



Universiteit
Leiden

The Netherlands

Data structures for quantum circuit verification and how to compare them

Vinkhuijzen, L.T.

Citation

Vinkhuijzen, L. T. (2025, February 25). *Data structures for quantum circuit verification and how to compare them*. IPA Dissertation Series. Retrieved from <https://hdl.handle.net/1887/4208911>

Version: Publisher's Version

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/4208911>

Note: To cite this publication please use the final published version (if applicable).

Data Structures for Quantum Circuit Verification and How To Compare Them

Lieuwe Thomas Vinkhuijzen

Data Structures for Quantum Circuit Verification and How To Compare Them

Proefschrift

ter verkrijging van
de graad van doctor aan de Universiteit Leiden,
op gezag van rector magnificus prof.dr.ir. H. Bijl,
volgens besluit van het college voor promoties
te verdedigen op dinsdag 25 februari 2025
klokke 14.30 uur

door

Lieuwe Thomas Vinkhuijzen

geboren te Alphen aan den Rijn

in 1993

Promotor: Prof.dr. H.H. Hoos

Co-promotores: Dr. A.W. Laarman

Dr. T.J. Coopmans

Promotiecomissie:

Dr. J. Tura Brugués

Prof.dr. H.C.M. Kleijn

Prof.dr. A. Plaat

Prof.dr. W.J. Fokkink (Vrije Universiteit Amsterdam)

Dr. K.S. Meel (University of Toronto)

Prof.dr. R. Wille (Technische Universität München)

Copyright © 2025 Lieuwe Vinkhuijzen.

IPA Dissertation Series 2025-05

The work in the thesis has been carried out under the auspices of the research school IPA (Institute for Programming research and Algorithmics).

This project was funded by the research program VENI with project number 639.021.649 of the Netherlands Organization for Scientific Research (NWO).



Cover art: *Colourful Linings*, Gouache on canvas, by Joyce Olivier

Acknowledgements

First, I wish to thank my copromotor Alfons Laarman for his supervision. I am grateful for the wide creative latitude you gave me in pursuing directions and ideas in my research. You always took the effort to understand these ideas at least as well as I did. As a result, your technical feedback was always superb, abundant and on point. I am indebted to Tim Coopmans for being an excellent copromotor in the final days of my PhD project. Our long collaboration on LIMDDs has been an extremely enjoyable part of my time here. I thank my promotor Holger Hoos for his mentorship, for many insightful discussions and many useful comments on drafts of all my papers.

I am greatly indebted to all the folks at the Johannes Kepler Universität in Linz, Austria. Robert Wille, Stefan Hillmich and Thomas Grurl, thank you for your hospitality and for being fantastic coauthors. I thank Richard Kueng for insightful discussions; Tom Peham for explaining about dolphins; and Aaron Sander and Lukas Burgholzer for the most opinionated discussions anyone will ever have about decision diagrams. The helpful and friendly staff at the JKU student accommodations made my stay especially pleasant.

I thank my coauthors Vedran Dunjko and David Elkouss for your involvement in our LIMDD paper, and for your patience during the long design phase.

I thank André Deutz for his invaluable support.

My time at LIACS would not have been the same without many other PhD students who were in the same boat and who provided lunch + fun; in no particular order, Marie, Marios, Koen, Arina, Tanjona, Anna, Anne, Lise, Sander, Thodoris, Hugo, Can, Dimitrios, Sebastiaan, António, Alexa, Leni, Daniela, Charles, Solomiia, Grace, Marshall, Yash, Furong, Thomas, Gerrit Jan, Casper, Rachel, Dalia, Niki, Hao, thank you for all the lunches, the music and the pastries.

Many anonymous reviewers have made insightful and valuable comments on drafts of our papers. I am indebted for their help both in rigorously checking the mathematical and technical details and in improving the presentation of our work, even when our paper wasn't accepted for publication. Therefore, I wish to thank the anonymous reviewers at QIP, SODA, STOC and Quantum for their feedback on drafts of [Chapter 3](#). I thank the anonymous reviewers at ASP-DAC and SPIN for their feedback on drafts of [Chapter 4](#). I thank the anonymous reviewers at KR, IJCAR, QIP and AAAI for their helpful feedback on drafts of [Chapter 5](#). I thank the anonymous NFM reviewers for helpful feedback on a draft of [Chapter 6](#). I thank the anonymous reviewers at CONCUR and SETTA for valuable feedback on drafts of [Chapter 7](#). I moreover thank the organizers of SETTA for sending a substantial amount of Chinese tea halfway around the world by way of a prize for the best video presentation award – I am still enjoying it.

Lastly, I want to thank all the functions, polynomials, Boolean circuits, quantum circuits, algebraic circuits, all the directed acyclic weighted graphs, and the estimates, inequalities and eigenvalues, without which this thesis would be all the poorer.

Contents

Acknowledgements	v
1 Introduction	1
1.1 Context	2
1.2 State-of-the-art data structures for quantum algorithm analysis	6
1.3 Research questions	12
1.4 Contributions	13
1.5 Outline	15
1.6 Publications and artefacts	16
2 Preliminaries	19
2.1 Mathematical preliminaries	20
2.2 Introduction to quantum computing	21
2.2.1 Quantum states and operations	24
2.2.2 How to simulate a quantum circuit	27
2.2.3 Clifford circuits, stabilizer states and Pauli operators	27
2.3 Decision Diagrams	30
2.3.1 Decision diagrams for quantum states	33

Contents

2.3.2	Comparing decision diagrams	37
2.4	Approaches to quantum software verification	39
3	LIMDD: a decision diagram for simulation of quantum computing including stabilizer states	45
3.1	Introduction	46
3.2	Preliminaries: decision diagrams and the stabilizer formalism	49
3.2.1	Decision diagrams	49
3.2.2	Pauli operators and stabilizer states	52
3.2.3	Matrix product states	55
3.3	Local Invertible Map Decision Diagrams	55
3.3.1	The LIMDD data structure	57
3.3.2	Succinctness of LIMDDs	62
3.3.3	Pauli-LIMDD manipulation algorithms for simulation of quantum computing	67
3.3.4	Comparing LIMDD-based simulation with other methods	79
3.4	Canonicity: Reduced LIMDDs with efficient MAKEEDGE algorithm	82
3.4.1	LIMDD canonical form	82
3.4.2	The MAKEEDGE subroutine: Maintaining canonicity during simulation	90
3.5	Efficient algorithms for choosing a canonical high label	93
3.5.1	Choosing a canonical high label	94
3.5.2	Efficient linear-algebra algorithms for stabilizer subgroups	97
3.5.3	Constructing the stabilizer subgroup of a LIMDD node	100
3.5.4	Efficiently finding a minimal LIM by multiplying with stabilizers	105
3.6	Numerical search for the stabilizer rank of Dicke states	110

3.7	Related work	111
3.8	Discussion	113
4	Efficient implementation of LIMDDs for Quantum Circuit Simulation	115
4.1	Introduction	116
4.2	Background	119
4.2.1	Quantum states and operations	119
4.2.2	Classical quantum circuit simulation	121
4.2.3	Verification of quantum circuits	123
4.3	Motivation	123
4.3.1	Quantum Multiple-valued Decision Diagrams (QMDDs)	124
4.3.2	Local Invertible Map Decision Diagrams (LIMDDs)	126
4.3.3	The need for a LIMDD implementation	127
4.4	Implementation of LIMDDs	128
4.4.1	Established techniques	129
4.4.2	Implementing Local Invertible Maps	130
4.5	Case study	133
4.6	Discussion	135
4.7	Conclusions	136
5	A Quantum Knowledge Compilation Map	137
5.1	Introduction	138
5.2	Data structures for quantum states	140
5.2.1	Quantum information	140
5.2.2	Data structures	141

Contents

5.3	Succinctness of quantum representations	145
5.4	Tractability of quantum operations	146
5.5	Rapidity of data structures	148
5.5.1	A sufficient condition for Rapidity	149
5.5.2	Rapidity between quantum representations	151
5.6	Related work	152
5.7	Discussion	154
6	The Power of Disjoint Support Decompositions in BDDs	157
6.1	Introduction	158
6.2	Background and related work	159
6.3	Succinctness separation between DSDBDD and BDD	163
6.4	Conclusion and future work	165
7	Model Checking with Sentential Decision Diagrams	167
7.1	Introduction	168
7.2	Background	170
7.2.1	Sentential Decision Diagrams	170
7.2.2	Model checking and reachability	173
7.3	Vtree heuristics	174
7.3.1	Two-phase vtree construction	174
7.3.2	Abstract variable tree heuristic	175
7.3.3	Folding out vtrees for large variable domains	176
7.4	Related work	177
7.5	Experiments	177
7.5.1	Results	179

7.5.2	SDD runtime profiles	182
7.6	Conclusion and future work	183
8	Conclusions and outlook	187
8.1	Research questions, revisited	187
8.2	Discussion	191
8.3	Future work	193
8.3.1	Future work on LIMDDs.	193
8.3.2	Future work on DSDBDDs	197
8.3.3	Future work on SDDs	198
8.3.4	Future work on quantum knowledge compilation	199
8.3.5	Future work on verification and decision diagrams	200
A	Proof that cluster states and coset states need exponentially large QMDDs	205
B	How to write graph states, coset states and stabilizer states as Tower-LIMDDs	211
C	Advanced LIMDD algorithms	221
C.1	Measuring an arbitrary qubit	221
C.2	Applying Hadamards to stabilizer states in polynomial time	223
D	LIMDDs prepare the W state efficiently	231
E	Proofs of Section 5.3	239
F	Proofs of Section 5.4	245
F.1	Easy and hard operations for ADD	245

Contents

F.2	Easy and hard operations for QMDD	247
F.3	Easy and hard operations for LIMDD	251
F.4	Easy operations for MPS	258
F.5	Easy and hard operations for RBM	259
G	Proofs of Section 5.5	261
G.1	Rapidity is a preorder and generalizes earlier definitions	261
G.2	A Sufficient Condition for Rapidity	265
G.3	Rapidity Relations between Data Structures	267
G.4	MPS is at least as Rapid as QMDD	268
G.5	Transformations between QDDs	271
G.5.1	Transforming LIMDD to QMDD	272
G.5.2	Transforming QMDD to LIMDD	272
G.5.3	Transforming ADD to QMDD	272
G.5.4	Transforming QMDD to ADD	273
	Bibliography	275
	Summary	305
	Samenvatting	309
	Curriculum Vitae	313