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Tangent fermions: massless fermions on a lattice

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Tangent fermions: massless fermions on a lattice

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Cover:

Two tulips, one faceted and one fully rendered, represent the concepts of discreteness and continuity. Their stems are interlaced and wrapped in a ribbon, forming a Möbius strip in reference to non-Abelian braiding and topology. Several key formulas accompany the lines of the painting.

By Yevheniia Cheipesh.

To my family

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