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Plato, Aristotle, or Both ?

Dialogues between
Platonism and Aristotelianism in Antiquity



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PRINCIPLES, CONVERSION, AND CIRCULAR PROOF
THE RECEPTION OF AN ACADEMIC DEBATE IN PROCLUS AND PHILOPONUS

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It is common knowledge that both Plato and Aristotle admired the mathematics of their day for the success of its research, the rigidity of its proofs, and the necessity of its conclusions. Plato considered mathematics as the ideal preparation for dialectic; Aristotle modelled his *Analytics* on mathematical terminology and argument. Both philosophers exploited numerous mathematical examples throughout their works.

Here, however, I would like to discuss a debate on which Plato and Aristotle found themselves opposed to the mathematicians. As we shall see, this debate may well have influenced their presentation of the mathematical practice of their day. The debate concerns the notion of 'element', the nature of deduction as entailed by different definitions of 'element', and the consequences of these views for the understanding of the practice of mathematical argument. In this debate the mathematician Menaechmus (among others) objected to the philosophers of the Academy that the practice of his profession did not match the epistemology and ontology proposed for it by Plato and Aristotle. In the first part of this paper I shall introduce the debate, and consider some of the arguments in detail. The dossier thus collected will be the starting point of our analysis of its reception in late antiquity.

In the second part of the paper I turn to Proclus, first of all because he preserved crucial material on Menaechmus in his *Commentary on Euclid's Elements*. I shall briefly discuss the question how Proclus' reports of the debate are related to his Neoplatonic concerns. I shall argue that his testimony is interesting evidence for the impact of the 'mathematical turn' in later Platonism and for the interpretation of historical evidence. If a true Platonist of late antiquity has to embrace mathematical method and mathematical practice whenever possible, no strong opposition between philosophers and mathematicians can be allowed to stand.

In the third part of my paper I will argue that Philoponus, and probably Ammonius before him, chose to implement the mathematical paradigm in their interpretation of Aristotle's *Posterior Analytics* – thereby

changing considerably its purpose and logic. Philoponus' interpretation of Aristotle's discussion of logical conversion has been condemned by modern commentators. They regard Philoponus as an amateur without any logical talent, who completely misrepresents the structure of Aristotelian logic and fills his pages with long digressions of empty talk. Although I am the last person to deny Philoponus his share of mistakes or a certain tendency for digressions, I refuse to believe that his interpretation of Aristotle's logic is merely the sign of a weak mind. On the contrary, as I hope to show, much becomes clear when we realize that Philoponus' commentaries testify to a remarkable project: the mathematization of Aristotelian logic.¹ In particular, the notion of *mathematical* conversion as defined by Proclus began to influence the understanding of Aristotle's *logical* conversion. Of course the result is not in any way a sound interpretation of Aristotelian logic as we understand it today – but it is not empty talk either.

The Academic debate²

The traces of the debate between philosophers and mathematicians have been preserved for us mainly in three famous sources: Plato's *Theaetetus*, Aristotle's *Analytics*, and Proclus' *Commentary on Euclid's Elements*. Our spokesman for the mathematicians will be Menaechmus.³ Menaechmus contributed to the theory of the three conic sections (parabola, hyperbola, ellipse) which he applied to the construction of mean proportionals.⁴ He is called a friend of Plato's, and a pupil of Eudoxus, who also taught Aristotle.⁵ Proclus associates Menaechmus with the mathematician Amphinomus, and opposes him to Speusippus. If we take Stobaeus seriously, Menaechmus may even have been invited to teach geometry to the young Alexander the Great—which would make him a

¹ For a more detailed description of Philoponus' project in his commentary on the *Posterior Analytics* see my paper "Philoponus and the mathematization of logic", forthcoming. Here I shall confine myself to part of his treatment of conversion.

² For the following reconstruction I am indebted to MORROW (1970), BARNES (1976) (with the modification in BARNES (1981), 40 n. 34), BROWN (1976), BOWEN (1983), and SMITH (1986); where my reconstruction differs from theirs this is signalled in the footnotes. None of them considered the significance of this debate for the philosophy of late antiquity. Most commentators on the *Theaetetus* tend to play down the mathematical background of the notion of element in the *Theaetetus*, as well as the fact that mathematicians act as interlocutors. For a different assessment, see e.g. BENITEZ and GUIMARAES (1993).

³ The sources on Menaechmus were first collected by SCHMIDT (1884).

⁴ Procl., *In Tim.* II, 34, 2 Diehl (= fr. IX Schmidt).

⁵ Procl., *In Eucl.* 67, 8-12 (= fr. IV Schmidt).

colleague of Aristotle's at Pella.⁶ The fragments to be discussed below suggest that Menaechmus was not only a competent mathematician but also had a keen interest in methodological and epistemological questions.

Let us first turn to the famous discussion of Socrates' dream in Plato's *Theaetetus*, which needs little introduction. In the third part of the dialogue we find Socrates in discussion with Theaetetus, who is staged as a promising young mathematician with more interest in philosophical issues than his old teacher Theodorus. They have come to reject Theaetetus' proposal that 'true judgment' (*alêthês doxa*) as such is an acceptable definition of knowledge. Then Theaetetus suddenly remembers a third definition: knowledge is belief with an account (*logos*). Socrates takes up this suggestion and offers 'a dream in return for a dream' (201b8). In this dream he heard some people saying that the primary elements (*prôta* or *stoicheia*) out of which we and everything else are composed, are perceivable but have no account, and can only be named. It is the complexes that are both knowable and expressible; they are the objects of true judgment (201e-202b). Only he who can give and take an account (*logon didonai*) of a thing has knowledge.

Twice Socrates uses the model of syllables and letters (*stoicheia*) to emphasize the notion of priority or cognitive asymmetry. In the first instance it turns out that the process of learning to read and write shows that 'it is absolutely necessary that anyone who is ever to know a syllable must first get to know the letters' (203d). But this yields a circular definition of knowledge: 'true belief with an account of the elements' presupposes *knowledge* of the elements. Hence 'in admitting this, we shall find that our beautiful theory has taken to its heels and got clean away from us' (203d). In the second instance the experience of learning to read and write suggests the following:

T1 Then if the proper procedure is to take such elements and complexes as we ourselves have experience of, and make an inference (*tekmairesthai*) from them to the rest, we shall say that the elements are much more clearly known (*enargesteran*), and the knowledge of them is more decisive (*kuriôteran*) for the mastery of any branch of study than knowledge of the complex. And if anyone maintains that the complex is by nature knowable, and the element unknowable, we shall regard this as tomfoolery (*paizein*), whether it is intended to be or not.⁷

⁶ Stob., *Strom.* 2, 31 (= fr. I Schmidt). BARNES (1981), 76, is inclined to take Stobaeus seriously.

⁷ *Th.* 206b5-11 (tr. LEVETT-BURNYEAT, 1990).

If we use a single concept of knowledge governing both knowledge of simples and knowledge of complexes, cognitive asymmetry must run into difficulties. By means of an aristotelian inference from what is known to us to what is known generally (206b), Plato provides what we might call a general framework of 'analysis'.⁸ It accounts for all of the physical, lexical, grammatical and mathematical compounds that are mentioned or alluded to in the third part of the *Theaetetus*.⁹ For each domain the same problem arises: if knowledge rests on analysis into simples (*prôta* or *stoiceia*), the simples themselves cannot be known; if they cannot be known, nor can the compounds they make up. On the other hand, if the simples can be known, i.e. by analysis, they are not simples after all and they are no different in this respect from the compounds they are supposed to explain.¹⁰

Plato's emphasis on cognitive asymmetry and the rejection of unknowable elements point in the direction he (and Aristotle) were going to take. In the *Republic* Plato provides a picture of the hypothetical method in mathematics as too limited for his concerns.

T2 I think you know that students of geometry, calculation and the like hypothesize the odd and the even, the various figures, the three kinds of angles, and other things akin to these in each of their investigations, as if they knew them. They make these their hypotheses and don't think it necessary to give any account of them, either to themselves or to others, as if they were clear to everyone. And going from these first principles through the remaining steps, they arrive in full agreement at the point where they started their investigation.¹¹

Here Plato suggests that mathematicians take the existence of the main objects of their science for granted, without even asking for a further account. With respect to mathematical objects, this further account in terms of the Forms is exactly what, according to Plato, his science of dialectic has to offer.¹² However, Plato also hints that this shortcoming in mathe-

⁸ The term as such does not occur in Plato.

⁹ Cf. MORROW (1970), 328.

¹⁰ Cf. *Rep.* 533c3-5.

¹¹ *Rep.* 510c2-d2 (transl. Grube/Reeve, modified).

¹² *Rep.* 511b2-c1. For Plato too, there is at least one ultimate 'un-hypothetical' principle (511b6); the Good, and the Forms, are knowable and definable but not as composites: the principle can be 'touched' (511b6 ἀγγόμενος). A similar view of philosophical method is developed in *Phd.* 100-102: each hypothesis should not be taken as a true principle (ἀρχή) but scrutinized for a *logos* of its own by positing further hypotheses, so as to lead to what the *Phaedo* calls an 'acceptable' (ικανόν) starting-point (101d6-8).

mathematical method is not just an oversight on the part of the mathematicians but a feature of mathematical epistemology. For Plato the third line segment represents

T3 ... the kind of thing that, on the one hand, I said is intelligible, and on the other, is such that the soul is forced to use hypotheses in the investigation of it, not travelling up to a first principle, since it cannot reach beyond its hypotheses.¹³

In so far as the first principle is not (yet) part of the discussion, the soul is forced to use hypotheses. Indeed, Plato's own investigations in e.g. the *Phaedo* and the *Meno* explicitly adhere to this format of assuming hypotheses for the purpose of the argument, as an intermediate step towards further insight.¹⁴

It is not likely, though, that mathematicians would agree that *in principle* they cannot reach beyond at least some of their hypotheses, that their discipline *as such* is marred by a structural incapacity to explain its starting-points in any way. There is some evidence in Plato that the mathematicians of his day would not have agreed.

T4 So we must, it appears, inquire into the qualities of something the nature of which we do not yet know [*sc.* virtue]. However, please relax your rule a little bit for me and agree to investigate whether it is teachable or not by means of a hypothesis. I mean the way geometers often carry on their investigations. For example, if they are asked whether a specific area can be inscribed in the form of a triangle within a given circle, one of them might say: "I do not yet know whether that area has that property, but I think I have, as it were, a hypothesis that is of use for the problem, namely this: If that area is such that when one has applied it as a rectangle to the given straight line in the circle it is deficient by a figure similar to the very figure which is applied, then I think one alternative results, whereas another results if it is impossible for this to happen. So, by using this hypothesis, I am willing to tell you what results with regard to inscribing it in the circle—that is, whether it is impossible or not.

¹³ *Rep.* 511a. I take it that the third line segment represents, among others, mathematical knowledge.

¹⁴ *Phd.* 101d3-e3, 107b4-9; *Meno* 86d8-87b5, quoted below; cf. also *Rep.* 534b8-c5 with respect to the Good.

So let us speak about virtue also, since we do not know either what it is or what qualities it possesses, and let us investigate whether it is teachable or not by means of a hypothesis, and say this: ..."¹⁵

The mathematician that is 'quoted' here shows himself far more aware of the difficulties involved than Plato cares to admit elsewhere. While accepting an hypothesis for the sake of determining the possibility of the solution of a problem,¹⁶ he is conscious that he has no knowledge of the true state of affairs (yet). His choice to continue from a hypothesis is simply a practical one, and he is full of hopes of solving the problem satisfactorily in the end.

A further indication that the mathematicians were not as naïve or unfortunate as Plato sometimes makes them out, can be derived from Proclus' report on two views of 'element' distinguished by Menaechmus:¹⁷

T5 The term "element", however, can be used in two senses, as Menaechmus tells us.

For (1) what proves is called an "element" of what is proved by it; thus in Euclid the first theorem is an element of the second, and the fourth of the fifth. In this sense many propositions can be called elements of one another, when they can be established reciprocally. From the proposition that the exterior angles of a rectilinear figure are equal to four right angles we can prove the number of right angles to which the interior angles of the figure are equal, and vice versa. An element so regarded is a kind of lemma.

But in another sense (2) "element" means a simpler part into which a compound can be analyzed. In this sense not everything can be called an element of anything, but only the more primary members of an argument leading to a conclusion, as postulates are elements of theorems. This is the sense of "element" that determines the arrangement of the elements in Euclid's work, some of them being elements of plane geometry, and some elements of stereometry. This also is the meaning the work has in numerous compositions in arithmetic and astronomy entitled "elementary treatises" (*stoicheiōseis*).¹⁸

¹⁵ *Meno* 86d8-87b5.

¹⁶ Note that here 'hypothesis' does not refer to an existence claim as in T2.

¹⁷ In view of the mixed vocabulary in this text I agree with BARNES (1976), 286 that this cannot be a literal quotation from Menaechmus.

¹⁸ Proclus *In Eucl.* 72, 23-73, 14 (tr. Morrow). Cf. Menaechmus (= fr. V Schmidt).

The first sense of 'element' rules out the presence of absolute priority in the relation between an element and what it is the element of. The second sense acknowledges the priority which Plato chose to emphasize. Interestingly, Proclus claims that the *second* sense of element is meant in the title of Euclid's *Elements*, although he finds both the first and second senses of 'element' applied in the content of the work. Proclus suggests that Menaechmus left both definitions standing, without preferring one to the other. The assertion that the first sense exists, and is relevant to mathematical proof (as Proclus confirms), suggests that Menaechmus would have resisted Plato in this respect.

This cluster of problems surfaces with renewed clarity and vigour in Aristotle's *Posterior Analytics* I 3 which informs us about further stages of the debate between philosophers and mathematicians.¹⁹ Let us follow him and consider the options in more detail, and thus gather the pieces of information that Proclus and Philoponus will use in their reception of this debate.

Aristotle distinguishes three different views. The first party claims that no knowledge exists because principles cannot be known; the search for priors yields an infinite regress which renders knowledge of the primitives impossible—and Aristotle agrees.²⁰ An infinite number of priors cannot be surveyed; if so, nothing can be known. In order to escape from this infinite regress one must assume primitives. Hence we are left with the second horn of the dilemma, that such primitives will be unknowable because by definition they cannot be known, i.e. by demonstration.²¹ One can only *assume* primitives, thus forfeiting knowledge in the strict sense. This statement may reflect the mathematical practice of hypothesis as it was presented by Plato. However, it is unlikely that any mathematician who trusts his proofs would like to forfeit his claim to knowledge. Therefore a further option was proposed.

T6 The other party agrees about knowledge, which, they say, arises only through demonstration. But they argue that nothing prevents

¹⁹ For various proposals for identification of the positions to be discussed, see BARNES (1976), 282, nn 8-9.

²⁰ *An. post.* I 3, 72b7-15.

²¹ In contradistinction to Plato in the *Theaetetus*, Aristotle formulates knowledge in terms of demonstration (*ἀπόδειξις*). His own theory of demonstration can be regarded as a more successful answer to the question what kind of *logos* is needed to turn true belief into knowledge. At the same time Aristotle picks up the mathematical concern with demonstration.

there being demonstrations of everything; for it is possible for demonstrations to proceed in a circle or reciprocally.²²

This party accepts, with Aristotle, the second horn of the dilemma, and agrees that knowledge only arises through demonstration. But, they claim, primitives are knowable all the same, if one accepts the possibility that demonstrations proceed in a circle or reciprocally. Aristotle disagrees, and devotes the second half of chapter I 3 to refute this solution to the problem; as we shall see, he had already been concerned with this issue in the *Prior Analytics*.²³

Aristotle himself clearly prefers a third answer:

T7 We assert that not all understanding is demonstrative: rather, in the case of immediate items knowledge is indemonstrable. And it is clear that this must be so; for if you must know the items which are prior and from which the demonstration proceeds, and if things come to a stop at some point, then these immediates must be indemonstrable.

We argue in this way; and we also assert that there is not only understanding but also some principle of knowledge by which we get to know the definitions.²⁴

Aristotle takes the same approach as Plato took: demonstration is based on absolute priority which cannot be reversed. In the previous chapter *An. post.* I 2 he has just laid down this starting point in resounding clarity.²⁵ In T7 Aristotle clearly spells out the consequences of his own

²² *An. post.* I 3, 72b15-18 (tr. BARNES, 1976).

²³ Also the rhetoric of Aristotle's rebuttal suggests he is dealing with an important rival opinion. E.g., it is 'clear' (72b25-6: δῆλον; cf. b35) that circular proof is 'impossible' (72b25: ἀδύνατον, cf. b 27), and he considers it 'obvious' (73a18: φανερόν) that it is 'empty and impossible' (73a18-19 κενόν τε καὶ ἀδύνατον) to say that everything can be proved because there is reciprocal proof.

²⁴ *An. post.* I 3, 72b18-25.

²⁵ *An. post.* I 2, 71b19-22, 26-29, 31-33: "If to understand something is what we have posited it to be, then demonstrative understanding in particular must proceed from items which are true and primitive and immediate and more familiar than and prior to and explanatory of the conclusions. (...) They must proceed from items which are primitive and indemonstrable because otherwise you will not understand unless you possess a demonstration of these items (to understand something of which there is a demonstration non-incidentally is to possess a demonstration of it). (...) and prior, if they are explanatory and we already know them not only in the sense of grasping them but also of knowing that they are the case." Cf. *An. post.* I 23, 84b19-22.

view: he declares that knowledge of immediate items²⁶ exists despite the fact that they are indemonstrable. Therefore he has to add that there is a different principle of knowledge by which one might obtain insight in definitions, which are (among) the starting points of demonstrations.²⁷

Aristotle's strong requirement that a demonstration also *explains* the conclusion entails non-reversible causal priorities which demonstration is supposed to bring out. Proclus claims that Aristotle gave rise to the idea that geometers are not concerned with (all) causes, and attributes this view to the mathematician Amphinomus.²⁸ This passage is evidence that in the Academy at least one mathematician chose to deny that mathematics aims at discovering causes. This point of view would match a mathematical practice which denies absolute priority in proofs.

In the meantime, the only obstacle to the acceptance of Aristotle's view remains circular or reciprocal proof. First of all Aristotle makes clear that this type of proof clearly sins against the priority requirement:²⁹

T8 That it is impossible to demonstrate simpliciter in a circle is plain, if demonstrations must proceed from what is prior and more familiar. For it is impossible for the same thing at the same time to be both prior and posterior to something – except in different ways (i.e. one in relation to us and the other simpliciter), ways which induction makes familiar. But in that case, knowing simpliciter will not have been properly defined. Rather, it will be ambiguous. Or else the one demonstration is not a demonstration simpliciter in that it proceeds from what is more familiar to us.³⁰

²⁶ I.e. items which cannot be further explained by a middle term according to Aristotle's theory of demonstration.

²⁷ Such insight was already hinted at at *An. post.* I 2, 71b16-17, and discussed in *An. post.* II 19. For definitions as proper starting points of demonstration see *An. post.* I 10, 72a14-24.

²⁸ HEATH (1956), vol. 1, 150 n. 1 disagrees and refers to *An. post.* 71b9ff, 79a2ff, 85b 23ff for Aristotle's position; one might add *Metaph.* III 2, 996a22-36; XIII 3, 1078a33-b 6. For the origin of Proclus' claim in Stoic and later ancient philosophy, see now HARARI (2008). Procl., *In Eucl.* 202, 9-25 ascribes the position to Amphinomus, a contemporary of Menaechmus, and adds Geminus' reply: even though not all arguments in mathematics display the cause, they do in case the argument is universal and applies to all similar cases.

²⁹ BARNES (1976), 280-1 classifies this as *ignoratio elenchi* on Aristotle's part—even though there can hardly be ignorance involved; this paragraph is a statement of Aristotle's position in which he distinguishes himself from his opponents, no more no less.

³⁰ *An. post.* I 3, 72b25-30.

It is interesting that Aristotle feels the need to explain that his own distinction between prior to us and prior *simpliciter* does not open the door to circular proof. Perhaps his opponents in the debate launched such an argument against him?³¹ Aristotle's answer leaves nothing to be desired: proof from what is prior *simpliciter* is not equivalent to proof from what is more familiar to us, so that the two together cannot constitute circular or reciprocal *proof* in the strict sense of the term.³² As we shall see, Philoponus will approach this text differently.³³ The issue gains further significance from the fact that the Platonic method of division and collection has often been compared to mathematical analysis and synthesis.³⁴ In that Platonic context, too, a mathematician might argue that a circular argument is at stake, and one which is not open to the solution here provided by Aristotle.

In a next step, Aristotle shows circular proof to be futile by analyzing it in his own terms.³⁵ Aristotle first explains that reciprocal proof and circular proof (i.e. through more than two terms) amount to the same statement: 'if A is the case A is the case' which is no proof at all. Not without sarcasm Aristotle repeats that it is easy to prove everything in this way.

However, circular proof is not completely ruled out by Aristotle.³⁶ He acknowledges that properties may follow one another, or, in other words, can be counterpredicated. In those cases one property (or definitional phrase) can be used to demonstrate another.³⁷ This, Aristotle says, he has proven in his account of deduction (see *An. pr.* II 5).³⁸ In addition he claims to have proven that in the other figures either no deduction at all comes about or else none concerning the assumptions.³⁹ No doubt some of the conversions discussed in *An. pr.* II 5-7 are useful for reducing second and third figure syllogisms to the first figure. However, this remark from the *Posterior Analytics* shows that the exhaustive survey

³¹ BARNES (1976), 278 n. 2 points to the sceptical charge of circularity brought against proof by Sext. *Adv. Math.* 8, 347; cf. more generally Sext. *PH* 2, 163-166, 193-204. Moreover, BARNES (1976), 283-4 suggests an affinity with Plato's method of analysis-cum-synthesis (or collection) as an impulse for 'an Academic geometer' to frame his argument as circular proof. See further below pp. 226 and 228.

³² *An. post.* I 3, 72b30-32.

³³ See below pp. 235-236.

³⁴ So e.g. HINTIKKA and REMES (1974).

³⁵ *An. post.* I 3, 72b32-73a 20. Cf. *An. pr.* II 16, 65a5-9.

³⁶ For the suggestion that Aristotle would himself need circular argument to explain cyclical natural processes, see BARNES (1976), 290-2.

³⁷ For the principle that properties (*ἰδιώτα*) and definitions (*ὁρίσι*) are counterpredicated and thus convert, see *Top.* I 8, 103b7-19. If A (*definiendum*) is B (*definiens*), then if B then A.

³⁸ See *An. pr.* II 5 on first figure syllogisms.

³⁹ See *An. pr.* II 6-7.

Aristotle undertakes in *Prior Analytics* II 5-7 also served the purpose of specifying under which conditions circular proof is admissible, if at all. *Only* in the first figure, and *only* in cases where the terms are strictly convertible is there room for circular proof. In the *Posterior Analytics* the morale of the survey is drawn in the clearest possible terms: *because* the terms are convertible in such a limited number of cases, circular proof cannot be the solution to the problem of knowledge. The drive for completeness in the chapters on conversion is at least partly to be explained by the importance of the issue in this Academic debate. Not without reason, Robin Smith has suggested that Aristotle developed this part of the *Prior Analytics* in order to deal on a metalogical level with the problem described in the *Posterior Analytics* I 3: in this way the syllogistic establishes that every premiss regress terminates.⁴⁰

Aristotle shows himself well aware that within the realm of mathematics things may look differently:

T9 If it were impossible to prove truth from falsehood, it would be easy to make analyses; for then the propositions would convert from necessity. Let A be something that is the case; and if A is the case, then these things are the case (things which I know to be the case – call them B). From the latter, then, I shall prove that the former is the case. In mathematics conversion is more common because mathematicians assume nothing incidental—and in this too they differ from those who argue dialectically—but only definitions.⁴¹

Mathematicians were prone to use more definitions and less accidental predications among their premises than the average dialectician. Since the terms of a definitional predication convert, they would have the benefit of more opportunities for proper conversion, and hence encounter more opportunities

⁴⁰ SMITH (1986), 54, 57, 59, 63. Note that in *An. post.* I 19, 82a15-19 Aristotle observes that in the context of counterpredicables there is no first and last term: all order disappears. Further trouble for Aristotle is hidden in *An. post.* II 17, 99a16-23, 36-37, for which see BROWN (1976), 270 n. 24 and 264 respectively.

⁴¹ *An. post.* I 12, 78a6-13. In *An. pr.* II 2-4 Aristotle argued that although it is impossible to prove a falsehood from truths, it is possible to deduce a truth from falsehoods. From this it is clear that not all propositions in a valid deduction convert from necessity. Cf. SMITH (1989), 185 for the suggestion that these chapters argue against people assuming that a deduction could be discovered by a process of 'analysis', which, I take it, includes the mathematicians. The restrictions that Aristotle imposes on the truth conditions of the premises and the conclusion in *An. pr.* II 2-4 suggest precisely the kind of natural priority which those who wished to apply analysis throughout would have to deny.

for reciprocal or circular proof.⁴² Indeed, in the context of the method of analysis and synthesis which has to come full circle, all means of conversion would have to be sought out.⁴³ This serves to explain the mathematicians' confidence in circular proof as a means of solving the *Theaetetus* problem—a confidence that Aristotle is showing to be unfounded in these very chapters of the *Prior Analytics*. Yet, at the same time, Aristotle's admission that in a limited set of cases all conversions are allowed can be considered a weak spot in his own absolute proof structure.⁴⁴ It is this weak spot that Philoponus will exploit to turn the tables against Aristotle.

Aristotle's point of view is clearly reflected in his division of the meaning of 'element' (*stoicheion*) in *Metaph.*, which does not make room for the reciprocal meaning that Menaechmus discerned (see T5 above):

T10 The elements of geometrical proofs (*diagrammata*), and in general the elements of demonstrations (*apodeixeis*), have a similar character; for the primary demonstrations, each of which is implied in many demonstrations, are called 'elements' of demonstrations; and the primary deductions, which have three terms and proceed by means of one middle, are of this nature.⁴⁵

It is clear that the notion of 'element' in demonstrations is here closely tied to Aristotle's syllogistic with its characteristic notion of priority. Geometrical proof is not allowed to constitute a domain of its own, rather it is assimilated to the Aristotelian model. We can now see that this short passage is not a harmless 'dictionary' entry: it encapsulates Aristotle's position in the debate.

Finally, it comes as no surprise that Proclus confirms for us that Menaechmus, too, was interested in problems of conversion. Since Philoponus will use this text it deserves to be quoted in full:

T11 Conversion among geometers has two meanings. In the strict and primary sense it occurs when two theorems interchange their conclu-

⁴² Hence Aristotle's point in *An. post.* I 12, 78a10ff is not that mathematics is somehow special, nor that "Aristotle has observed that mathematical theorems often do 'convert'; but he has not got to grips with his observation", BARNES (1976), 154. The point is that Aristotle mentions mathematics because his discussion is part of the debate between philosophers and mathematicians in the Academy, with which he got to grips only too well.

⁴³ Cf. BROWN (1976), 269-273.

⁴⁴ For this weakness in Aristotle's theory see esp. SMITH (1986), 60.

⁴⁵ *Metaph.* IV 3, 1014a35-b 3.

sions and their hypotheses with each other, that is, when the conclusion of the first becomes the hypothesis of the second and the hypothesis of the first is adduced as the conclusion of the second. For example: "In an isosceles triangle the angles at the base are equal" (here the hypothesis is "isosceles triangle", and the equality of the angles at the base is the conclusion); and "Triangles having equal angles at the base are isosceles". The latter is precisely what the sixth theorem asserts, taking as hypothesis the equal angles at the base and as conclusion the equality of the sides that subtend those equal angles.

The other form of conversion involves only a certain interchange among the component parts. If, for example, a theorem is composite and arrives at a conclusion from several hypotheses, we take the conclusion and one hypothesis and reach a conclusion consisting of one or more of the other hypotheses. It is in this sense that the eighth theorem will be the converse of the fourth. The fourth states "When the sides and angles are equal, the bases that subtend them are equal"; the other "On equal bases equal sides contain equal angles." Of these components "on equal bases" was the conclusion of the former, while the positing of equal sides was one of its hypotheses.

Of these two forms of conversion the primary type is uniform and determinate, but the other is varied and can run to a great number of theorems; there is not a single converse, but many, because of the plurality of hypotheses in the composite theorem. Often, however, we make a single converse of a theorem whose hypothesis consists of two or more members, when they are not all determinate, but some indefinite.

But we must also note in this connection that many conversions are made fallaciously and are not true converses. For example, every hexagonal number is triangular, but it is not true that every triangular number is hexagonal. The reason is that the former character is of more general, the latter of more particular application; and one can be asserted of the other only as true in every instance. But propositions about attributes that a subject has primarily and essentially can be converted. These matters also have engaged the attention of the mathematicians in the circle of Menaechmus and Amphinomus.⁴⁶

It appears that Menaechmus and his fellow mathematician Amphinomus were engaged in the same investigations into conversion as Aristotle in the *Prior Analytics*. What is more, with Aristotle they acknowledged that

⁴⁶ Proclus *In Eucl.* 252, 5-254, 5 (tr. MORROW, 1970).

only propositions about primary and essential attributes can be converted properly.⁴⁷ It seems likely that Aristotle's sensitivity to the mathematicians' position can be attributed to discussions in the Academy in which Menaechmus also participated. But can we discern a similar sensitivity to the philosophers' position on the part of the mathematicians?

There is an important difference between the types of conversion that Proclus ascribes to geometry in T11, and Aristotle's definition of the conversion of premises and syllogisms in the *Prior* and *Posterior Analytics*.⁴⁸ In the first type of geometrical conversion the conclusion of one theorem becomes the hypothesis of the other and the hypothesis of the first is adduced as the conclusion of the second. Stated as such, no conversion of terms is intended. Although the wording of the example perhaps suggests otherwise, it should be read as the conversion of antecedent and conclusion, with 'hypothesis' as term for the antecedent.⁴⁹ The second type of geometrical conversion occurs within a single complex argument, and means no more than that the conclusion plus one or more of the antecedents can be used to prove another antecedent within the same argument. Again, there is no mention of conversion of any proposition as required by Aristotle.

Such conversions are precisely what we would expect from a mathematician like Menaechmus who was willing to adopt a meaning of 'element' that entails a full exchange of propositions between the roles of premise and conclusion (T5). But it is also what we would expect from a mathematician who has come to realize that convertibility of terms is not as ubiquitous as some of his colleagues might have thought. Such a mathematician might well emphasize a different aspect of mathematical practice, in addition to the proper conversion of primary and essential attributes. This is the conversion of premises (or antecedents) and conclusions between or within arguments, *without the troublesome conversion of terms*. If so, this mathematician could accept Aristotle's claim that conversion is a restricted phenomenon, and use it where applicable, but continue to hold that reciprocal or circular proof (i.e. without conversion of terms) may serve to solve the *Theaetetus* problem in other cases. Thus the versatility of mathematical practice ignores the strictures on conversions as prescribed by Aristotle's syllogistics. Despite Aristotle's direct and indirect attempts to reduce

⁴⁷ BARNES (1976), 287 rightly warns that the Aristotelian terminology of Proclus' report on Menaechmus at *In Eucl.* 25, 2 may well be due to its transmission from Menaechmus through Eudemus and Geminus to Proclus.

⁴⁸ Cf. also SMITH (1986), 60-61.

⁴⁹ Here I have to pass over the details of the Neoplatonic appropriation of Peripatetic and Stoic logic.

mathematical proof to his theory of demonstration, we can see that the mathematicians did not accept the straight-jacket of the *Analytics*.⁵⁰

Proclus and the mathematical turn

We are now in a better position to evaluate Proclus' reports on the debate. There can be no doubt that Proclus, as a Neoplatonist, stood by the priority requirement voiced by both Plato and Aristotle. Indeed, the author of the *Elements of Theology* pictured the entire universe as a stable structure built on ontological and (therefore) epistemological priorities. However, as the title of his metaphysical pièce de résistance suggests, Proclus regarded Euclid's *Elements* as the perfect model to present this structure. This is true only if Euclid's *Elements* embodies the demands for demonstrative priority as laid down by Aristotle.⁵¹ It is incompatible with the relative notion of an 'element' of proof such as Proclus reports Menaechmus to have proposed. In this respect it is noteworthy that Proclus feels himself obliged to acknowledge that the relative notion of 'element' is found to be applied in Euclid's *Elements* (see T5). The echoes of ancient mathematical practice in Euclid could not be erased, and the famous mathematician Menaechmus is called upon to vouch for this potentially problematic feature of Euclid's *Elements*. Nevertheless, Proclus emphasizes, as we have seen, that the title of Euclid's *Elements* (and, indeed the title of all kinds of *stoicheiôseis*, no doubt including his own *Elements of Theology* and *Physics*) preserves the non-relative notion of 'element' defended by the philosophers—which he *also* finds embodied in Euclid's *Elements*. From the reconstruction above it is clear that Euclid, as well as Menaechmus, may have had good reason to keep *both* the relative notion of 'element' (for proofs *without* conversion of terms) *and* the non-relative notion of 'element' (for proofs *with* conversion of terms) in order to keep up circular proof. Proclus uses this careful result of the debate to picture Euclid as a true mathematician (with Menaechmus) *and* a true philosopher (with Plato and Aristotle).

If we accept this as a model of Proclus' approach to the discussions handed down to him we can begin to see why he also presents us with a harmonic picture in other reports of controversy between philosophers and mathematicians. In another passage that mentions Menaechmus, we are informed about a discussion in which Speusippus and Amphinomus

⁵⁰ On the troublesome relation between Aristotle's syllogistic and mathematical proof see e.g. MUELLER (1974), with NOVAK (1979), MCKIRAHAN (1992), ch. XI.

⁵¹ This is now confirmed by HARARI (2008) who shows how Proclus introduces causality in mathematical demonstration.

sided against Menaechmus.⁵² The first party claimed that all propositions should be called ‘theorems’ rather than ‘problems’. For the objects of the theoretical sciences, viz. eternal things, cannot be brought into being or made as if they did not exist previously as the term ‘problem’ might suggest. In such cases mathematical ‘construction’ should be regarded as understanding mathematical objects by taking them *as if* they were in the process of coming to be (*ou poiêtikôs alla gnôstikôs*). Menaechmus opposed them by claiming that all inquiries are problems, but differentiated between problems that seek to provide something sought for, and problems which determine the qualities or relations of a determinate object.

Proclus decides the issue simply by claiming that both parties are right.⁵³ Speusippus is right because mechanics is concerned with perceptible objects, whereas mathematics is not. Menaechmus is right because the discovery of theorems has recourse to matter, that is, intelligible matter. This matter is shaped by the movement of thought in projecting its own ideas in constructions, sectionings, superpositions, etc. In other words, by invoking his own metaphysics and psychology of mathematics Proclus gives both parties their due.⁵⁴ Mathematics is not concerned with mechanical production and perceptible matter (emphasized by ‘theorem’), but still there is some kind of production at stake, as well as some kind of matter, viz. intelligible matter (which supports ‘problem’). Each party recognized true characteristics of (the philosophy of) mathematics, and thus the ancient quarrel evaporates in the light of Neoplatonism.⁵⁵

These examples may serve as further indications that Proclus’ philosophical commentary on Euclid was part of his implementation of what one might call ‘the mathematical turn’ in Neoplatonism. In the early centuries AD the paradigmatic role of mathematics in Platonic philosophy and its Pythagorean ancestry were joined with the paradigmatic role of mathematics in Aristotle’s *Analytics* in order to construct a Pythagorean descendance of all parts of philosophy. In recent scholarship the precise steps of this development have been traced, not least the crucial role of Iamblichus in the establishment of the mathematical approach to phi-

⁵² This discussion is the main topic of BOWEN (1983).

⁵³ Procl., *In Eucl.* 78, 13-79, 2.

⁵⁴ On Proclus’ theory of the projection of the innate *logoi* of mathematical figures onto the imagination see MUELLER (1990), and the texts from Syrianus and Proclus in SORABJI (2004), vol. 1, 76-79.

⁵⁵ It is impossible to ascertain whether it is Proclus or one of his sources that first sought to harmonize philosophy and mathematics in this way. Here we are dealing with Proclus’ presentation.

losophy.⁵⁶ This mathematization was not limited to Pythagorean designs of metaphysics, physics, ethics, and politics as in Iamblichus’ *On Pythagoreanism*. It also gave Iamblichus’ work on Aristotle and the Peripatetic tradition a distinctive approach, which has been studied for instance in relation to the *Categories*⁵⁷ and *De anima*.⁵⁸ In Syrianus and Proclus the mathematization of physics and metaphysics was implemented in increasing detail. Proclus’ *Timaeus* commentary is a defense of the scientific character of Plato’s physics (or *phusiologia*) by showing its affinity to both Euclidean and Aristotelian methodology.⁵⁹ Less attention has been paid to developments in Neoplatonic logic, method, and epistemology,⁶⁰ let alone to the late ancient commentaries on the *Prior* and *Posterior Analytics* which remain largely unexplored today.⁶¹ The final section of this paper constitutes some tentative steps towards a further elucidation of this part of the philosophical tradition.

Philoponus and the *Analytics*

The previous sections set the stage for Philoponus’ treatment of Aristotle’s *Analytics* in his commentaries on the *Prior* and *Posterior Analytics*, which derive from Ammonius’ lectures.⁶² These commentaries allow us a unique view into the late ancient understanding of Aristotelian logic. As I mentioned at the outset, Philoponus has been considered a poor guide to logic precisely on the basis of his commentary on the *Prior Analytics*.⁶³ Here I

⁵⁶ See e.g. O’MEARA (1989), CENTRONE (2000), and BECHTLE (2006).

⁵⁷ Cf. DILLON (1997).

⁵⁸ Cf. e.g. STEEL (1978), FINAMORE and DILLON (2002).

⁵⁹ See now MARTIJN (2010) for an illuminating and exhaustive treatment of this issue; for Proclus’ relation to Euclid cf. MUELLER (1987), SCHMITZ (1997), HARARI (2006) and (2008).

⁶⁰ Notable exceptions are LEE (1984), SCHRENK (1994), BECHTLE (2000) and (2002), both reprinted in (2006), and HARARI (2008).

⁶¹ This state of affairs is changing after the publication of the translation of Alexander of Aphrodisias *On Aristotle Prior Analytics* and Philoponus *On Aristotle Posterior Analytics*, in Duckworth’s *Ancient Commentators on Aristotle* series. DETEL (1993) contains useful summaries of late ancient comments but makes no attempt at an appraisal of their more general philosophical outlook.

⁶² In our manuscripts the title of Philop., *In An. pr.* (CAG 13.2) speaks of ΣΧΟΛΙΚΑΙ ΑΠΟΣΗΜΕΙΩΣΕΙΣ ΕΚ ΤΩΝ ΣΥΝΟΨΕΩΝ ΑΜΜΩΝΙΟΥ ΤΟΥ ΕΡΜΕΙΟΥ to which the title of Philop., *In An. post.* (CAG 13.3) adds ΜΕΤΑ ΤΙΝΩΝ ΔΙΔΩΝ ΕΠΙΣΤΑΣΕΩΝ. This might suggest that Philoponus allowed himself a bit more freedom in the latter report, of which only his work on the first book survives; the commentary on the second book in our edition appears to be a more concise report of the lectures of Ammonius by a contemporary author. We also possess another *reportatio* of Ammonius lectures on the *Prior Analytics* I, known under the name of Ammonius himself (CAG 4.6).

⁶³ See LEE (1984) discussed below.

do not want to rescue him from any logical mistakes he may have made, or to defend his interpretation of Aristotle's logical treatises. I only wish to suggest an explanation for some of his moves.

In line with the general development sketched above, mathematics also exercised its influence on more detailed Neoplatonic interpretations of logic, whether from Stoic or Peripatetic provenance. There is every reason to expect that this tendency would also be noticeable in Alexandria, where Philoponus worked. Ammonius, his teacher, was a pupil of Proclus in Athens, and Ammonius' successor as head of the school of Alexandria, the mathematician Eutocius, was taught mathematics by Marinus, another pupil of Proclus. Perhaps it is not irrelevant to add that we derive a substantial part of our knowledge of Menaechmus' innovations in mathematics from Eutocius' commentary on Archimedes' *On the sphere and cylinder*.⁶⁴ Ammonius lectured on Nicomachus' *Introductio arithmetica*, as we know from the reports of his teaching by the hands of Asclepius and Philoponus. Philoponus himself wrote a treatise on the astrolabe. In Philoponus' report of Ammonius' teaching on the *Prior Analytics*, Iamblichus is named once as being among "the more exact interpreters".⁶⁵ We may surmise that Plato, Aristotle, Proclus, and Menaechmus were all well-known in Alexandria, and that the mathematical turn had its influence on the Alexandrian curriculum as well as the position of mathematical studies.

On reading Philoponus' commentary on *Posterior Analytics* I, we find that Philoponus inserts numerous mathematical illustrations and elucidations in the commentary, and sometimes even full-blown mathematical arguments, often taken or adapted from Euclid. This may count as a first indication that a mathematical model is gaining ground over the Aristotelian framework.

Here I would like to single out for more detailed examination the discussion which prompted T.S. Lee to write his most devastating criticisms of Philoponus, viz. the latter's commentary on Aristotle's chapters on conversion in the *Prior Analytics*.⁶⁶ Lee notes that in contrast to Alexander and Boethius, both Ammonius and Philoponus discuss conversion quite elaborately and systematically.⁶⁷ This might suggest an enhanced

⁶⁴ Eutoc. *In Archim. sph. cyl.* 78, 13-80, 24 (= fr. XI Schmidt who includes 82, 1-84, 7 as Menaechmian); 90, 4-11 (=fr. X Schmidt); 96, 16-19 (from a verse by Eratosthenes, cf. fr. III Schmidt with Procl., *In Tim.* II, 34, 2 Diehl, *in Eucl.* 111, 20ff).

⁶⁵ Philop., *In An. pr.* 26, 4-6; cf. *In Cat.* 9, 12.

⁶⁶ See LEE (1984), 79-94. Cf. MCKIRAHAN (2008), 3-5 for more recent criticism of Philoponus' (lack of) understanding of mathematics and logic.

⁶⁷ LEE (1984), 79.

interest in the topic, as a result from their enhanced interest in mathematics for which conversion was important, as we have seen. Lee expresses surprise at Philoponus' explanation of the conversion of syllogisms at *In An. pr.* I 2, 40, 15ff. and tries to understand it in logical terms, without success. He refrains from attributing a plain logical mistake to Philoponus only for the formal reason that the passage constitutes a stipulation of meaning (*Begriffsbestimmung*).⁶⁸ He fails to see, though, what purpose this stipulation might have, especially since Philoponus does not seem to use this type of conversion later on in his commentary, not even in the context of *An. pr.* II 8-10 which treats of the conversion of syllogisms. Because Philoponus' explanation does not have its counterpart in Ammonius' commentary, he assumes incompetence (*unzulängliche Kenntnisse*), sloppiness (*Nachlässigkeit*) together with characteristic long-windedness as explanations. Finally, Lee suspects that *An. pr.* II was not often studied in Philoponus' days for lack of interest in formal logic, and continues to deal with the commentators' far more elaborate distinctions concerning the conversion of premises.⁶⁹

Let us have a closer look at Philoponus' explanation of the conversion of syllogisms:

T12 [Aristotle] taught about conversion of syllogisms in the second book of the [*Prior Analytics*], I mean the one after this. For one has to know first what a syllogism is so as to learn how it converts. They say that syllogisms convert when we take the conclusion, add the major premise and establish the minor. For instance, there is a syllogism saying: "the soul is self-moving, what is self-moving moves eternally, what moves eternally is immortal, so the soul is immortal." So we have the conclusion that the soul is immortal. We take it and turn it into the minor premiss; then we take the remaining premises and construct the minor premiss as conclusion, as follows: the soul is immortal, what is immortal moves eternally, what moves eternally is self-moving, so the soul is self-moving. It is also possible to combine the minor premiss with the conclusion and make the major premiss in the previous syllogism the conclusion. For instance, the soul is immortal, what is immortal is self-moving, what is self-moving moves eternally, so the soul moves eternally.⁷⁰

⁶⁸ LEE (1984), 81-83.

⁶⁹ LEE (1984), 83-84.

⁷⁰ Philop., *In An. pr.* 40, 14-27.

Lee notices that the terms in the example “accidentally have the same extension” (*Begriffsumfang*), and supposes that the characteristics of the example led Philoponus astray.⁷¹ Philoponus should have phrased the example in terms of identity relations, which would have made clear that each chain *implies* the others. Lee rightly objects that such an implication, however, does not constitute a syllogistic inference of the kind that Aristotle had in mind.⁷²

Although this is quite right, it ceases to be a problem when the example, including the convertibility of all terms involved, is not regarded as incidental to Philoponus’ aims. After our examination of Proclus’ definition of geometrical conversion it will be easy to recognize here the second type of geometrical conversion (T11): “The other form of conversion involves only a certain interchange among the component parts. If, for example, a theorem is composite and arrives at a conclusion from several hypotheses, we take the conclusion and one hypothesis and reach a conclusion consisting of one or more of the other hypotheses.” The two possible conversions, which Philoponus distinguishes by referring to the ‘major’ and the ‘minor’ premise, confirm Proclus’ statement that “there is not a single converse, but many, because of the plurality of hypotheses in the composite theorem.” A reference to this type of conversion in a commentary on the *Prior Analytics* makes sense if one assumes that the author aims at showing in every possible way that the *Analytics* shares common ground with mathematical method and logic.

In this light it is significant that the example Philoponus uses is not mathematical in character, nor of Aristotelian provenance, but a reflection of the argument in Plato’s *Phaedrus* for the immortality of the soul.⁷³ As it happens, Proclus referred to this argument in similar terms in his commentary on the *Parmenides*, where it serves as an example of Parmenidean dialectic.⁷⁴ Philoponus is quite right to spot the potentialities of this prime Platonic argument as an illustration of logic. It allows geometrical conversion, and at the same it meets Aristotle’s requirements for the conversion of terms, since all its terms are co-extensive. Thus the argument qualifies for the only type of circular proof that Aristotle was willing to allow, and the famous Platonic argument for the immortality of the soul is accounted for within the framework of Aristotle’s logic. Be-

cause Philoponus invokes a definition of geometrical conversion in explaining a metaphysical argument, it is a perfect example of the ‘mathematical turn’, seeking to extend mathematical logic beyond mathematics. Instead of incompetence, Philoponus shows Platonic ingenuity, and testifies to the significance of Neoplatonic logic for the understanding of such matters as the immortality of the soul.⁷⁵

From this text alone one might begin to suspect that circular proof and the ubiquity of conversion may have appeared less inappropriate to Philoponus than to Aristotle. Did this have an effect on Philoponus’ commentary on *An. post.* I 3, where Aristotle rejected circular proof by asserting priority in demonstration and by reducing circular proof to absurdity?

Here it must suffice to single out two modifications of Aristotle’s argument that may confirm our suspicion. We have seen (T8) that at *An. post.* I 3, 72b25–28 Aristotle stipulates that demonstration must be based on things that are prior and better known. He then explains that his own distinction between prior to us and prior to nature cannot be used to create circular proof, as long as the term ‘proof’ is used in its strict sense. Philoponus agrees with the emphasis on priority, and he even seems to regard the aim of ‘establishing effects from their causes’ as the hallmark of demonstration. He explains:

T13 Now, [Aristotle] says, perhaps we did not do well to give the definition of demonstration saying that it is based on things that are primary in nature, since in fact there are two ways of demonstration, one based on things prior in nature and one based on things prior to us, as we said that the spherical [shape] of the moon is established through things that are primary to us. But this, he says, is false, and our definition was given well. For establishing prior things through posterior things is not always necessary, except in cases where the posterior things are irrefutable signs, such as ‘since there is ash, fire was once here’, or ‘since the moon is illuminated in this way, it is spherical’. However, if from the fact that a woman is pale it is established that she has given birth, since the indicator is refutable, such [an argument] would not be called a demonstration in any way. And in the case of irrefutable signs, we will not say that such [a demonstration] is demonstration in the strict sense, but that this whole thing is a demonstration

⁷¹ LEE (1984), 82.

⁷² LEE (1984), 83.

⁷³ *Phdr.* 245c5–246a2.

⁷⁴ Procl., *In Parm.* 1014, 15ff.

⁷⁵ The argument returns, in whole or in part, in *In An. post.* 6, 19–23; 7, 18–25; 24, 7–12; 28, 12–17; 30, 16–7, 21–4; 127, 33–128, 4; 143, 14ff.

from a sign, since it is necessary to establish effects from their causes, which is a property of demonstration in the strict sense, and not causes from their effects.⁷⁶

We have seen that Aristotle does not consider a possible mistake, but clarifies that his distinction between prior to us and prior by nature cannot be used to construct circular proof. Instead, before commenting on Aristotle's statement proper (72b28-9), Philoponus here inserts his remarks on sign inferences, which he had mentioned earlier.⁷⁷ Such inferences from effect to cause are necessary only when the signs are irrefutable; only inferences from irrefutable signs suggest necessary proof, and indeed only they were deemed worthy of the name 'demonstration'. One aspect of his earlier mention of sign inference merits quotation:

T14 For demonstration in the strict sense, as I said, is one that confirms things that are secondary on the basis of things that are prior, *when being primary and known in nature, and being better known to us coincide*. But when this does not obtain, but we are compelled to confirm things that are prior on the basis of things that are posterior, this kind of proof is called [a proof] from a sign, and because of the irrefutability of signs, it has been deemed worthy of the name 'demonstration'. (my italics)

Philoponus clearly suggests that before we know the priors we may well be compelled to use sign inferences. It becomes clear that the distinction prior by nature/prior to us is far more important in Philoponus' discussion than it was for Aristotle. It is no longer a possibly misleading distinction, but 'prior to us' is believed to contain a clear reference to signs, and to the awareness that inference from signs, especially from irrefutable signs, may well have to precede demonstration. Is it a coincidence that Plato's vocabulary in the *Theaetetus* passage quoted above (T1) also contains the verb *tekmairesthai* which in late Antiquity refers to necessary signs?

We know that for the commentators of late antiquity sign inference and

⁷⁶ Philop., *In An. post* I 3, 72b25, 49, 5-14 (transl. MCKIRAHAN, 1992).

⁷⁷ See e.g. 31, 17-32, 7. For sign inference in Philoponus and Simplicius see MORRISON (1997), with DE HAAS (1999); texts in SORABJI (2004), vol. 3, 265-268. Because the textual basis for sign inference in Aristotle is slim (cf. *Rhet.* 1357b 3-5, 1402b12-21, *An. pr.* 70a6-9, b1-6) it is likely that the importance of sign inference in Hellenistic philosophy has prepared the way for its popularity in Philoponus and Simplicius; cf. MANETTI (1993).

induction constituted modes of analysis.⁷⁸ If we pursue this line of argument, the pair of sign inference and demonstration we have encountered in Philoponus' commentary begins to take the shape of circular proof as constituted by the Platonic pair of analysis and synthesis—equally necessary, and equally reliable. Hence, Philoponus can only side with Aristotle in *An. post* I 3 by having recourse to a particular definition of proof: if 'proof' properly speaking is defined as establishing effects from causes, sign inference is not proper proof (end of T14).

An even more obvious correction of Aristotle's meaning is to be found further down in the commentary on *An. post* I 3, where Aristotle referred to his treatment of conversion in *Prior Analytics* I 3, 73a14-16. Philoponus seizes the occasion to insert his own view on the matter:

T15 The Philosopher says that nothing is inferred by circular proof in the figure because everything inferred in the third figure is particular, and supposing that the conclusion is particular, if we want to prove one of the premises which is universal, it is not possible by a circular proof (*An. pr.* II 6, 58b33-II 7, 59a3). For if we assume the conclusion, which is particular, and the particular premise, we will infer the remaining one in this way [i.e. as a particular].

But I say that just as in the examples I gave in the first figure⁷⁹—*because he takes terms that are coextensive*, even though a universal affirmative [premise] does not convert with itself, likewise, as I said, *because the terms are coextensive* Aristotle granted that the affirmative [premise] converts with itself—it is not absurd that in the third figure too, *when the terms are coextensive*, it is possible to infer the conclusion as universal and when it has been inferred as universal to demonstrate the premises too by circular proof.

For example 'man [is predicated] of everything that is capable of laughing, that which is receptive of intelligence and knowledge [is predicated] of everything capable of laughing, therefore man [is predicated] of everything that is receptive of intelligence and knowledge'. I prove the minor premise by analysis into the first figure, assuming the conclusion and the major [premise]. But it is not possible to prove the major

⁷⁸ For analysis see SCHRENK (1994); texts in SORABJI (2004), vol. 3, 268-271. For a similar acknowledgement from Proclus cf. *In Eucl.* 206, 12-26, discussed in HARARI (2006), 384-385.

⁷⁹ Cf. Philop., *In An. post.* I 3, 72b 32-35, 51, 21-52, 14, where he also used the terms 'man', 'capable of laughing', and 'receptive of intelligence and knowledge'.

[premise] either by reduction to the first figure or through the third [figure] itself. (my italics)⁸⁰

We have seen how Aristotle's chapter was geared towards minimizing the occurrence of circular proof. In this way he could show it was too rare to be a significant solution to the problem of the knowledge of elements. Philoponus claims that the example he himself (!) used in explaining conversion in the first figure can be stretched to apply to third figure syllogisms as well. He argues that if conversion of a first figure syllogism is allowed because the terms are coextensive, conversion of a third figure syllogism may also yield reliable results, if the same condition is met that the terms are coextensive.

First Philoponus lays out a (fallacious) third figure syllogism (a) in which he takes the conclusion to be established universally, precisely because the terms involved are coextensive. The terms 'capable of laughing' and 'receptive of intelligence' count as stock examples of a property and a differentia of 'man', and are thus convertible with it and with each other.

(a) Major	Man – capable of laughing
Minor	Receptive of intelligence – capable of laughing
Conclusion	Man – receptive of intelligence

He then claims he can prove the minor by reduction to the first figure, which one might reconstruct as in (b).⁸¹

(b) Former conclusion converted	Receptive of intelligence – man
Former major	Man – capable of laughing
Former minor	Receptive of intelligence – capable of laughing

Nevertheless Philoponus agrees with Aristotle in denying that there is no proof of the major by means of reduction to the first figure. This is true if one adheres to Aristotle's method for such a reduction, as applied in (b), which entails the conversion of the conclusion or one of the premises.⁸²

⁸⁰ Philop., *In An. post.* I 3, 73a15, 56, 32-57, 13 (tr. MCKIRAHAN, 1992).

⁸¹ Cf. MCKIRAHAN (2008), 131 n. 304.

⁸² MCKIRAHAN (2008), 131 n. 305 rightly notes that Philoponus should have allowed a similar proof for the major. However, by accident he refers to the *former* proof as the proof of the *major* and repeats the syllogism he also gave in n. 304, see previous note. The required proof would be: 'Man is predicated of receptive of intelligence and knowledge (conclusion), receptive of intelligence and knowledge is predicated of everything capable of laughing (minor premise), therefore man is predicated of everything capable of laughing (major premise).' This, however, would keep all propositions intact without the conversion of terms that Aristotle's reduction to the first figure requires. Cf. SMITH (1986), 60-61.

Philoponus also denies that the major can be proved through the third figure itself. This is also true, if such a proof would have to result from reshuffling the propositions of the third figure syllogism (a) as they are, i.e. without converting any of them.⁸³

At this point it is clear that Philoponus allows himself extraordinary freedom with Aristotle's syllogistic figures on the basis of the assumption that all terms of a syllogism convert. However, he seems anxious not to deviate too far from the text he is commenting on, and tries to give the impression that he agrees with part of Aristotle's restrictions on circular proof.

However, from T12 above it has become clear that Philoponus is prepared to interpret Aristotle's conversion along the lines of Proclus' geometrical conversion. What is more, the example of man's intelligence in T15 has the same characteristics as the argument about the immortality of the soul in T12: all terms convert with each other, so that each one can be substituted for the other. In these contexts any sense of natural order disappears, and the syllogistic figures are reduced to nothing but formal configurations.⁸⁴ Also the use of 'major' and 'minor' loses its significance, and seems to amount to nothing more than '1st premise' and '2nd premise'.⁸⁵ In other words, the arguments assume the shape of geometrical arguments in so far as premises and terms are elements without natural priority which receive their place in the argument for practical reasons only.

The more Aristotle's logic complies with *geometrical* conversion, the more it confirms the paradigmatic role of *mathematics* for all of philosophy. As a result, the manoeuvres we have identified in Philoponus are quite similar to the manoeuvres Menaechmus may have made to escape

⁸³ However, on Philoponus' understanding of Aristotle's logic one could construe these proofs as follows. The first figure proof, albeit *without converting* the terms of the conclusion, would be like (c):

(c) Former conclusion	Man – receptive of intelligence
Former minor	Receptive of intelligence – capable of laughing
Former major	Man – capable of laughing

Given the conditions under which Philoponus operates, it is also possible to construct a (fallacious) third figure syllogism to prove the major, as in (d). This, however, requires the conversion of the minor.

(d) Former conclusion	Man – receptive of intelligence
Former minor converted	Capable of laughing – receptive of intelligence
Former major	Man – capable of laughing

⁸⁴ For the significance of this result for our understanding of the development of 'traditional logic' in later centuries, see LEE (1984), 116-137.

⁸⁵ More research is needed to explain the emergence of these terms and their use by Philoponus.

from Plato's priority and Aristotle's conversion rules. Aristotle's weak spot, the allowance of conversion when all terms are counterpredicable, is exploited to the utmost.

Let it be clear that I do not recommend Philoponus to acquire insight into Aristotelian logic. But it is fascinating to see how these aspects of his interpretation of the *Analytics* serve to make Aristotelian logic match the mathematical model adopted by Platonism. As a result, Philoponus' comments on the Aristotelian sources of the debate between Menaechmus and the philosophers can be read as a re-enactment, and an extension of, the part played by Menaechmus.

Conclusions

A remarkable debate took place in the Academy. Plato and Aristotle, despite their admiration for mathematics, found themselves opposed by Menaechmus, despite his keen interest in epistemology and logic. The stakes were high: is it possible to know the elements that render knowledge possible? It turned out that in all likelihood this debate influenced the presentation of mathematical argument in Plato and Aristotle.

It is striking that all testimony concerning Menaechmus has been preserved in late ancient texts only. A closer look at the context of this testimony in Proclus' commentary on the first book of Euclid's *Elements* suggested that Proclus' interest in the debate is connected to the so-called 'mathematical turn' initiated by Iamblichus. This 'mathematical turn' involved a thorough re-interpretation of Platonic and Aristotelian philosophical texts in the light of their alleged Pythagorean roots. The importance of Euclid's *Elements* for Proclus' own philosophy is itself an example of the increasingly intimate relationship between philosophy and mathematics. This background explains why Proclus made attempts at reconciling the parties in the Academic debate.

Finally, we saw how Philoponus' commentary on Aristotle's *Analytics* also displays the mathematical turn, this time in the area of Aristotle's epistemology and logic. Philoponus adopts one of Proclus' definitions of geometrical conversion to understand Aristotle's discussion of syllogistic conversion. In order to corroborate this manoeuvre, he introduced a number of well-chosen examples so as to extend the domain of valid conversion and circular proof far beyond mathematics and the boundaries Aristotle meant to set for them. Philoponus' use of the examples of the immortality of the soul and the intelligence of mankind in this context secured these maxims of Neoplatonism in Peripatetic logic thus conceived.