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Data structures for quantum circuit verification and how to compare them

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Data Structures for Quantum Circuit Verification and How To Compare Them

Lieuwe Thomas Vinkhuijzen

Data Structures for Quantum Circuit Verification and How To Compare Them

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