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Strongly correlated electrons in Sachdev-Ye-Kitaev models and twisted bilayer graphene

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Strongly interacting electrons in Sachdev-Ye-Kitaev models and Twisted Bilayer Graphene

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The cover represents two ways in which Hyperbolic geometries play a role in the low energy properties of electrons. The yellow curve represents the fermi surface near a van-Hove singularity and the random colored dots represent the SYK model. The graphics were made using the manim python package.

To my mentor, Prof. Lars Fritz

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