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Separating quantum and classical computing: rigorous proof and practical application

Marshall, S.C.

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Separating quantum and classical computing: rigorous proof and practical application

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Simon Callum Marshall

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Promotores: Prof.dr. V. Dunjko
Prof.dr. T.H.W. Bäck

Co-promotor: Dr. E.P.L. van Nieuwenburg

Promotiecommissie: Prof.dr. M. Bonsangue
Prof.dr. A. Plaat
Dr. H. Basold
Prof.dr. S. Jeffery (Universiteit van Amsterdam)
Prof.dr. I. Kerenidis (Université Paris Diderot)

Contents

1	Introduction	1
1.1	Quantum Computing	3
1.1.1	Limited modern machines	5
1.1.2	Circuit Cutting	5
1.2	Machine Learning	6
1.3	Complexity Theory	7
1.3.1	Computational Models	8
1.3.2	Formal Languages	8
1.3.3	Complexity classes	8
1.3.4	Proof Relativisation	12
1.4	Research questions	12
2	Improved separation between quantum and classical computers for sampling and functional tasks	15
2.1	Main theorem	17
2.2	PostBQP and PostBPP	27
2.3	Improved Hardness of BosonSampling, IQP, and DQC1 . .	29
2.4	Conclusion and Open Problems	32
3	On Bounded Advice Classes	35
3.1	Introduction	35
3.2	Background	37
3.2.1	General notation and definitions	37

3.2.2	Oracle-machines and double oracles	38
3.2.3	Bounded and unbounded advice classes	39
3.2.4	Previous work	41
3.3	Connection to sparse and unary languages	42
3.4	When is bounded advice useful?	45
3.4.1	Useful classical advice	46
3.4.2	Useful Quantum advice	47
4	High Dimensional Quantum Machine Learning With Small Quantum Computers	51
4.1	Introduction	51
4.2	Related work	52
4.3	Model Motivation and Specification	53
4.3.1	Parameterised Quantum Circuits	54
4.3.2	Circuit Partitioning	55
4.3.3	Reduced Partition Model	59
4.4	Generalisation Error	61
4.4.1	Encoding Dependent Generalisation Gap	62
4.4.2	Term-Based Generalisation Gap	64
4.5	Evaluation and training of reduced partition model	65
4.5.1	Evaluation	65
4.5.2	Training	67
4.5.3	Barren Plateaus	68
4.6	Numerics	69
4.6.1	A Large Problem: Handwriting	69
4.6.2	Tests on Synthetic Data	72
4.7	Conclusion and future work	75
	Appendices	76
4.A	Example of increased complexity after the removal of a 2-qubit gate	76
4.B	Proofs of theorems	77
5	All this for one Qubit?	79
5.1	Introduction	79
5.2	Background	81
5.3	Bounds on the optimal scheme	83
5.4	Generalizations to other schemes	87
5.5	Conclusion	88
6	Shadows of quantum machine learning	91
6.1	Introduction	91

6.2	The flipped model	94
6.2.1	Flipped model definition	95
6.2.2	Properties of flipped models	96
6.2.3	Quantum advantage of a shadow model	98
6.3	General shadow models	100
6.3.1	Shadow models beyond Fourier	101
6.3.2	All shadow models are shadows of flipped models .	102
6.3.3	Not all quantum models are shadowfiable	104
6.4	Discussion	106
	Appendices	107
6.A	Formal definitions	107
6.A.1	Linear models	107
6.A.2	Shadow models	107
6.A.3	Complexity classes	109
6.B	Properties of flipped models	111
6.B.1	Sample complexity of evaluating quantum models	111
6.B.2	Generalization performance	113
6.B.3	Flipping bounds	117
6.C	Quantum advantage using shadow models	121
6.C.1	Discrete cube root learning task	121
6.C.2	A simple shadow model	126
6.D	Relations between shadow models	127
6.D.1	Flipped models are universal	127
6.D.2	BQP and P/poly	129
6.D.3	Shadow models beyond Fourier	131
6.D.4	Shadowfiability	133
7	Conclusion	139
7.1	Research Overview	139
7.1.1	Future work	140
	Bibliography	143
	Acknowledgments	157
	Summary	159
	Samenvatting	161
	Curriculum Vitæ	165
	List of publications	167

