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Schaar, M.S. van der

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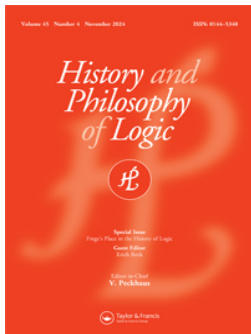
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The Kantian Background of Frege's Notion of Judgement

Maria van der Schaar 

Leiden University, Leiden, Netherlands

ABSTRACT

The paper gives, in the first place, a performative reading of Frege's account of non-alethic modalities. Second, it asks the question whether there is an influence of Kant's writings on Frege's notion of assertive force. It is generally acknowledged that the Neo-Kantian tradition has been essential to Frege's development. The paper shows that it can also be fruitful to look at more direct influences of Kant himself, especially on the early Frege. The paper shows that Kant's notion of assertoric modality must have been an inspiration for Frege to make the distinction between judgement, including the assertoric force, and judgeable content. Kant's *logical I* may also have played a role in Frege's account of judgement in terms of the judging agent, the first person. The conclusion of the paper, though, is that Frege is not a Kantian philosopher, as his idea of logic as providing a positive standard of objectivity is completely new.

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1. Introduction

In Frege's *Begriffsschrift* there is a unique role for the judgement stroke, both in the explanation of the inference rules and in the presentation of its axioms and theorems. In order to make clear whether the occurrence of a sentence is used to make an assertion or not, Frege introduces the judgement stroke, the vertical part of '⊢'. As Frege puts it in an early publication: 'When I want to assert a content as correct, I put at the left side of the content stroke the judgement stroke: $\vdash 2 + 3 = 5$ ' (1883, 5). For modern readers, it seems merely a sign indicating that Frege asserts the content to be true. We tend to see the judgement stroke along empirical-pragmatic terms, and we might even be tempted to use a small token of *a* or *b* as subscript next to the judgement stroke, to keep track of who asserted what. For, we give in general an account of judgement by describing someone's judgement from an external point of view. We thus use a name for the judging subject, consider the subject to be part of the world as an empirical subject, and consider judgement as an empirical relation in the world. Frege, though, does not understand the judgement stroke in empirical terms. For him, the essence of logic lies in the assertive force indicated by the judgement stroke (1915, 272). As I have argued elsewhere (Schaar 2018), in Frege's account of the judgement stroke, the *I* who is making the judgement is not to be understood in empirical terms, nor is the judgement to be understood that way. The first person is rather a non-empirical,

CONTACT Maria van der Schaar  m.v.d.schaar@phil.leidenuniv.nl  Leiden University, Leiden, Netherlands

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logical I. It is not relevant that Frege as empirical subject is making the judgement. As a truth-claim is made in the assertions of the *Begriffsschrift*, anyone who reads it is to make these judgements by herself. The I is thus the logical agent in us, engaged as a first person in understanding the *Begriffsschrift*.

In the history of logic, there seems to be no precedent to the judgment stroke.¹ Still, a historical background may help us to understand the non-empirical notions of judgement and judging agent that are needed to understand the role of the judgement stroke in the *Begriffsschrift*. Although most logicians of the nineteenth century made ample use of the notion of judgement in their logic, Frege accuses them of not making a clear distinction between logic and psychology. The Neo-Kantian tradition may help us in understanding Frege, as this tradition is well-known for its anti-psychologism. Hans Sluga (1980) and Gottfried Gabriel (2002) have shown the importance of the Neo-Kantian tradition for Frege. Regarding the notions of judgement and judging agent we might also turn to Kant himself, as Frege directly engages with Kant, and Kant makes use of a non-empirical notion of the I in his *Kritik der Reinen Vernunft*.

To what extent can we assume that Frege was familiar with Kant's writings? As the references in the *Begriffsschrift* and the *Grundlagen* show, Frege has read the beginning of Kant's *Kritik der reinen Vernunft* (1781/1787) and part of the *Jäsche Logik* (1800), as well. One may wonder to what extent the *Jäsche Logik* contains Kant's words, but in Frege's time it was considered to be a fully Kantian work. Frege thought of Kant as a great philosopher, using the Hartenstein edition *Immanuel Kant's sämtliche Werke* (1867–1868).² He may have obtained knowledge of Kant directly through his writings; through a seminar on Kant by Kuno Fischer that Frege attended as a student in Jena (Cf. Kreiser 2001, 64); or, in discussions with Kantian philosophers. Kant was a major figure in Germany in the last quarter of the nineteenth century. Because Frege directly relates to Kant in his early writings, and because the judgement stroke is already there in the early *Begriffsschrift*, I will mainly focus on Frege's earlier writings, neglecting for example the *Sinn/Bedeutung* distinction. As we will see, the agreement between Kant and Frege often hides deeper-lying differences between the two philosophers.³ This does not imply, though, that Frege was not inspired by Kant. We will also see that a confrontation with Kant's writings will help us in obtaining a sharper view on Frege.

How can a confrontation between Frege and Kant help us in understanding Frege's account of judgement and the judgement stroke in particular? In Section 2, we will see that the Kantian distinction between the different modalities of judgement is taken up by Frege in the opening sections of the *Begriffsschrift*. Are these different modalities relevant to the distinction between judgement and judgeable content that Frege introduces in these early sections? And are they relevant to the elucidation of the judgement stroke in terms of 'the writer' and 'the reader' that Frege gives there? We will see that one of the

¹ Leibniz, the other philosopher besides Kant that Frege is willing to mention in a positive way, can be considered as an exception. Elsewhere I have explained that Leibniz has a sign of assertive force (Latin: *Ita*, Yes) in his universal language. This reflects his interest in the logical uses of language (Schaar 2019, 60). The idea can be found in a small text, 'Analysis Particularum' (Leibniz 1686), not available in 1879, as far as I know.

² The two volumes of Baumann that Frege used extensively as a source in the *Grundlagen* do not contain a chapter on Kant. Although it does present a Kantian view at the end of volume II, there are no specific references to Kant's writings.

³ Joan Weiner (1990, 48 ff.) has shown that Kant's writings have played an inspiring role in Frege's understanding of arithmetic and the notion of analyticity, although Frege's notion of analyticity is different from Kant's. Furthermore, both Kant and Frege endorse the thesis that geometry consists of synthetic a priori truths. Frege's view, though, is not Kantian in the strict sense, for it does not allow for *pure* intuition ('reine Anschauung', *Grundlagen*, §26).

Kantian modalities, the assertoric one, may have played an inspiring role for Frege in making the distinction between judgement and judgeable content. Although Frege's notion of judgement differs considerably from Kant's, in both accounts of judgement the judging agent plays a central role. In relation to this point, the question is raised, in Section 3, to what extent there is a counterpart to Frege's notion of judging agent in Kant's *I think* and the transcendental ego. In Section 4, we will see that there is an agreement between Kant and Frege insofar as both give an account of objectivity in terms of judgement. Frege's account of objectivity and truth in terms of assertive force may have been inspired by the way Kant accounts for objectivity without neglecting the role of the judging agent. Frege departs, though, from Kant and the Kantian tradition in his new idea of logic as a science. For Frege, logic has a foundational role in relation to the sciences and to philosophy as well.

2. The Epistemic Modalities and the Judgement Stroke

In his *Begriffsschrift* (1879, §4) Frege mentions the three modalities that we know from Kant's first *Kritik* and from the *Jäsche Logik*. For Frege, they concern the distinction between the problematic, the assertoric and the apodictic use one can make of a sentence. After he has made the distinction between judgement as act and judgeable content, Frege argues that the traditional distinction between general and particular judgements concerns the judgeable content, not the act of judgement. This likewise holds for the traditional distinction between affirmative and negative judgements; for Frege, negation pertains only to the content. The situation is different for the modalities, for they do not concern the judgeable content, according to Frege. How does Frege formulate the distinction between these modalities?

The apodictic judgement differs from the assertoric judgement insofar as the existence of general judgements is indicated from which the sentence (*Satz*) can be inferred, while the assertoric judgement does not give such an indication. When I put forward a sentence as necessary, I give a hint concerning my grounds of the judgement.

[...]

When a sentence is put forward as possible (*möglich*), the speaker suspends the judgement by indicating that he does not know any laws from which its negation would follow.⁴

As Frege's explanation of the apodictic judgement makes clear, these modalities concern the presence or absence of grounds for one's judgement, or for one's withholding the judgement. The grounds that play a role in these explanations are the general judgements, containing laws. These modalities may therefore be called *epistemic*.⁵ They are not

⁴ *Bs*, §4. I have deleted the last part of the sentence, 'or he says that the denial of the sentence is false', for this presents a special case of the assertoric judgement, as Frege writes.

⁵ Frege's a priori / a posteriori and analytic / synthetic distinction in *Grundlagen* §3, are also epistemic, as they are explained in terms of justification for making the judgement. A truth is a priori if it can be demonstrated by general laws alone that themselves neither need nor are capable of demonstration. For Frege, analyticity is a stricter notion, as it demands a demonstration by means of logical laws and definitions alone. Just as the distinction between the different modalities, none of these distinctions pertain to the content. As the distinctions from the *Grundlagen* do not have a performative nature, they differ from the modal distinctions in the *Begriffsschrift*. There is a connection, though, between putting forward a sentence as necessary (the apodictic judgement) and the judgement being a priori. If one knows the judgement to be a priori, think of a geometrical theorem, one is entitled to put forward the sentence expressing its content as necessary. The notion of analyticity, being essential to Frege's logicist project, does not have a counterpart in the modalities.

to be understood as epistemic modalities in the modern sense, for the latter concern the content only. The modern epistemic modalities are alethic in the sense that they present truth-relations between propositions concerning knowledge.⁶

On Frege's account, these modalities are ways in which we put forward a *Satz*, a sentence expressing a judgeable content: 'When I put forward a sentence as necessary, I give a hint concerning my grounds of judgement [...] When a sentence is put forward as possible, the speaker suspends the judgement'.⁷ I propose a *performative* reading of Frege's modalities. This means that I understand the 'putting forward' ('hinstellen') of a sentence as a certain use one is making of a *free-standing* sentence. This gives a natural relation of the passage on the modalities to the distinction between judgement and judgeable content introduced in *Begriffsschrift*, §2. The 'assertoric judgement', in which there is *no* indication that the sentence can be derived from general judgements, can be understood as a special case of judgement, including its assertive force. The 'apodictic judgement' is for Frege, I assume, a special case of judgement, as well. One uses the free-standing occurrence of a sentence to make an assertion or judgement in Frege's sense, and, at the same time, one indicates, one gives a hint, that the judgement can be derived from general laws. The putting forward of a sentence as possible is not a special case of assertion or judgement, since the speaker withholds judgement, indicating that one does not know any laws from which the denial could be derived.⁸ What is at issue in Frege's modalities is a certain type or variant of illocutionary force, not in the sense that we have here three different variants of assertive force. It is not that the putting forward of a sentence as possible would contain the weakest assertive force, while putting it forward as necessary would contain the strongest degree of assertive force. Frege does not allow for degrees of assertive force, just as he does not allow for degrees of truth. It is rather that two of the modalities have assertive force, while the modality of possibility has no *assertive* force at all, at least, in the one meaning Frege assigned to this modality that we selected. Still, when the occurrence of a sentence is put forward as possible, thereby indicating that one does not know any laws from which its negation would follow, the utterance *has* a certain illocutionary force. If one says 'I hereby put this sentence forward as possible', and one accepts Frege's convention about what kind of use is intended, the utterance is performative.⁹

The modal use is to be indicated by a certain 'hint', for as soon as one describes how one puts forward the sentence, one expresses a new content by means of a new sentence. In a mere description, one misses the performative aspect of these modalities. For Frege, only free-standing occurrences of sentences may have such an epistemic-performative use. A sentence that is part of a complex sentence cannot be used this way. The occurrence of a sentence clause $p \ \& \ \neg p$ as part of $\neg (p \ \& \ \neg p)$, where we can substitute any declarative for p , is not put forward as possible. It has a certain linguistic use, but not, on its own, a

⁶ The box and diamond in modal logic are operators, allowing for de dicto ($\Diamond p$), and de re modalities ($\Box x \Diamond Fx$); they thus express part of the propositional content. The box and diamond may be given an epistemic reading. Although Frege does not allow for such alethic modalities, I assume that we need both the alethic and the Fregean modalities.

⁷ *Bs*, §4. I use 'sentence' as a translation of 'Satz', using it in the sense of 'declarative sentence', as Frege speaks of the linguistic notion, not of the proposition as thought. The term is not a technical one in the *Begriffsschrift*. Later, in the *Grundgesetze* (I, §5), Frege does use 'Satz' in a technical sense, as short-hand for 'Begriffsschriftsatz': concept-script assertion.

⁸ A: 'It may rain on that day. What will we do then?' B: 'If it rains, we cancel everything.' When used as antecedent, the sentence loses the epistemic modality.

⁹ Austin 1975, 57. Frege's performative account of epistemic modality provides a unique answer to the problems raised by Seth Yalcin (2011) for a descriptivist, factualist account of epistemic modal terms. Yalcin proposes an expressivist account in terms of belief, and thus differs from the Fregean, performative account.

performative use. In Frege's system of 1879, the equivalent of such clauses are preceded by the horizontal, indicating, in our example, that $p \ \& \ \neg p$ forms a unity, to which the first occurrence of the negation-sign in $\neg (p \ \& \ \neg p)$ applies. The function of the horizontal can thus be compared to the role brackets have for us. The horizontal helps us in determining the scope of the logical signs. The horizontal is thus not an indication that what follows is to be taken as possible in the epistemic sense of the modalities. In Frege's 1879 system, the horizontal always precedes sentences and sentence-clauses (including dependent clauses). It will thus precede $p \ \& \ \neg p$ in $\neg (p \ \& \ \neg p)$. This does not mean that we put $p \ \& \ \neg p$ forward as *possible*, for we do know 'laws from which its negation would follow'.¹⁰

Sanford Shieh (2019, 52–53) gives an account of Frege's modalities in terms of Gricean implicatures. Implicatures, though, do not *especially* pertain to free-standing sentences; they are rather modifiers of the content. Compare the use of 'but', a conventional implicature, in the antecedent of the following sentence: 'If you come to the party but stay only for fifteen minutes, he will be insulted'. As the modal terms indicate a certain performative use, they are radically different from standard cases of conventional implicatures.¹¹

I do not think we have to explain Frege's modalities as meta-judgements, as Gabriel and Schlotter (2017, 84) have proposed. They understand Frege's epistemic modalities as judgements about judgements,¹² and then rightly explain that such a meta-judgement cannot be represented in Frege's system. This reading describes the performative aspect of these modalities by means of a new content of a meta-judgement. It thereby misses the performative element. Furthermore, it has difficulties to explain the putting forward of a sentence as possible; for the latter cannot be a judgement about a *judgement*. If one believes that Frege's epistemic modalities only make sense as meta-judgements, it becomes difficult to see the connection between the notion of assertoric judgement as one of the epistemic modalities and the new notion of judgement Frege introduced in the earlier *Begriffsschrift* sections.

In the explanation of the epistemic modalities Frege uses the first-person pronoun and a term like 'the speaker'. Such formulations may be read in a subjective sense. Leila Haaparanta (1988, 253) reads Frege's epistemic modalities this way: they relate to 'our private grounds of judgement'. Perhaps, though, these terms can also be understood in a non-subjective sense. That may also help us in understanding Frege's use of the first-person indexical and of the term 'the writer' in the explanation of the judgement stroke. Perhaps we can understand Frege's use of these terms when we confront his account of these modalities with Kant's account of them.

¹⁰ In *Begriffsschrift* (§2) it seems that the horizontal has a different role, to raise in the reader a mere presentation of a judgeable content. This is a *specific* use of the horizontal, when it precedes a judgeable content, and is contrasted with the role of the full judgement stroke. As the horizontal precedes Fx in $\forall x(Fx \vee \neg Fx)$, it may precede *non*-judgeable contents as well (cf. *Bs*, §§11–12). In the *Grundgesetze* (1893) the horizontal has a different role; there it refers to the function or concept ... *is the true*.

¹¹ There are several interesting hint-cases in Frege's writings. The others seem to relate to a non-alethic, communicative content, and the colouring cases, for example, can be understood along Gricean terms. Regarding the epistemic modalities, the hint indicates a certain *use*. The hint-cases thus agree only in a negative sense: *none* of them relates to the alethic content.

¹² That they mean 'judgement' in the modern, Fregean sense is clear from their criticism of the position they attribute to Frege.

The three-fold distinction between problematic, assertoric and apodictic has its origin in Kant, and was well-known in the nineteenth century.¹³ For Kant, the distinction applies to ‘judgements’. The term ‘judgement’ in Kant refers to the product of an act that orders and unifies the subject- and predicate-concept, the *matter* of the judgement. Kant’s use of ‘judgement’ is in agreement with the logical tradition, and differs from the modern, Fregean use, in which assertive force is essential to judgement. Kant speaks of the ‘modalities of judgement’. Frege’s terminology prevents him to use the term ‘judgement’ in all three cases, for the ‘problematic or possible judgement’ cannot refer to a judgement in Frege’s sense of the term. Instead, Frege uses the term ‘Satz’ as a replacement of the Kantian term ‘judgement’ in his account of the modalities. As we have seen above, for Frege, the distinction concerns the different ways in which we put forward a sentence (*Satz*). Today we are used to Frege’s sense of the term judgement. Therefore, I will write ‘judgement’ if Kant’s non-assertoric sense is intended.

For Kant, each ‘judgement’ consists of a matter and a form. The different types of acts of judgement provide for different logical forms or functions unifying the matter in a certain way. Three of the four unifying functions are quantity (all, some, this), quality (affirmative, negative, ...), and relation (disjunctive, ...). These three forms will combine, so that we may have forms like *Every... is...*, or *Some... are not...*. Together with the matter these forms determine the content (*Inhalt*) of the ‘judgement’. As neither the affirmative nor the negative form in Kant brings in the assertive aspect, Frege’s judgemental force cannot be compared with Kant’s affirmation as a form of quality.

The fourth function or form consists of the logical modalities; in contrast to the other functions, they do not contribute anything to the content of the ‘judgement’, as Kant writes.¹⁴ These modalities indicate *the way in which* a ‘judgement’ is affirmed or denied (*JL*, §30, A169): whether the content is thought of as possible, the ‘problematic judgement’, whether one determines something about the case, the ‘assertoric judgement’, or whether one asserts the truth of the content with necessity, the ‘apodictic judgement’ (*JL*, §30).

This is not to say that, for Kant, the modalities have no relation to the content at all. The content is connected to the laws of the understanding, and this connection *determines* whether our judgement is apodictic, assertoric or merely problematic. The modalities express how the ‘judgement’ is determined by the faculty of knowledge (*das Erkenntnisvermögen*, *JL*, §30). Or, as Kant puts it in the first *Kritik*, they ‘relate the value of the copula to thinking as such’ (*KrV*, A74 / B100). In the apodictic judgement, the judgement is thought of as ‘determined by the laws of the understanding’ (*KrV*, A76 / B101); in the assertoric judgement, the judgement is thought of as somehow connected to these laws, but not as determined by them.

Whether Kant formulates it in terms of the faculty of knowledge or in terms of the laws of the understanding, he takes these modalities to be epistemic. Frege thus agrees with

¹³ Hermann Lotze (1874, I, §§41–46) discusses the three-fold distinction, and raises the issue of their ambiguity: the distinction may either be epistemic, relating to the ground for the act of judgement (Lotze 1874, I, §43, p. 65), or it may relate to what is asserted, to the content (Lotze 1874, I, §44, p. 65). Lotze’s account may have influenced Frege, as Lotze uses the term ‘Grund’, ‘ground’, here. Kant himself distinguishes the modalities of judgement from the categories of modality. The latter express the relation of the experienced object to the faculty of knowing (*KrV*, B266). The categories of modality depend, for Kant, on the modalities of judgement. The categories of modality play no role in Frege’s writings.

¹⁴ The modality of judgement is a special function of judgement, being special in the sense that it adds nothing to the content (*Inhalt*) of the judgement (for, besides quantity, quality and relation there is nothing that contributes to the content of the judgement) (*KrV*, A74 / B100).

Kant in taking these non-alethic modalities in an epistemic sense. In Kant, the laws of the understanding are valid for any judging agent insofar as they form the necessary conditions for any judgement whatever, where these necessary conditions may be purely formal or relate to any possible experience (cf. Boyle 2020). Kant's account of the epistemic modalities is thus not to be understood in a private sense.

In Kant, *every* 'judgement' has one of the three modalities; 'judgements' that are part of other 'judgements' have a modality, as well; the antecedents of conditional judgements are for Kant problematic 'judgments'. In a case of *Modus Ponens*, for example, the content of the antecedent is taken as merely possible in the first premise, and as asserted in the second one. As Béatrice Longuenesse (1998, 160) writes in *Kant and the Capacity to Judge*, 'a judgment is problematic [...] only if *used* as a merely problematic judgment'. The general kind of use that is introduced here differs from Frege's account of the modalities in terms of use. Although one could say that in the Kantian modalities a certain use is made of a 'judgement', or sentence, this need not be the performative use that one can make of the occurrence of a free-standing sentence that we saw in Frege's account of the epistemic modalities. For Frege, the epistemic modalities concern uses of free-standing sentences only. For Frege, the modalities concern performative use; they are cases of, or variants of, different kinds of illocutionary force. Still, we can note an important agreement between Frege and Kant. If we focus on the assertoric judgement in Kant, one can see there a distinction between the content of 'judgement' and the assertoric function as one of the epistemic modalities. And this may have been an inspiration for Frege to distinguish between act of judgement and judgeable content.

To conclude, in his account of the modalities, Kant may have been an inspiration for Frege as both understand these non-alethic modalities in epistemic terms. It is not yet decided whether these modalities in Frege concern private grounds, or not. Although Frege speaks of 'my grounds', the indexical term need not be understood in a subjective sense. When Frege writes that in the apodictic judgement the existence of general judgements or laws is indicated from which the sentence can be inferred, this does not point to a private justification. If, as I have explained, there is a close relation between the judgement stroke in the Begriffsschrift and the assertoric modality, this issue is related to the question whether the assertive force indicated by the judgement stroke is to be understood in a private sense (see Section 3).

One may thus say that Frege, like Kant, understands these modalities in terms of the use we can make of a judgement or 'sentence'. Frege has a specific notion of use in mind, though: these epistemic modalities concern a performative use. The judgement stroke in the Begriffsschrift precisely points to such use, by indicating assertive force.

A comparison between Frege and Kant regarding the modalities is complicated by their different notions of content, and the different role the act of judgement and the judging agent have in relation to the content. Some readers of Kant have pointed to a crucial difference between the two philosophers here. For Kant, the modalities and the I as thinking agent are not external to the content (cf. Newton 2019). One may agree that in Kant thoughts or judgements do not have a unity independently of the I that thinks this unity. In this sense, there is a crucial difference with Frege: for him, judgeable contents have a unity independent of the act of judging or asserting. I do not think, though, that this means that for Frege there is an *external* relation between the judgeable content and the judging agent who acknowledges the content as true. It is true that, on Frege's account, one may

look at the judgement from an external point of view and describe someone's judgement as an empirical phenomenon. We could describe this empirical phenomenon by the form *Jab*, and we would thus understand judgement as an external relation comparable to any other relation in the empirical world. The judgement stroke is not a sign of judgement in this empirical sense, though. Judgement in the logical sense is, for Frege, not an external relation between a judging subject and a judgeable content. The judging subject, the non-empirical I, forms an essential aspect of the logical judgement. Furthermore, on Frege's account, one cannot fully understand what the content is, without invoking the notion of inference or judgement, for the inferences we can make show us the relevant parts of the content (*Bs*, §3).

3. The Judgement Stroke and the Kantian *I Think*

In the *Begriffsschrift*, Frege writes that a judgement is 'expressed' by means of the sign '┊'. In the next step, Frege explains what is left if the judgement stroke is deleted. '— *A* does not express a judgement, but is rather to awaken in the reader the mere presentation [of the content expressed by *A*]'. For, 'the writer does not express whether he ascribes truth to it or not' (*Bs*, §2). The explanation of '—' is thus given in terms of the *absence* of ascription of truth, that is, in terms of judgement. This is surprising if one thinks of the later 'Der Gedanke' (Frege 1918, 62), where an account of judgement is preceded by an independent account of merely thinking a thought.

In the passage from the *Begriffsschrift* just mentioned, it is implicated that the presence of the judgement stroke indicates that the writer ascribes truth to the content, that the writer has made the judgement. How are we to understand 'the writer' in the elucidation of the judgement stroke? In other writings, Frege uses the indexical 'I': 'With this judgement stroke I close a sentence, [...] and the content of the sentence thus closed I assert as being true by the same sign' (Frege 1896, 377; cf. Frege 1883, 5). As we have seen in the introduction, this occurrence of the first-person pronoun is to be understood in a non-empirical sense, in which it is not relevant that Gottlob Frege is making the judgement. How, then, are we to understand the role of the judgement stroke in the *Begriffsschrift*?

In the *Grundgesetze* (Preface, xvi) the logical laws are called the laws of truth. In the earlier *Grundlagen* (§87), though, these laws are understood as laws of judgement. As judgement is an acknowledgement of the *truth* of a thought (1893), or an ascription of *truth* to a judgeable content (1879), the laws of truth can be understood as laws of judgement, as laws of assertive force. Although most of our logical laws can be demonstrated by logical means, there comes an end to demonstration when we reach the logical axioms and the inference rules such as *Modus Ponens*. Denying any of these results in confusion, as Frege puts it in the *Grundlagen* (§14; cf. *Grundgesetze*, Preface, xvi, xvii). We couldn't make sense of someone denying the law of identity, or of someone who doesn't accept *Modus Ponens* as a rule of inference. These laws form part of what it is to judge. The logical laws can be understood as a development of the word 'true' (Frege 1879–1891, 3). This means that they can be understood as a development of the notion of assertive force, as well, for Frege understands the notions of truth and assertive force to be internally related (cf. Frege 1915, 272). In one sense, the logical laws can be understood as describing universal relations between truth-values, concepts and objects in the world; in another sense, they form part of the assertive force. For the early Frege, these two theses do not exclude

each other. Objects, concepts, truth-values and their logical relations cannot be understood independently of reason (*Vernunft*), that is, they cannot be understood independently of our logical capacity, our capacity to judge: ‘to answer the question what things are independent of reason means judging without judgement, wash the fur without making it wet’ (*Grundlagen*, §26).¹⁵

The role that the laws of logic have for Frege may here be compared to the role that the laws of the understanding have in Kant’s transcendental project. Both Kant and Frege account for the objectivity of the basic principles without neglecting the role of the act of judgement and the judging agent. In a certain sense, the laws of logic are for Frege necessary conditions for judgement, as well, as they form part of the assertive force, not in the sense that we cannot make logical mistakes, but in the sense that these laws constitute the norms for our judgements. We cannot grasp something as a judgement or inference unless we see it as being subject to evaluation by the laws of logic (cf. *Tascheck* 2008). The difference between Kant and Frege can then be found in their different answers to the question what is to provide the basic principles of philosophy: transcendental philosophy or logic in Frege’s sense. I come back to this difference in the final section.

Can we elucidate the role of the judgement stroke in Frege’s logic by comparing and contrasting it with the *I think* in Kant that may accompany all our judgements? For Kant, the *I think* has a unifying aspect. In principle, it accompanies all our judgements and all our thinking (*KrV*, B131–32), for without it the content of the judgement could not form a unity. Without it, the logical forms or functions of the judgement that were mentioned in the former section could not do their unifying work, according to Kant. For Frege, the assertive force is not needed to unify a judgeable content, and the horizontal, in the early Frege, is merely to indicate that there is a unity, to which the logical signs may apply. Frege starts with a notion of judgeable content, as what can be judged, and then distinguishes different concepts and relations depending on the inferences one needs to make. In the *Begriffsschrift*, the judgeable content is conceptually prior to these concepts and relations. The general theme of the priority of judgement over concepts is a Kantian one, but, in Frege, the priority of the judgemental content in relation to concepts allows for something completely new. Frege is able to distinguish within the same judgeable content different concepts and relations. In *Cato kills Cato* the concepts *killing Cato*, *being killed by Cato*, or the relation of *killing*, can be discerned, depending on the inference one needs to make.¹⁶

The *I think* has a second unifying function in Kant. It unifies the different thoughts as *my* thoughts. There seems to be a partial agreement, at least, with the role of the judgement stroke in Frege’s explanation of the inference rules, or inference schema’s (*Bs*, §6; *Grundgesetze*, §14). Through the presence of the judgement stroke, the premises are connected to the conclusion, not only as asserted by the same agent, but also in the sense that the conclusion is grounded by the premises. ‘When we infer, we acknowledge a truth on the ground of other truths that are acknowledged by us in accordance with a logical law’ (Frege 1917, *WB*,

¹⁵ In Schaar (*forthcoming*) I consider Frege’s thesis that if the truth of a judgeable content is evident to the agent, (s)he is entitled to assert it as a logical axiom. As self-evidence is here not to be understood in a psychological sense, but purely in terms of our capacity to judge, I consider this view compatible with the view presented in this paper. The forthcoming paper explains Frege’s idea of logical capacity more extensively.

¹⁶ At first, Frege understands these functions to be linguistic (*Bs*, §9, p. 16), but on the next page he acknowledges that they concern concepts and relations as part of the judgeable content as soon as quantifiers and variables are introduced (*Bs*, §9, p. 17). The thesis is more explicitly expressed a few years later: ‘I start out from judgements and their contents, and not from concepts. [...] I only allow the formation of concepts to proceed from judgements’ (Frege 1880/81, 17).

30). This point is formulated in a more Kantian terminology in an earlier manuscript on logic: 'Judging insofar as one is conscious (*bewusst ist*) of other truths as providing a justification for it (*Rechtfertigungsgründe*) means INFERRING (*schliessen*)' (Frege 1879–1891, 3).¹⁷ The judgement stroke in Frege thus accounts for the unity of premises and conclusion in inference. Thereby it accounts for the fact that a demonstration or proof is a proof for me, the judging agent.

Whereas in Kant, the *I think* is to account both for the unity of the judgeable content and for the unity of one's thoughts and judgements, Frege separates these two kinds of unity. The presence of the judgement stroke in inference indicates the unity of the premises, and the unity of the premises and conclusion. The horizontal indicates a unity of content: what follows forms a unity to which the logical signs '¬', '→' or '∀x' may apply to express a new content.

Can Kant's account of the *I think* also be helpful in the elucidation of the judgement stroke in terms of the first-person pronoun? Kant takes the I that thinks to be a transcendental subject to which we cannot attribute any predicate, because it is presupposed in any form of predication, in any form of judgement. The I thus lacks all content (*KrV*, A346 / B404). We do not know the transcendental subject, for there is no object falling under a concept. In contrast to the I in the Cartesian *I think*, for Kant, the I is not a thing or substance, but the transcendental unity of consciousness. The I is thus non-empirical. This non-empirical I is the I of reason. The I in Kant, is 'the logical I, the capacity for intellectual synthesis shared by any rational thinker' (Newton 2019, 80).

In Frege, as we have seen, the I does not have this thoroughly unifying function, as the judgeable content is presupposed to have a unity. There is an agreement with Kant, though, insofar as the 'I' in Frege's account of the judgement stroke is not to be understood as indicating an empirical agent. Any reader, any logical agent, is to acquire the basic terms and concepts as Frege has explained them, and, on the basis of a grasp of these concepts, make the *Begriffsschrift* assertions step by step. The *Begriffsschrift* demands a first person engagement of the reader. The I who is making the judgements in the *Begriffsschrift* is a *logical, non-empirical I*, presupposed by any judgement we make about the world. It can be called a *transcendental subject*, as long as we do not take the term to be referring, for then we look at the agent as an object, and it has lost the special logical role it plays in relation to the judgement stroke.

There is some dispute whether the 'I' in Kant's 'I think' is to be taken as referring. According to Longuenesse (2017, 107, 109) it refers. According to Alexandra Newton, the 'I' in Kant's 'I think' is not a referring term (Newton 2019, 80 and 88, note 39). In the elucidations of the judgement stroke, Frege's 'I' might seem to refer to himself as judging agent. The role of the judgement stroke in the *Begriffsschrift* makes it clear, though, that nothing is referred to when it is used to make an assertion. Here it is crucial to distinguish the use of 'I' in the elucidation of the judgement stroke and in 'I (hereby) assert ...' from the use of 'I' in a sentence such as 'I am wounded'. In the latter case, 'I', together with the circumstances, is a referring expression.¹⁸

¹⁷ Longuenesse (2017, 26, 27) explains Kant's account of the *I think* here in terms of the judge's accountability for the assertion that suits Frege's use of the judgement stroke as well.

¹⁸ Frege's later thesis that one is given to oneself in a unique way (Frege 1918, 66) presupposes the idea of an I as object in the world, referred to by 'I' as in 'I am wounded', and is thus unrelated to the thesis that the act of judgement is a first person's act.

Notwithstanding the fact that, in contrast to Frege's judgement stroke, the Kantian *I think* has a unifying function on all levels of thought, we may conclude that Kant's *logical I* may have played an inspiring role for Frege in understanding the judging agent as a non-empirical I. In the early *Grundlagen*, the basic laws of logic and the inference rules are, for Frege, necessary conditions for judgement in the sense that they form the constitutive norms for judgement. This more general Kantian theme suits Frege's later remark, from 1915, that the assertive force contains the essence of logic. It thus seems that there is both in the early writings and in the later writings an internal relation between judgement and the laws of logic, the laws of truth.

4. Judgement, Logic and Reason

We have seen, in Section 3 (third paragraph), that Frege, in the early *Grundlagen*, points to a connection between objectivity and reason (*Vernunft*), and that the latter can be understood in terms of judgement. Does the analysis of the early Frege proposed help us in understanding the notion of judgement in Frege's mature writings, as well? In Kant and the (Neo-)Kantian tradition the notion of objective validity (*objective Gültigkeit*, KrV, A28 / B44) plays a crucial role in the account of objectivity and truth. For Kant, the concepts of objective validity and that of necessary general validity (for everyone) have the same extension (*Prolegomena*, §19). Before we ask the question whether a grasp of these notions in Kant may help us in understanding Frege's account of objectivity in terms of judgement, we may first note some important differences between Frege's notion of objectivity and truth, on the one hand, and the account of objectivity given by neo-Kantian contemporaries, on the other hand.

In the later *Grundgesetze*, Frege criticises the position defended by Benno Erdmann. In his *Logik* (1892), Erdmann proposes a naturalist reading of Kant's account of objective or general validity. For Erdmann, the necessity of the general validity of our logical laws is a psychological necessity: 'We cannot help assenting to them – such is the nature of our [...] thinking'.¹⁹ The problem, Frege notes, is that Erdmann explains truth in terms of a general consensus among individuals (*Grundgesetze*, Preface, xv). It is in this context that Frege claims that logical laws are not 'psychological laws of taking to be true, but laws of being true' (*Grundgesetze*, xvi). If we were to find other beings whose laws of thought directly contradict ours, the psychologistic logician has no means to ask the question 'Who is right?' (*Grundgesetze*, xvi). Frege's own reaction is: 'here we have a hitherto unknown kind of madness'. We cannot understand their laws of thought as *logical laws*.²⁰ For, we cannot acknowledge them as such, given that we have acknowledged the logical laws as laws of truth, as laws of assertive force. 'Whoever has once acknowledged a law of

¹⁹ 'Wir können nicht anders, als ihnen [the logical laws] zustimmen – nach der Natur unseres Vorstellens und Denkens' (Erdmann 1892, 378). Daniele Mezzadri (2018, 731–733) has given a well-documented account of Frege's confrontation with Erdmann's psychologism.

²⁰ 'What, however, if beings were even found whose laws of thought directly contradict ours' (*Grundgesetze*, Preface, xvi). Frege speaks here of 'laws of thought' (*Denkgesetze*), not of 'logical laws'. These aliens are thus not *logical* aliens in the strict sense. My reading of Frege here agrees with the reading Tyke Nunez (2019) has given of Kant's account of pure general logic. Like Kant, Frege allows for the possibility of logical mistakes that do not threaten our acknowledgement of the logical laws, but rejects the apparent possibility of logical aliens. In contrast to Kant, though, Frege *does* allow for a certain psychological reality in the *Grundgesetze*: we can imagine beings who have different laws of *thought*.

being true has thereby also acknowledged a law that prescribes what ought to be judged, wherever, whenever, and by whomsoever the judgement is made'. (*Grundgesetze*, xvii). In these pages, Frege acknowledges the possibility of a certain psychological reality; he does not acknowledge the laws of thought of these beings as *logical* laws. What is imagined is a psychological reality, a certain form of madness, not a logical reality. The general validity of the logical laws is not restricted to a validity for human beings as we have known them until now, for the general validity of the logical laws forms the constitutive norm for judgement.

The explanation of validity by the Neo-Kantians of the Southwest German school around Windelband seems closer to Frege's understanding of the logical laws as laws of truth than Erdmann's explanation. If one distinguishes between act and content of judgement, one may argue that the content is objective, independent of any judging agent, whereas the act of judgement is subjective, and merely object of study for psychology. Sometimes, the later Frege seems to endorse such a position. This would be in line with the Platonism one may find in Lotze and Neo-Kantians such as Rickert.²¹ As for them, for Frege, truth is a value, a goal in logic, accounting for the normative dimension in judgement as an acknowledgement of the truth of a thought, a judgeable content. In contrast to a Platonic understanding of objectivity, though, Frege does not consider the act of judgement to be a psychological notion. We need to make a distinction between the empirical notion of holding as true (*Fürwahrhalten*), or belief as we would call it, and the logical notion of judgemental act (cf. Schaar 2018). Objectivity is to be understood, not in terms of belief, but in terms of reason, our capacity to judge, as we have seen in Section 3. Judgement plays an important role in Frege's account of truth and objectivity (see also Ricketts 1986 and Reck 2007).

Lotze and the Neo-Kantians just mentioned give a merely psychological account not only of the act of judgement, but also of falsity. As they have a more strictly epistemic approach to philosophical problems, they acknowledge truths as objective. Not clearly distinguishing between falsehood and error, they consider falsehoods to be merely subjective. A logician, like Frege, is in need of both objective truths and objective falsehoods (Frege 1919, 147; Frege 1897, 150). Although Frege only allows for true premises in an inference, parts of these premises may be false. For Frege, there is thus a symmetry between true and false thoughts. The true and the false are both logical objects; they have equal right to be called a 'truth-value'. This is not to say that Frege does not account for the asymmetry of truth and falsehood: the logical laws are the laws of truth.

Can the understanding of objective validity in Kant give us a background for understanding Frege's account of objectivity in logic? In *Grundlagen*, §26, Frege introduces a distinction between the notion of objectivity and that of actuality (*Wirklichkeit*). Things in space, like the earth, can in principle be perceived; they are actual. Things that we cannot perceive may very well be objective: numbers, concepts, judgeable contents. 'I understand by objectivity an independence of our senses, intuitions, and presentations [...] but

²¹ Lotze speaks of the validity of truths ('die Geltung von Wahrheiten') with reference to Plato (Lotze 1874, §317, p. 513). I have put forward some arguments against Gabriel's thesis that Frege would give, like Windelband, a transcendental account of the logical laws in Schaar 2018, 233. This paper gives a more nuanced view on the issue, perhaps because it has taken Frege's early writings as starting-point.

not independence of reason (*Vernunft*)' (*Grundlagen*, §26). The objectivity within reason accounts for the objectivity of logic and arithmetic (*Grundlagen*, §105). It is an objectivity that holds for any judging agent. What is objective is in this sense generally valid, as Kant put it, not insofar as there is a consensus, as Erdmann thought, but insofar as it is valid for any judging agent as non-empirical I.

Like Kant, Frege understands reason in a non-psychological sense. In his later writings, reason (*Vernunft*) is called the logical source of knowledge (*logische Erkenntnisquelle*, Frege 1896, 362; cf. Frege 1924–25, 286). For Kant, logic gives us the laws of understanding in general; it is a self-cognition of the understanding and reason (*JL*, A14; cf. Boyle 2020, 121). In a similar way, Frege writes that the object of reason is reason (*Grundlagen*, §105). We will see, though, that for Frege there is a unique role for reason, our logical capacity, which makes it impossible to understand Frege as a Kantian philosopher.

For Kant, in our judgements, the two powers of understanding and sensibility always work together. We could never grasp objects that are immediately given to reason; this is precisely why, for Kant, traditional metaphysics is impossible. In every judgement, the two powers come together. This means, for Kant, that general logic is concerned with the laws of the understanding, the laws of thought, in abstraction from its relation to sensibility. It abstracts from all knowable content (*KrV*, A53–4 / B 77–78; *JL*, A3–4; cf. MacFarlane 2002). General logic is thus merely concerned with the forms of judgement, the logical functions of unity, not with its matter. The understanding has a unifying, synthesising function, where what is synthesised comes from our power of sensibility. For Kant, the different powers or faculties thus form a unity; each is grounded in an underlying unity (Boyle 2020, 133). Because for Kant logic thus abstracts from what is known in our judgement, its laws provide a merely formal condition for knowledge, as a negative standard, for the condition is necessary but not sufficient (*KrV*, A59 / B84; cf. *JL*, A3–4).

Frege explicitly rejects Kant's thesis that without sensibility objects cannot be given to us (*Grundlagen*, §89). No such abstraction is needed for Frege in determining the laws of logic. Our logical capacity can exercise its function without the use of any of the other capacities: sensibility and the geometrical source of knowledge. The objects of reason are immediately given to reason, our logical capacity (*Grundlagen*, §105). We can thus grasp these objects completely. This means that the logical laws, the laws of truth, have content and provide a positive standard for objectivity. For Frege, logical functions such as negation are not the result of an act of abstraction. As the later Frege (1906, 428) puts it, logic cannot be formal regarding its own concepts: negation, identity, implication, subsumption, and subordination of concepts. For Frege, logic is a true science with its own subject-matter, whose laws hold for all objects and functions in the world. If we realise that we are entitled to make the assertions of the *Begriffsschrift* by our logical capacity alone, we know that others will be able to make the same judgements, because all reasoning agents share this logical capacity, the capacity to judge. This may settle the question in what sense the laws of logic may form the standard of objectivity, and differ from other scientific laws: the ground of the laws of logic lies on a deeper level, as these laws are grounded in nothing but the capacity to judge.

Because our logical capacity can thus be exercised without the other capacities, there is no need for abstraction. Frege would thus agree with Hegel's criticism of Kant: if logic were to abstract from all content, we would merely obtain empty forms (*tote Formen*, *WL*, 41). Frege did not have any knowledge of Hegel's writings, as far as we know. Some Hegelian

ideas, though, may have come to Frege through Trendelenburg.²² Trendelenburg's thesis that there is a unity of logic and metaphysics, and that logic is the foundational science, must have strongly appealed to Frege (cf. *Trendelenburg* 1840, 12–14). For Frege, the ontological distinction between objects, functions and functions of different order are grounded in his logic, in the possible inferences we make, as Ricketts (1986) has shown. Logic is thus not only prior to the natural sciences, but prior to psychology and metaphysics, as well, in the sense that these sciences are in need of the logical principles (*Grundgesetze*, Preface, xix). The metaphysician, who asks a question about being or identity, has to take into account what logical role the concept of existence has, or what the formal properties of identity are.

As in Kant, Frege's account of objectivity can be understood as an objectivity for all rational agents. In contrast to Kant, though, Frege allows for judgements grounded in our logical capacity, in reason, alone. For Frege logic thus provides a positive standard of objectivity, and is not merely providing formal criteria for the correctness of our judgements. It is thus possible for Frege to see logic, rather than transcendental philosophy, as a foundational science. Given, though, that Frege understands the laws of logic as constitutive norms for judgement, one may note a transcendental element in Frege's conception of logic.

5. Conclusion

Kant's modalities of judgement must have been an inspiration for Frege. In Kant, the modalities relate to a difference in use, and this may have inspired Frege to distinguish between assertive and non-assertive uses of a sentence. Both Kant and Frege take these non-alethic modalities to be epistemic, and explain the apodictic modality in terms of general laws. In this sense, these modalities do not merely concern subjective justification. As Frege may be understood as giving a *performative* account of these modalities, his explanation of the epistemic modalities is also different from Kant. I take Frege's account of the epistemic modalities to be original and relevant for philosophy today.

Singular first-person pronouns and terms like 'the writer' and 'the reader' are used by Frege in the elucidation of the judgement stroke, and in the explanation of the epistemic modalities, as well. These terms are not to be understood in a subjective sense; Kant's notion of logical I may have inspired Frege to use the first-person pronoun here. As in Kant, the *I* in the elucidation of the judgement stroke is not an empirical subject, but a *logical I*, presupposed in the logical assertions of any Begriffsschrift. The role of the judgement stroke in the Begriffsschrift is different, though, from the role of the *I think* in Kant's philosophy. For, the judgement stroke indicates assertive force.

We have noted some important differences between Frege and the Neo-Kantians regarding the notions of objectivity and judgement. The crucial role that judgement and assertive force have in Frege's writings means that Frege is neither a Platonist nor a psychologist.

²² Frege most likely has discussed his ideas with his colleague in the philosophy department in Jena, Rudolf Eucken, who was Frege's neighbour, and had a thorough knowledge of Trendelenburg (cf. *Gabriel & Schlotter* 2017, 29). The thesis that Frege here connects to the pre-Kantian metaphysics of the neo-Leibnizian tradition, as Gabriel and Schlotter (2017, 176–178) have pointed out, should be qualified. For Wolff, for example, first philosophy is to be metaphysics: *Philosophia Prima Sive Ontologia* (1730). For Frege, logic is to be the foundational science.

Frege understands objectivity and judgement in terms of each other; objectivity is understood in terms of reason, our capacity to judge. Here it makes more sense to relate Frege directly to Kant.

Finally, a confrontation of Frege's idea of logic as science with Kant's thesis that general logic is purely formal, may give us an understanding of the importance logic has for Frege. Logic is the foundation for all the other sciences, and gives a positive standard of objectivity grounded in nothing but the capacity to judge. The presence of the judgement stroke is essential to Frege's new logic. As indicator of assertive force, of a first person's judgement, it expresses that logic is to give the laws of assertive force, the laws of truth. This conception of logic is completely new. We thus see that Frege was inspired by Kant, but did not stay within the Kantian tradition. One may conclude, though, that the *first person* cannot be neglected in a history of early analytic philosophy, and that this idea has a Kantian background.

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ORCID

Maria van der Schaar  <http://orcid.org/0000-0003-4944-2806>

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