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Leiden
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Sweeping vacuum gravitational waves under the rug

Negro, A.

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Summary

With the first direct detection of GWs from the Laser Interferometer Gravitational-Wave Observatory on September 14, 2015, the gravitational wave detection era started. Since then, several other signals from black holes and neutron stars events have been detected, and the possibility of studying the universe by means of GWs represents a new frontier.

How about GWs from cosmological origin? Detecting GWs from early universe phenomena would allow us to uncover mysteries that were previously inaccessible through traditional electromagnetic observations. The possibility of opening this new window on the early universe comes with challenges, not only on the experimental side, but also on the theoretical side, with the goal of having an accurate description of such early universe GWs signals. In this thesis we focus our attention on vacuum GWs: vacuum tensor quantum fluctuations which inevitably come in the form of stochastic backgrounds. It is important to be careful in distinguishing vacuum GWs from primordial gravitational waves sourced by dynamical processes involving energy and momentum transfer, such as GWs from phase transitions, particle production or decay of topological defects. The latter involve propagating gravitons sourced by some physical processes while vacuum GWs are by definition vacuum polarization effects whose imprint on various observables is no less real, but comes with important subtleties that motivate the work presented in this thesis.

To contextualize our work, in the **Introduction** we reviewed the thermal history of the universe, the foundations of the cosmological model and we motivated the need to include vacuum quantum fluctuations in the early stages of our universe. After that, we shifted our attention to introduce the tools needed to quantify the contribution of vacuum quantum fluctuations. We reviewed the basics of the renormalization procedure as a method to meaningfully reabsorb UV divergences arising in computing quantum corrections. We remarked the possibility of obtaining divergent vacuum energies and highlighted that the splitting into classical quantities and quantum corrections can be misleading: one has access to fully dressed observables and cannot separately measure quantum corrections and classical quantities.

Can vacuum GWs be constrained by N_{eff} bounds? This is the question from which our work started. **Chapter 2** is a detailed review of the typical expressions that one can find in the literature to connect the energy density of vacuum GWs to the bounds on the effective number of relativistic species, N_{eff} , at the time of Big Bang nucleosynthesis. We concluded the chapter by pointing out that whether one

can meaningfully constrain vacuum GWs with N_{eff} bounds is intrinsically bound to the definition of the stress energy tensor of GWs and to how the divergences that inevitably arise in computing the energy density of vacuum GWs are renormalized. The latter caveat motivated the work presented in **Chapter 3**, where we applied well-established renormalization techniques to the case of a massless, non-interacting scalar field on a Friedmann–Lemaître–Robertson–Walker background. We showed how dimensional regularization can be used to extract UV divergences from scaleless integrals, the independence of logarithm divergences on the choice of the regularization method and the need to use regularization methods that preserve covariance in order to find counterterms that are proportional to geometric invariants. Furthermore, by studying a background evolution that after a pre-inflationary radiation dominated era transitions to a pure de Sitter phase which is followed by a second post-inflationary radiation dominated era, we explicitly demonstrated that IR/UV scales connected with the beginning/end of inflation do not cure UV divergences and that IR divergences are an artifact of the idealization of a past infinite de Sitter geometry. By applying the lessons learned in the apparently simple example of a scalar field to the case of vacuum GWs, in **Chapter 4** we addressed the question whether N_{eff} bounds can be used to infer constraints on vacuum GWs. We started by deriving a formula for the energy density of GWs suitable for regularization, we addressed the caveats regarding the definition of the stress energy tensor of GWs and derived an improved formula that does not rely on prior scale separation. We then followed through the renormalization procedure, we isolated the divergent structure and after having subtracted the divergences in consistently defined counterterms, we commented on the renormalization conditions that must be imposed to fix the scheme-dependent finite leftover and obtain a meaningful answer.

In the second part of the thesis we revisited the study of vacuum GWs in a covariant formulation. To do so in **Chapter 5** we introduced the formalism needed to address the study of vacuum GWs as a spin-2 particle on curved spacetime. We then applied such tools in **Chapter 6**, where the renormalized stress energy tensor for GWs is obtained from the variation of the effective action with respect to the background metric. We derived the effective action for GWs, we regularized and consistently subtracted the divergences by adding counterterms at the level of the action. Finally, we specialized our otherwise general result to a radiation dominated background to fix the renormalization conditions, connect to the foliation dependent derivation and comment on the consequences on N_{eff} bounds.

Among the results that we presented in tackling, both in a covariant and foliation specific formulation, the renormalization of divergences in primordial observables, we conclude that N_{eff} bounds cannot be used to constrain vacuum GWs. Such constraint represents an attempt to compute and measure the absolute value of quantum corrections. The latter, by definition, cannot be measured separately from the classical background, as all physical observations are necessarily of fully dressed quantities.