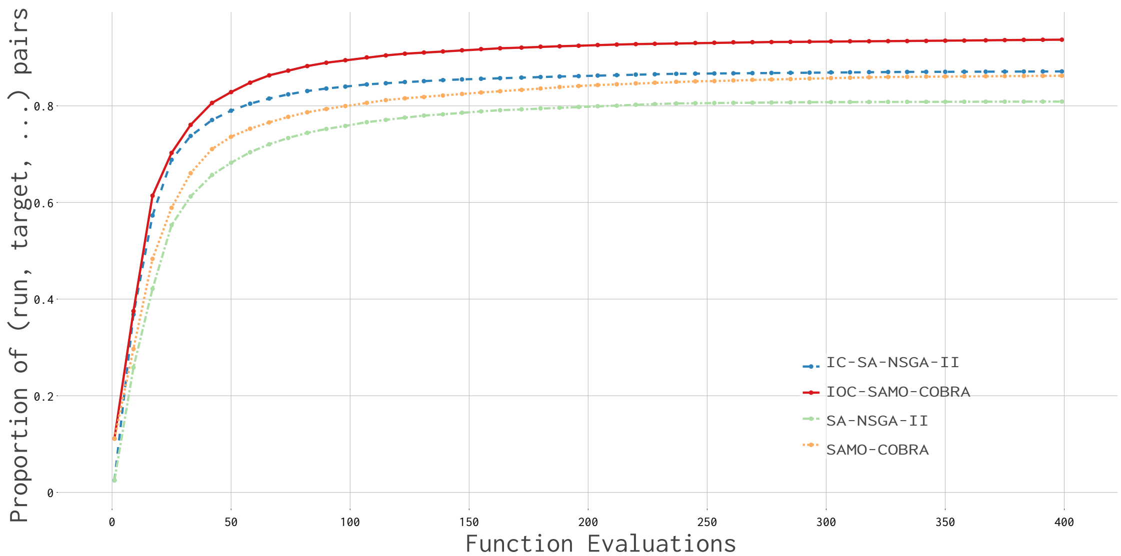

Figure 1: Empirical Cumulative Distribution Function for IC-SA-NSGA-II, IOC-SAMO-COBRA, SA-NSGA-II, and SAMO-COBRA.

functions and the RBF surrogates with the smallest approximation error for identifying promising solutions. COBYLA [7] is used to find the set of promising solutions by optimizing the multi-point hypervolume acquisition function [5]. The multi-point acquisition function optimizes the joint hypervolume of multiple solutions and will therefore automatically prefer a set of diverse solutions with little overlapping hypervolume over a set of similar solutions that have a lot of overlapping hypervolume. The solution set that is predicted feasible and predicted to contribute a lot of hypervolume according to the inexpensive functions and surrogate models is then evaluated on the computationally expensive functions. After evaluation, IOC-SAMO-COBRA refits the RBF surrogate models, the best RBF surrogates and kernel are reconsidered, and the search for promising solutions continues until the evaluation budget is exhausted.

3 EXPERIMENTS

IOC-SAMO-COBRA is tested on 22 different constraint multi-objective optimization problems. 15 of these problems are artificially created test functions, while 7 of these are real-world-like problems [4]. The IOC-SAMO-COBRA algorithm is compared to the IC-SA-NSGA-II algorithm [2], and the counterparts that do not exploit inexpensive functions but use RBF surrogates for all objective and constraint functions. For a fair comparison between IC-SA-NSGA-II and IOC-SAMO-COBRA, it is assumed that the constraints of the test functions are inexpensive, while the objectives are expensive.

4 RESULTS AND CONCLUSION

The hypervolume performance metric [1] convergence results of the four algorithms, obtained from 10 independent runs on all 22 different functions, are aggregated and presented in an Empirical Cumulative Distribution Functions (ECDF) [8] plot in Figure 1. As can be interpreted from this figure, the IOC-SAMO-COBRA and the IC-SA-NSGA-II algorithm outperform their counterparts that use surrogates instead of exploiting inexpensive functions. A deeper analysis of the experimental outcome reveals that the results on test functions with a very low feasibility ratio benefit the most from exploiting the inexpensive constraint functions. Additionally, IOC-SAMO-COBRA outperforms the IC-SA-NSGA-II algorithm, making it a suitable choice for optimizing constraint multi-objective problems with parallel evaluation opportunities and a mix of expensive and inexpensive evaluation functions. For more details and an example of a real-world ship design optimization problem, please refer to the original paper [4].

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