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Jiang, J-Q.; Cai, Y.; Ye, G.; Piao, Y-S.

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Broken blue-tilted inflationary gravitational waves: a joint analysis of NANOGrav 15-year and BICEP/Keck 2018 data

Jun-Qian Jiang,[a,b] **Yong Cai**,[c] **Gen Ye**[d] **and Yun-Song Piao**[a,b,e,f]

[a]*School of Fundamental Physics and Mathematical Sciences,*
Hangzhou Institute for Advanced Study, UCAS,
Hangzhou 310024, China
[b]*School of Physical Sciences, University of Chinese Academy of Sciences,*
Beijing 100049, China
[c]*School of Physics and Microelectronics, Zhengzhou University,*
Zhengzhou, Henan 450001, China
[d]*Leiden University, Instituut-Lorentz for Theoretical Physics,*
2333CA, Leiden, Netherlands
[e]*International Center for Theoretical Physics Asia-Pacific,*
Beijing/Hangzhou, China
[f]*Institute of Theoretical Physics, Chinese Academy of Sciences,*
P.O. Box 2735, Beijing 100190, China

E-mail: `jiangjq2000@gmail.com`, `caiyong@zzu.edu.cn`,
`ye@lorentz.leidenuniv.nl`, `yspiao@ucas.ac.cn`

Abstract: Recently, the pulsar timing array (PTA) collaborations have reported the evidence for a stochastic gravitational wave background (SGWB) at nano-Hertz band. The spectrum of inflationary gravitational wave (IGW) is unknown, which might exhibit different power law at different frequency-bands, thus if the PTA signal is primordial, it will be significant to explore the underlying implications of current PTA and CMB data on IGW. In this Letter, we perform a joint Markov Chain Monte Carlo analysis for a broken power-law spectrum of IGW with the NANOGrav 15-year and BICEP/Keck 2018 data. It is found that though the bestfit spectral tilt of IGW at PTA band is $n_{\mathrm{T}}^{\mathrm{PTA}} = 2.42^{+0.32}_{-0.91}$, at CMB band the bestfit is $n_{\mathrm{T}}^{\mathrm{CMB}} = 0.55^{+0.37}_{-0.10}$ while a detectable amplitude of r with $n_{\mathrm{T}}^{\mathrm{CMB}} \simeq 0$ is still compatible. The implication of our results for inflation is also discussed.

Keywords: Bayesian reasoning, gravitational waves / sources, gravitational waves and CMBR polarization, inflation

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Contents

1 Introduction

It is well-known that the detection of inflationary or primordial gravitational waves (IGW) will not only solidify our confidence on the inflation scenario [1–5], but also bring us significant insight into the physics of very early universe.

The ultra-low-frequency IGW with $f \sim 10^{-18} - 10^{-16}$Hz will source the B-mode polarization in the cosmic microwave background (CMB) [6, 7], which has been still searched for by BICEP/Keck [8]. Recently, the PTA experiments [9–12], have found a stochastic GW background (SGWB) at $f \sim 10^{-10} - 10^{-8}$Hz [13, 14]. Though such a SGWB is compatible with that brought by inspiralling supermassive black holes binaries [13–15], see also [16, 17] for the supermassive primordial black holes, it might be just IGW but with $n_{\rm T} > 0$ [13, 14, 18–20].[1]

However, the spectrum of IGW is actually unknown. The amplitude of IGW at CMB band is usually quantified as the tensor-to-scalar ratio $r = \frac{A_{\rm T}}{A_s}$, we have

$$P_{\rm T}(k) = rA_s \left(\frac{k}{k_{\rm pivot}}\right)^{n_{\rm T}}, \tag{1.1}$$

where $k_{\rm pivot}$ is the pivot scale. It is possible for inflation to yield a blue-tilted SGWB with $n_{\rm T} > 0$ [20, 38–51], however, the primordial scalar perturbation is nearly scale-invariant at CMB band [52], which seems to be in favor of a slow-roll model of inflation with $n_{\rm T} \simeq 0$ [53]. Inspired by [54, 55], see also [56–60], it might be more reliable to consider a broken power-law IGWB at $f \sim 10^{-18} - 10^{-8}$Hz,

$$P_{\rm T}(k) = rA_s \left(\frac{k}{k_{\rm pivot}}\right)^{n_{\rm T}^{\rm CMB}} \left(1 + \left(\frac{k}{k_{\rm break}}\right)\right)^{-n_{\rm T}^{\rm CMB}+n_{\rm T}^{\rm PTA}}, \tag{1.2}$$

where $f_{\rm break} \ll 10^{-8}$Hz is the scale at which power-law is broken, and the conversion between k and f is

$$f = \frac{kc}{2\pi a_0} = 1.5 \times 10^{-15} \left(\frac{k}{{\rm Mpc}^{-1}}\right) {\rm Hz}. \tag{1.3}$$

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[1] In addition, see also recent other possibilities e.g. [21, 21, 22, 22–37].

In refs. [61–63] such a broken SGWB has been studied but with $f_{\rm break} > 10^{-7}$Hz. According to eq. (1.2), we have $P_{\rm T} = rA_s\left(\frac{k}{k_{\rm pivot}}\right)^{n_{\rm T}^{\rm CMB}}$ when $k \ll k_{\rm break}$, while

$$P_{\rm T} = rA_s\left(\frac{k_{\rm break}^{n_{\rm T}^{\rm CMB}-n_{\rm T}^{\rm PTA}}}{k_{\rm pivot}^{n_{\rm T}^{\rm CMB}}}\right)k^{n_{\rm T}^{\rm PTA}}, \tag{1.4}$$

when $k \gg k_{\rm break}$. Actually, a period of inflation might be complex so that at different bands of $f \sim 10^{-18} - 10^{-8}$Hz we will have different power-law IGW, while eq. (1.2) is just the simplest of such possibilities.[2]

It has been found that for power-law IGW (1.1), recent NANOGrav data favors $n_{\rm T} \simeq 2$. As a result, IGW at CMB band is negligible. Thus recent CMB data has not been included in the analysis of NANOGrav [13] (a hard prior $r \lesssim 0.03$ is actually sufficient [19]). However, this is not valid for broken power-law IGW (1.2), which allows a non-negligible r at CMB band. Thus it is significant to perform a joint analysis of both latest PTA and CMB data with full exact likelihood to explore the underlying impact of current data on IGW.

However, a joint Markov Chain Monte Carlo (MCMC) analysis of NANOGrav 15-year and BICEP/Keck 2018 data has been still open. In this Letter, we present the first such analysis, and find that the bestfit spectral tilt of IGW at PTA band is $n_{\rm T}^{\rm PTA} \simeq 2$, however, different from the results of NANOGrav [13], $n_{\rm T}^{\rm CMB} \simeq 2$ is not favored at CMB band, instead the bestfit is $n_{\rm T}^{\rm CMB} \simeq 0.5$ while a detectable amplitude of r with $n_{\rm T}^{\rm CMB} \simeq 0$ is still compatible. The implication of our results for inflation is also discussed.

2 Dataset and method

NANOGrav. We use the NANOGrav 15-year dataset [9] at PTA band, assuming that the signals observed in NANOGrav [9], EPTA [10], PPTA [11] and CPTA [12] are mutually consistent. The likelihoods are calculated with `ceffyl` [64].

BICEP/Keck (BK18). We use the BICEP/Keck 2018 official likelihood[3] [8] at CMB band, taking dust, synchrotron and noise into account.

As in the analysis of NANOGrav [13], we fix the parameters of standard ΛCDM model to the bestfit values of the Planck 2018 baseline results:[4] $\Omega_b h^2 = 0.02238$, $\Omega_c h^2 = 0.12011$, $100\theta_{MC} = 1.040909$, $\tau = 0.0543$, $\ln(10^{10}A_s) = 3.0448$, $n_s = 0.96605$. In addition to the nuisance parameters in the BICEP/Keck likelihood, our MCMC parameters set is $\{r_{0.05}, n_{\rm T}^{\rm CMB}, n_{\rm T}^{\rm PTA}, \log_{10} k_{\rm break}\}$, where the subscript 0.05 for r indicates that it is calculated at the pivot scale $k = 0.05{\rm Mpc}^{-1}$. The uniform priors are set, and the unit of $k_{\rm break}$ is ${\rm Mpc}^{-1}$.

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[2] Actually, beyond PTA band such an IGW spectrum will be conflicted with the BBN bound on relativistic components, however, we only consider (1.2) at $f \sim 10^{-18} - 10^{-8}$Hz, since at higher-frequency band (1.2) might have been modified [61–63], see also section 3.

[3] http://bicepkeck.org/bk18_2021_release.html.

[4] The corresponding bestfit values might shift in light of the early resolution of the Hubble tension [65], in particular the spectral index n_s of primordial scalar perturbation will shift towards $n_s = 1$ [66–68], however, such shifts will not essentially alter our results.

Parameter	Best fit	68% limits
$r_{0.05}$	0.0296	$< 0.0549\,(< 0.107)$
$r_{0.01}$	0.0141	$< 0.0199\,(< 0.0338)$
$n_{\rm T}^{\rm CMB}$	0.428	$0.55^{+0.37}_{-0.10}$
$n_{\rm T}^{\rm PTA}$	2.08	$2.42^{+0.32}_{-0.91}$
$\log_{10} k_{\rm break}$	4.19	$5.0^{+2.0}_{-1.5}$

Table 1. The bestfit values and the 68% limit of posterior distributions of the parameters in the broken power-law model (1.2) of IGW with $n_{\rm T,CMB}$ free. We also show the 95% limit for r in parentheses.

And we use `CLASS` [69] to calculate the evolutions of GWs and other components, such as photons and baryons, and `cobaya` [70] with the MCMC Metropolis sampler and oversampling the nuisance parameters and $\{n_{\rm T}^{\rm PTA}, k_{\rm break}\}$ to speed up our calculation.

Here, we also need to calculate the energy spectrum $\Omega_{\rm GW}(f)$ of IGW. As in the analysis of NANOGrav [13], around the PTA scales we have

$$\Omega_{\rm GW}(f) = \frac{\Omega_{\rm r}}{24}\left(\frac{g_*(f)}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}(f)}\right)^{4/3} P_{\rm T}(f), \tag{2.1}$$

where $\Omega_{\rm r}$ is the ratio of radiation to critical energy density at present, and g_*^0 and $g_{*,s}^0$ are the effective number of relativistic degrees of freedom for energy and entropy, respectively, while $g_*(f)$ and $g_{*,s}(f)$ correspond to the qualities when the modes with comoving wavenumber $k = 2\pi a_0 f$ reentered the Hubble horizon, see e.g. ref. [71].

3 Results

In this section, we will focus on the broken power-law model (1.2) of IGW with $n_{\rm T}^{\rm CMB}$ free and $n_{\rm T}^{\rm CMB} = -r/8$, respectively.

3.1 $n_{\rm T}^{\rm CMB}$ is free

The results are presented in figure 1 and table 1. As expected, the bestfit value $\log_{10} k_{\rm break} = 4.19$, which is just between the CMB and PTA bands. The posterior of the spectral index $n_{\rm T}^{\rm PTA}$ at PTA band is $n_{\rm T}^{\rm PTA} \simeq 2$, in agreement with the results of NANOGrav [13, 19],[5] since it is constrained mainly by the PTA itself. According to eq. (1.4), we have $P_{\rm T} \sim r k^{n_{\rm T}^{\rm PTA}} k_{\rm break}^{n_{\rm T}^{\rm CMB} - n_{\rm T}^{\rm PTA}}$ for fixed $k_{\rm pivot}$, which suggests that $n_{\rm T}^{\rm CMB}$ is correlated positively with $\log_{10} k_{\rm break}$, as showed in figure 1.

The 95% upper limit of $r_{0.05}$ is $r_{0.05} < 0.107$, higher than that of Planck+BK18 (e.g. $r \lesssim 0.03$ in refs. [8, 72]), while the 95% upper limit of $r_{0.01}$ is $r_{0.01} < 0.0338$, However, this is a natural result, since for a blue-tilted IGW with the bestfit $n_{\rm T}^{\rm CMB} = 0.428$, the amplitude of IGW must be lower at larger scale (smaller $k_{\rm pivot}$). Accordingly, the constraint will be notably different at different $k_{pivot} \lesssim 0.05\ {\rm Mpc}^{-1}$.

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[5] Here, our results corresponds to the $T_{\rm rh} \gtrsim 1\,{\rm GeV}$ part of ref. [13].

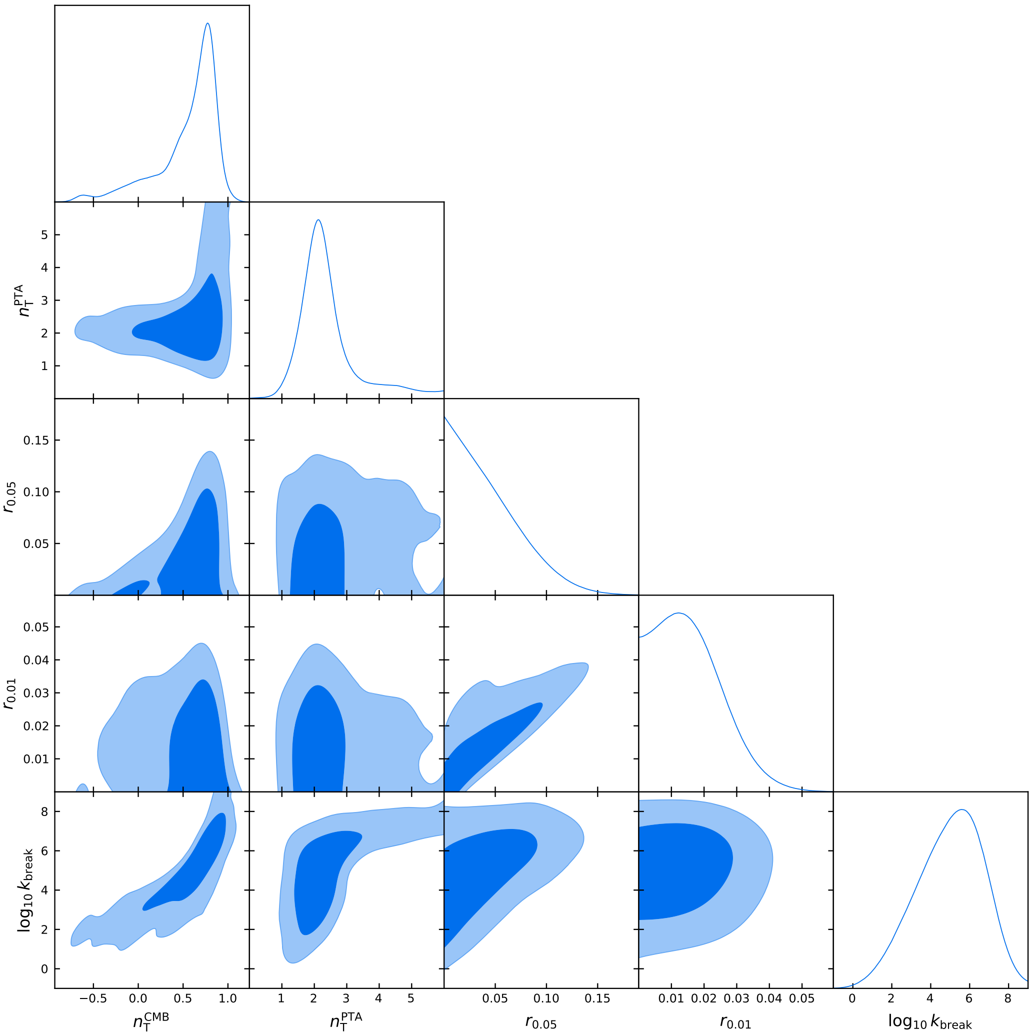

Figure 1. Marginalized posterior distributions of the parameters in the broken power-law model (1.2) of IGW with $n_{\mathrm{T,CMB}}$ free.

However, it is significant to find that unlike that of NANOGrav [13] at CMB band $n_{\mathrm{T}}^{\mathrm{CMB}} \simeq 2$ is not favored, instead $n_{\mathrm{T}}^{\mathrm{CMB}} = 0.55^{+0.37}_{-0.10}$ with the bestfit $n_{\mathrm{T}}^{\mathrm{CMB}} = 0.428$, while $n_{\mathrm{T}}^{\mathrm{CMB}} \simeq 0$ is still in the 95% CL. range, which suggests that a slow-roll period of inflation at CMB band is not excluded. Thus it is interesting to see what if we fix $n_{\mathrm{T}}^{\mathrm{CMB}} = -r/8$.

3.2 $n_{\mathrm{T}}^{\mathrm{CMB}} = -r/8$

Next, we fix $n_{\mathrm{T,CMB}} = -r/8$, i.e. standard slow-roll inflation is not broken at CMB scale. The results are presented in figure 2 and table 2. As expected, the broken scale is $k_{\mathrm{break}} = 2.72^{+0.92}_{-0.40}$, which is well between the CMB and PTA bands, and the posterior of the spectral index $n_{\mathrm{T}}^{\mathrm{PTA}}$ at PTA band is still $n_{\mathrm{T}}^{\mathrm{PTA}} \simeq 2$.

The 95% CL. range of r is $r_{0.05} = 0.019^{+0.020}_{-0.019}$, which is similar to that of $r_{0.01}$, since unlike in section 3.1, we have $n_{\mathrm{T}}^{\mathrm{CMB}} = -r/8 \simeq 0$ nearly scale-invariant. This upper limit, $r \lesssim 0.039$, is comparable with (but slightly higher than) that of Planck+BK18 [8, 72]. The

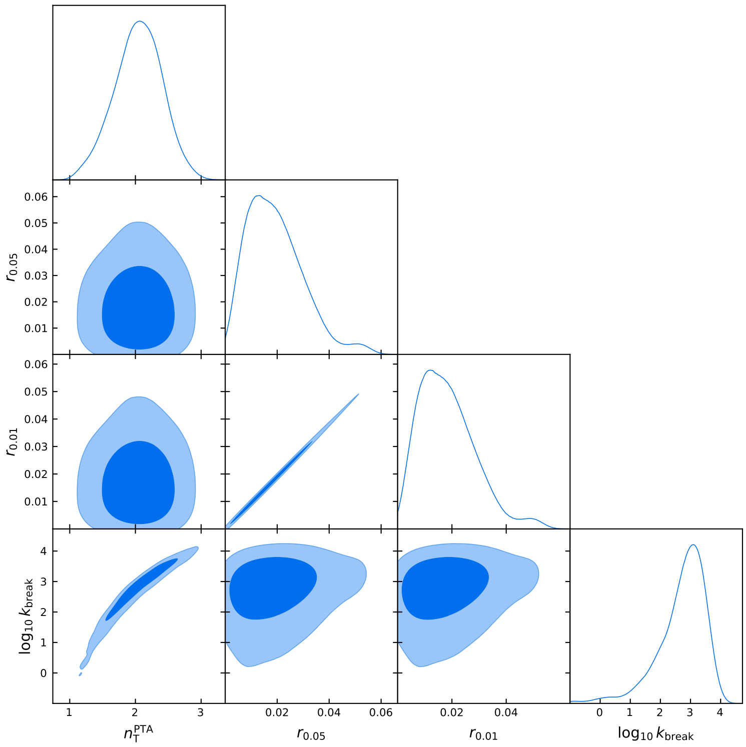

Figure 2. Marginalized posterior distributions of the parameters in the broken power-law model (1.2) with slow-roll $n_{\rm T,CMB}$.

Parameter	Best fit	68% limits
$r_{0.05}$	0.0091	$0.0194^{+0.0069}_{-0.014}$ $\left(^{+0.020}_{-0.019}\right)$
$r_{0.01}$	0.0086	$0.0184^{+0.0065}_{-0.013}$ $\left(^{+0.019}_{-0.018}\right)$
$n_{\rm T}^{\rm PTA}$	2.198	$2.04^{+0.39}_{-0.34}$
$\log_{10} k_{\rm break}$	3.01	$2.72^{+0.92}_{-0.40}$

Table 2. The bestfit values and the 68% limit of posterior distributions of the parameters of the broken power-law model (1.2) with slow-roll $n_{\rm T,CMB}$. We also show the 95% limit for r in parentheses.

bestfit r is $r_{0.05} = 0.0091$, however, it is unexpected that $r = 0$ is at 2σ level. According to eq. (1.2), the bound on r seems to be indirectly affect the NANOGrav results ($n_{\rm T}^{\rm PTA}$, $k_{\rm break}$).

4 Discussion

4.1 Conclusion

In conclusion, we find that though the bestfit spectral tilt of IGW at PTA band is $n_{\rm T}^{\rm PTA} \simeq 2$, unlike that of the NANOGrav [13] $n_{\rm T}^{\rm CMB} \simeq 2$ is not favored at CMB band, instead the bestfit is $n_{\rm T}^{\rm CMB} = 0.55^{+0.37}_{-0.10}$, while a detectable amplitude of r with $n_{\rm T}^{\rm CMB} \simeq 0$ is still compatible. In particular if we fix $n_{\rm T}^{\rm CMB} = -r/8$ in light of standard slow-roll inflation, the upper bound on r is $r \lesssim 0.039$, slightly higher than that of Planck+BK18 [8]

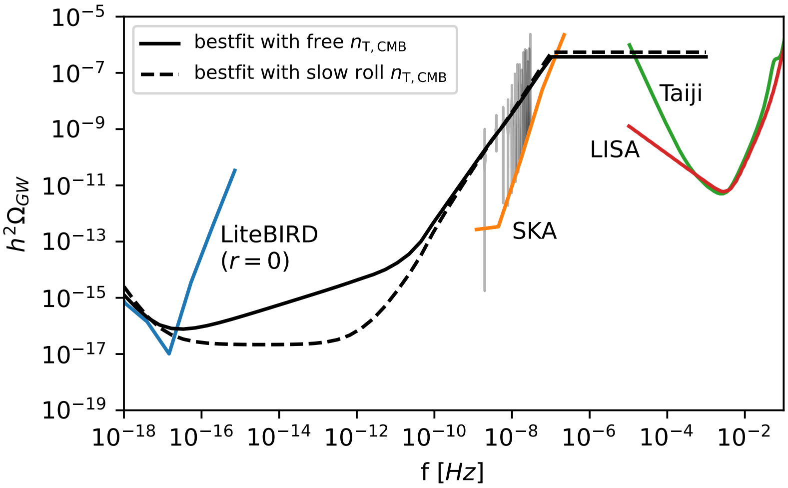

Figure 3. The bestfit energy spectrum $\Omega_{GW}(f)$ of IGWB with $n_{\rm T}^{\rm CMB}$ free and $n_{\rm T}^{\rm CMB} = -r/8$, respectively. We also show the sensitivity curves for LiteBIRD, SKA (Square Kilometre Array), space-based laser interferometers: LISA and Taiji. Here, we use the transfer function showed in ref. [73], which can return to eq. (2.1) at the high frequency limit, see also refs. [61, 74–77]. As commented in section 4, when $f > 10^{-7}$ the highly-blue ($n_{\rm T}^{\rm PTA} \simeq 2$) tilt needs to be cut off, though we plot $\Omega_{GW}(f > 10^{-7})$ with a straight line, which but is actually model-dependent, e.g. [62, 63].

In figure 3, we show our bestfit energy spectrum $\Omega_{GW}(f)$ of IGWB with $n_{\rm T}^{\rm CMB}$ free and $n_{\rm T}^{\rm CMB} = -r/8$, respectively. The sensitivity curves for LiteBIRD and SKA are based on the results of ref. [78]. Here, our bestfit $\Omega_{GW}(f)$ covers the band at $f \sim 10^{-18} - 10^{-8}$Hz, which naturally offers a guide for building relevant models of inflation and exploring new physics at corresponding band, since such a bestfit $\Omega_{GW}(f)$ can simultaneously explain current observations at both CMB and PTA bands.

The GW energy density contributes to the effective number of relativistic degrees of freedom $N_{\rm eff}$ at the time of BBN, which can be constrained by observations (see e.g. [79, 80]):

$$N_{\rm eff}^{\rm GW} = \left(3.046 + \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\right)\frac{1}{12}\int {\rm d}\ln k P_T \tag{4.1}$$

The IR cutoff for the integral is the comoving horizon at the time of BBN $f_{\rm IR} \simeq 10^{-10}$ Hz. Our expression of the power spectrum is assumed to be valid until $f = 2.97 \times 10^{-8}$ Hz, which is the frequency of the 14th bin of the NANOGrav 15-yr result. We find a contribution of $\Delta N_{\rm eff} = 0.012$ for a free $n_{\rm T}^{\rm CMB}$ and $\Delta N_{\rm eff} = 0.014$ for $n_{\rm T}^{\rm CMB} = -r/8$, which is consistent will current constraint $\Delta N_{\rm eff} \lesssim 0.4$ (95% CL., see e.g. [52]).

However, beyond the PTA band the highly-blue ($n_{\rm T}^{\rm PTA} \simeq 2$) tilt must be cut off at a certain $k_{\rm cutoff} = 2\pi f_{\rm cutoff}$ to avoid the confliction with the BBN bound. For example, if the power spectrum maintains flat till some $f_{\rm UV} \sim 10^{8}$[6] when $f > 10^{-7}$, it will lead to $\Delta N_{\rm eff} = 1.01$ for a free $n_{\rm T}^{\rm CMB}$ and $\Delta N_{\rm eff} = 1.27$ for $n_{\rm T}^{\rm CMB} = -r/8$. We find a cutoff frequency $f_{\rm cutoff} \simeq 10^{-3}$ can produce $\Delta N_{\rm eff} \simeq 0.4$, which has not been excluded by the current constraint. Besides, if the power spectrum is kept flat till $f =$20-76.6 Hz, the energy density of the GWs

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[6] The choice of $f_{\rm UV}$ has many assumptions. Here we consider the value in refs. [79, 81].

(3.72×10^{-7} for a free $n_{\rm T}^{\rm CMB}$ and 5.45×10^{-7} for $n_{\rm T}^{\rm CMB}=-r/8$) will violate the upper limit of the LIGO/Virgo constraint $\Omega_{\rm GW}\lesssim 5.8\times10^{-9}$ in the band 20-76.6 Hz [82]. It also required the cutoff to not conflict with these results. In figure 3, what is $P_{\rm T}$ at $k>k_{\rm cutoff}$ is not relevant with our MCMC results, which actually is model-dependent, e.g. [62, 63], see also discussion in section 4.2. Thus the spectrum of IGWB at that band is open, however, the space-based laser interferometers, LISA and Taiji, might detect the corresponding IGWB signal.

In next decade, with the accumulations of PTA and CMB data, SGWB at PTA band would be confirmed eventually, and also upcoming CMB observations, such as CMB-S4 [83], LiteBIRD [84] would bring us more information on IGWB at CMB band. In view of this, our work is suggesting that a joint MCMC analysis of PTA and CMB data might be indispensable, which would open a unforeseen door into our very early Universe.

4.2 Implications for inflation

It is interesting to see what our results will imply for inflation. As commented in ref. [19], a blue-tilted IGW suggests that inflation must violate null energy condition (NEC), i.e. $T_{\mu\nu}n^\mu n^\nu<0$ (equivalently $\dot H>0$ or $\epsilon=-\frac{\dot H}{H^2}<0$), e.g. [38, 39], see also [43, 44], if the initial Bunch-Davis state of perturbation modes is not modified. Though it is not difficult to achieve $n_{\rm T}^{\rm PTA}\simeq 2$, which requires $\epsilon\ll -1$, e.g. [40, 56], such a highly blue tilt of IGWB implies that inflation must end at a low scale [13, 19], or else it will be conflicted with the BBN bound on relativistic components.

However, it might be possible that the violation of NEC happened before a (standard) slow-roll inflation, so that the blue-tilted $P_{\rm T}$ is cut off at certain $k_{\rm cutoff}$ beyond the PTA band (when $k>k_{\rm cutoff}$ the spectrum is flat). In corresponding models, the energy spectrum of IGW will be shown as the black solid $\Omega_{GW}(f)$ curve in figure 3, and at CMB band for such a period of NEC violation the scale-invariant primordial scalar perturbation required by Planck observations can be harvested in light of [38, 43, 56–59, 85].

It is also possible that the violation of NEC might be just short-lived, which happened between two (slow-roll) periods of inflation with $H\simeq H_{\rm inf1}$ and a higher scale $H\simeq H_{\rm inf2}\gg H_{\rm inf1}$, respectively. In corresponding model [54, 86, 87], we approximately have

$$P_{\rm T}(k)\sim\frac{H_{\rm inf1}^2+\mathcal{A}\left(\frac{k}{k_{\rm cutoff}}\right)^{n_{\rm T}^{\rm PTA}}H_{\rm inf2}^2}{1+\mathcal{A}\left(\frac{k}{k_{\rm cutoff}}\right)^{n_{\rm T}^{\rm PTA}}}\,,\tag{4.2}$$

where $\mathcal{A}=\frac{\Gamma^2(\nu)}{\pi}\left(\frac{2\nu-1}{4}\right)^{1-2\nu}\sim\mathcal{O}(1)$ with $\nu=1/2+\frac{1}{1-\epsilon}$,

$$n_{\rm T}^{\rm PTA}=-\frac{2\epsilon}{1-\epsilon}\simeq 2,\quad for\quad \epsilon\ll 1.\tag{4.3}$$

According to eq. (4.2), we have $P_{\rm T}\simeq H_{\rm inf1}^2$ for $k/k_{\rm cutoff}\ll(\frac{H_{\rm inf1}}{H_{\rm inf2}})^{2/n_{\rm T}^{\rm PTA}}\ll 1$ corresponding the CMB band, while $P_{\rm T}\sim k^{n_{\rm T}^{\rm PTA}}\simeq k^2$ for $(\frac{H_{\rm inf1}}{H_{\rm inf2}})^{2/n_{\rm T}^{\rm PTA}}\ll k/k_{\rm cutoff}\ll 1$ corresponding the PTA band. Thus when $k/k_{\rm cutoff}\ll 1$, we have

$$P_{\rm T}(k)\sim H_{\rm inf1}^2\left[1+\mathcal{A}\left(\frac{k}{k_{\rm cutoff}}\right)^{n_{\rm T}^{\rm PTA}}\frac{H_{\rm inf2}^2}{H_{\rm inf1}^2}\right]\,.\tag{4.4}$$

It is just eq. (1.2) for $k_{\rm break} \simeq k_{\rm cutoff}(\frac{H_{\rm inf1}}{H_{\rm inf2}})^{2/n_{\rm T}^{\rm PTA}}$ and $n_{\rm T}^{\rm CMB} \simeq 0$. Thus the energy spectrum will be showed as the black dashed $\Omega_{GW}(f)$ curve in figure 3. Here, $k_{\rm cutoff}$ is a cutoff scale, see also section 4.1, beyond which $P_{\rm T} \sim H^2_{\rm inf2}$ is flat, or might be modified as in e.g. refs. [61–63]. Thus the IGWB at $k > k_{\rm cutoff}$ ($f > 10^{-7}$Hz in figure 3) is model-dependent, which is unknown and needs to be explored, which suggests that the requirement of BBN that inflation must end at a low scale [13, 19] is not workable.

It is also interesting to investigate other mechanisms, in which blue-tilted IGW at PTA band is sourced by other components during slow-roll inflation, such as the gauge fields [88–94], the non-Bunch-Davis initial states [95, 96], modified gravity, e.g. [97–100], and the collisions of bubbles nucleated [101, 102]. In light of our MCMC results on r, $n_{\rm T}^{\rm CMB}$ and $n_{\rm T}^{\rm PTA}$, and also bestfit $\Omega_{GW}(f)$ in figure 3, it is interesting to resurvey the relevant models.

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