Tangent fermions: massless fermions on a lattice

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Summary

Paul Dirac came up with the equation that describes the motion of a relativistic particle in vacuum. At low energies, the Dirac equation simplifies to the Schrödinger equation because the relativistic effects fade away. Since condensed matter physics studies electrons at very low energies, the Schrödinger equation is adequate for it most of the time. However, there is a case in which this simplification breaks down. If the particle is massless, the Dirac equation is qualitatively different from the Schrödinger equation no matter how low the energy is. In other words, the massless Dirac equation is still meaningful, while there is no such thing as the "massless Schrödinger equation". In some condensed matter systems, such as the surface of a 3D topological insulator, the electrons are effectively massless and we must necessarily use the massless Dirac equation to describe them.

A very convenient way to numerically solve these equations is to discretise them. By this we mean to lay a lattice over space — and possibly also time — and only allow the particle to exist on the points of this lattice. When we do this, differential operators become finite difference ones and this allows us to transform our differential equation into an algebraic one that can be readily solved by a computer. If one does this carefully, the solution of the discrete version approximates the continuous one if the lattice is fine enough.

Unlike the Schrödinger equation, the Dirac equation cannot be trivially discretised. The Nielsen-Ninomiya theorem proves that if we try to do it naively, extra unphysical massless fermion species appear, giving rise to a

number of undesired artefacts. This is known as *fermion doubling*, and the main focus of this thesis is to tackle this problem via the discretisation method of *tangent fermions*.

Chapters 2, 3 and 4 are devoted to developing various aspects of this method. In chapter 2, we introduce a way to use it to solve the Dirac equation for massless fermions in a space-time lattice. We show that this method is unique in that it avoids fermion doubling and preserves the topological protection of the Dirac cone.

In chapter 3, this approach is used to simulate the dynamics of a massless electron that moves towards a potential barrier. Theory predicts that massless particles cannot be stopped by the barrier, this phenomenon is known as Klein tunneling. We contrast our method with others and show that it reproduces the effect with excellent accuracy.

In chapter 4, we extend the tangent fermions method to account for the effect of magnetic fields on massless fermions. We show how our approach prevents the broadening of the zeroth Landau level in presence of magnetic disorder, an artefact that otherwise arises due to fermion doubling.

Chapters 5 and 6 are not directly related to the method of tangent fermions but still describe processes that arise in materials with a Dirac-like dispersion relation. In chapter 5, we study the effect a non-zero net supercurrent parallel to the edges of a topological superconductor. We find that the supercurrent can induce a “chirality inversion” of the Majorana edge modes that exist in this system.

In the last chapter, we numerically simulate the injection of “edge-vortices” into the edges of a topological superconductor. These are a type of quasiparticles that can theoretically be used to realise a fault tolerant quantum computer.