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Separating quantum and classical computing: rigorous proof and practical application

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Separating quantum and classical computing: rigorous proof and practical application

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